Semi-Markov PEPA:

Compositional Modelling and Analysis with General Distributions

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- i.e. allowing the use of general distributions in PEPA models

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- i.e. allowing the *limited* use of general distributions in PEPA models

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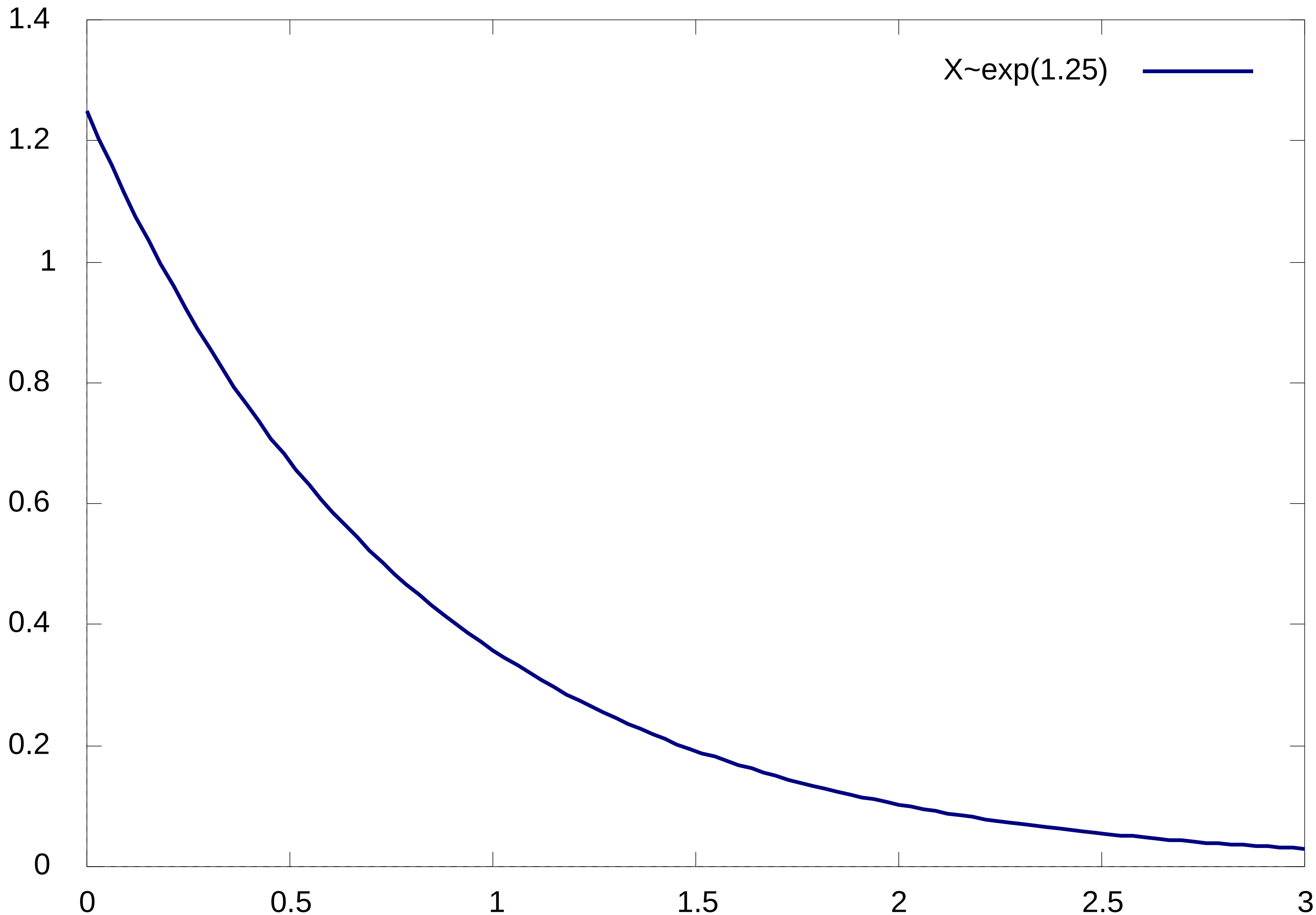
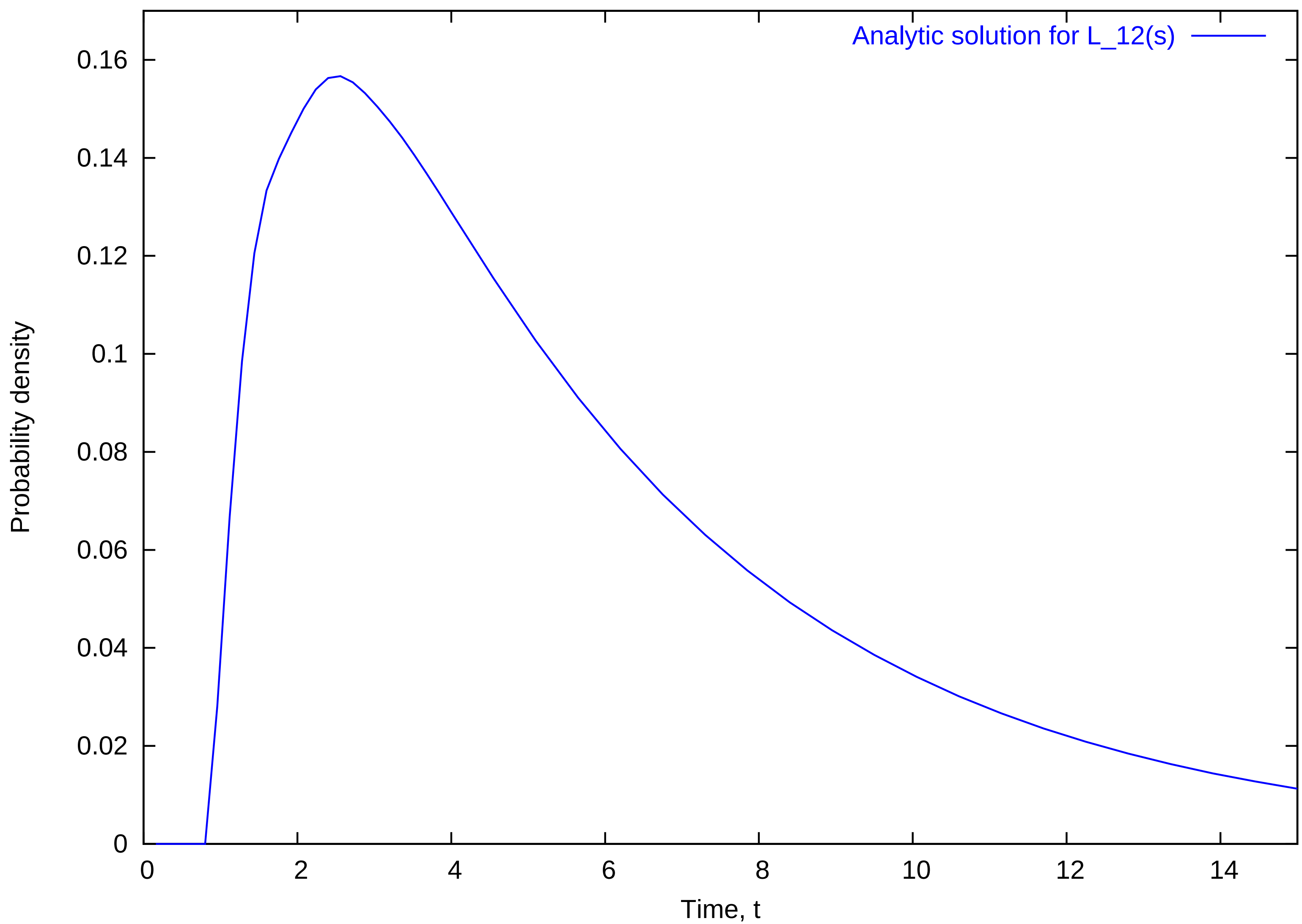
- In the beginning there was PEPA...!
- PEPA is an entirely Markovian process modelling formalism
- Users would like to use actions which take other types of distribution...
- ...not just the exponential distribution
- *We have some cool tools that can analyse semi-Markov Processes with $\sim$20 million states*

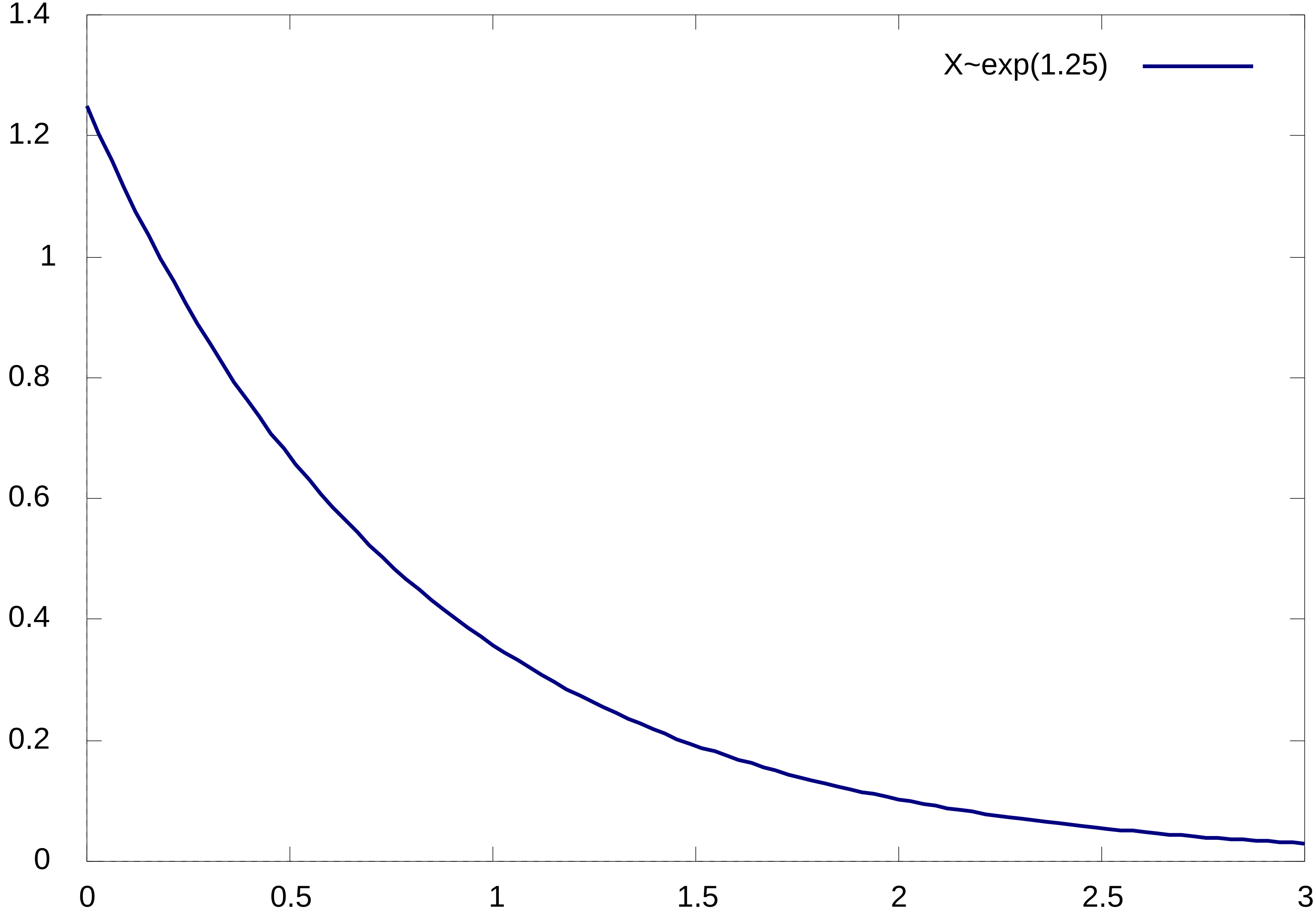
An exponential distribution



A non-exponential distribution



Markov property



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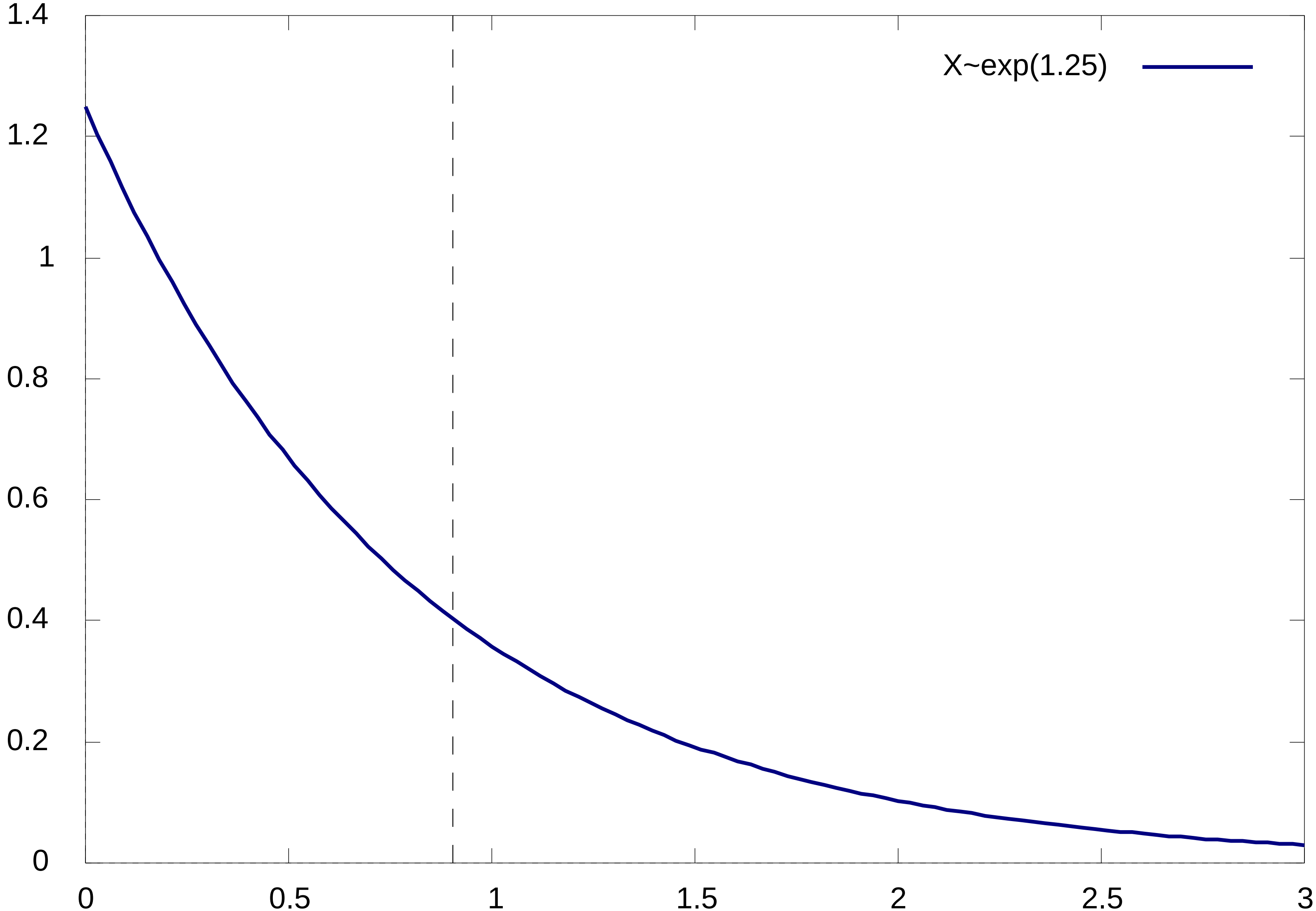

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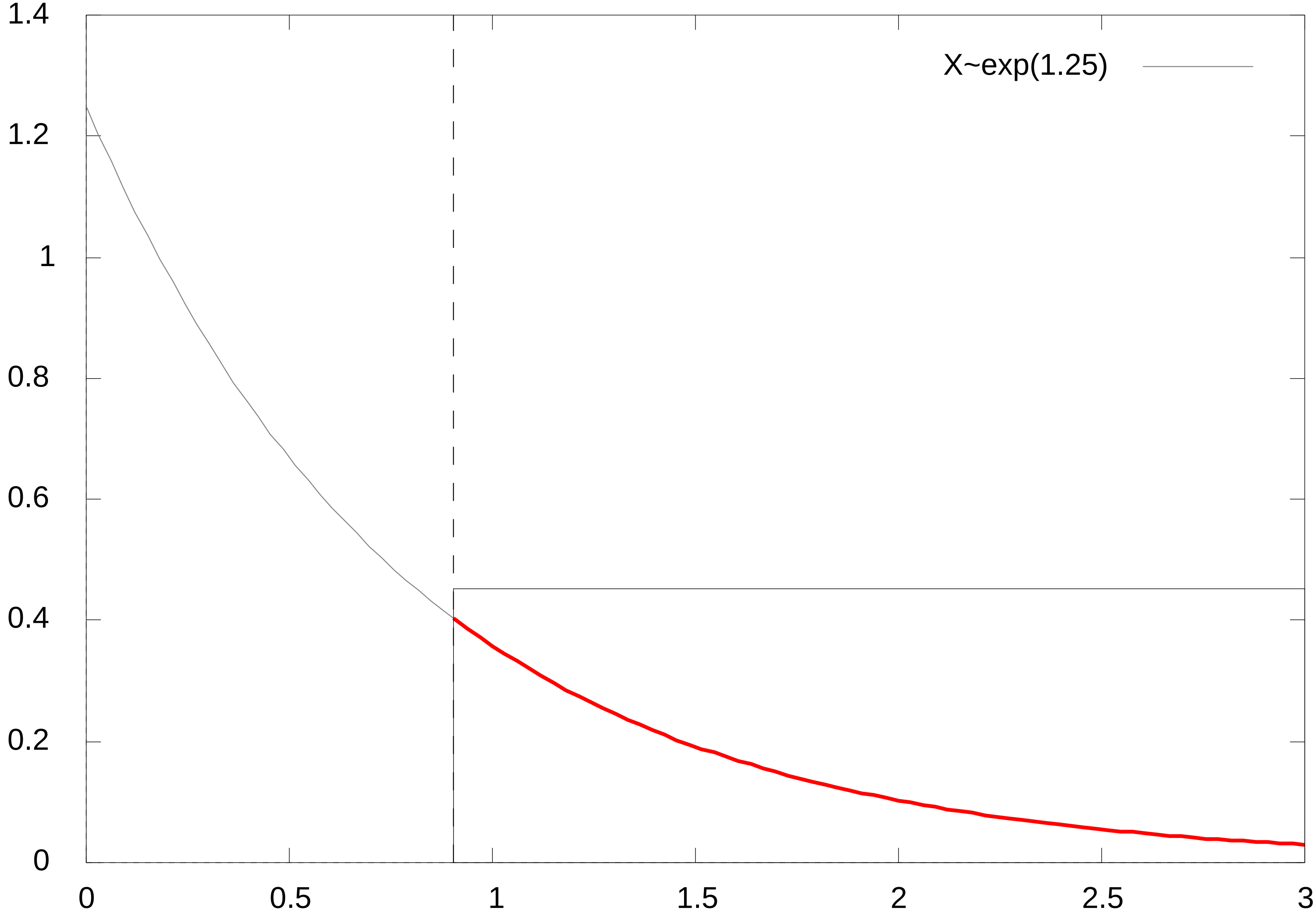

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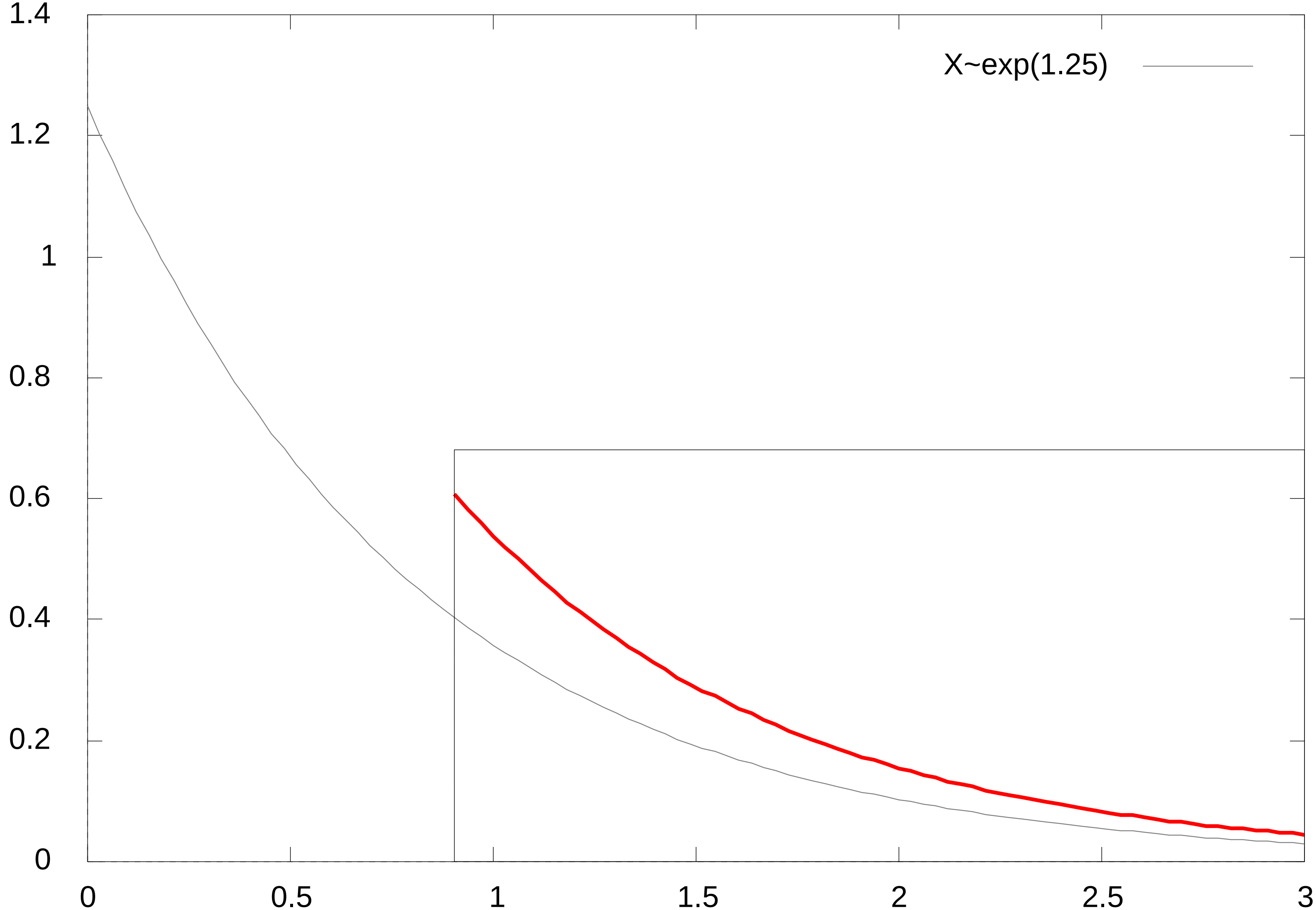

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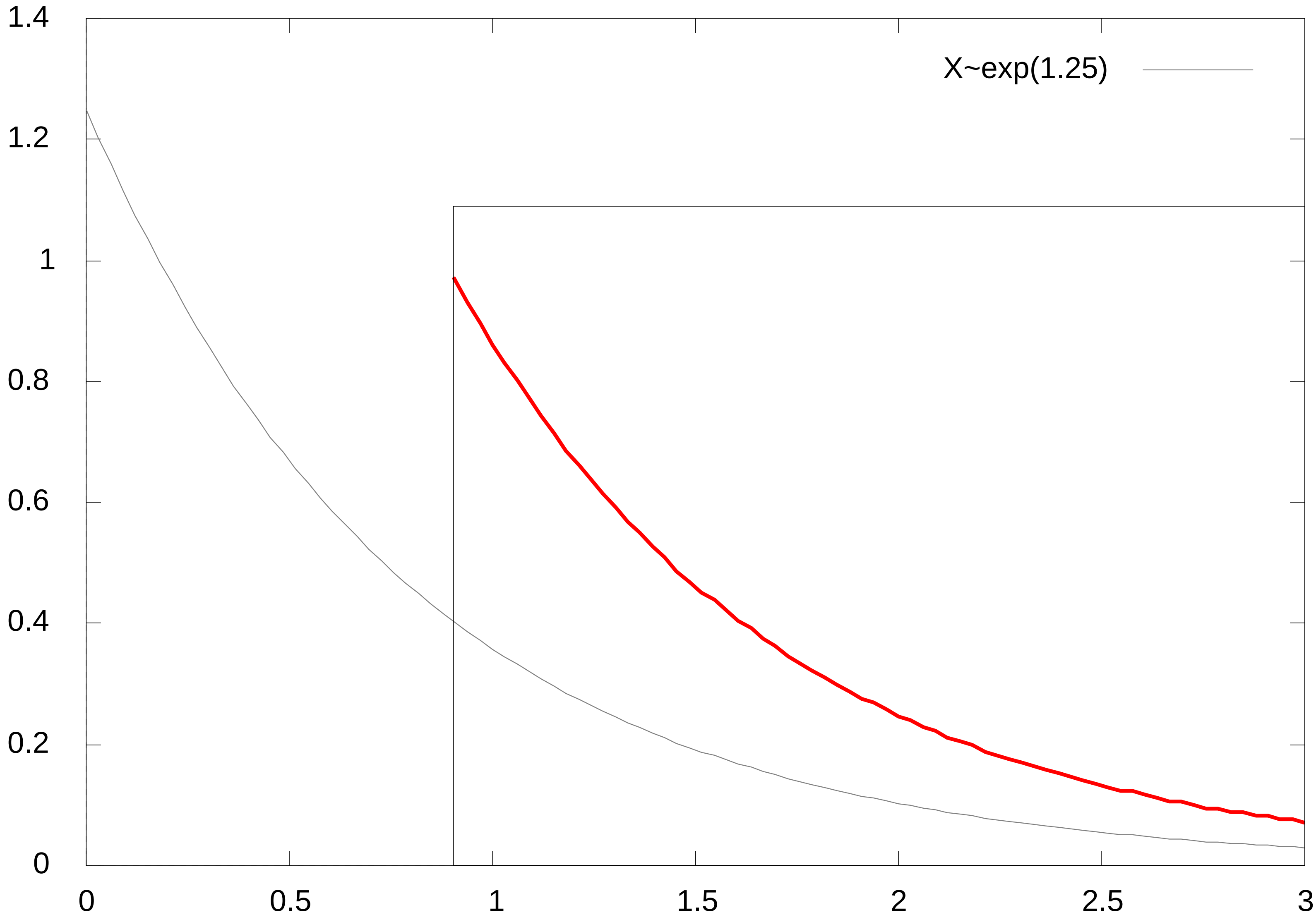

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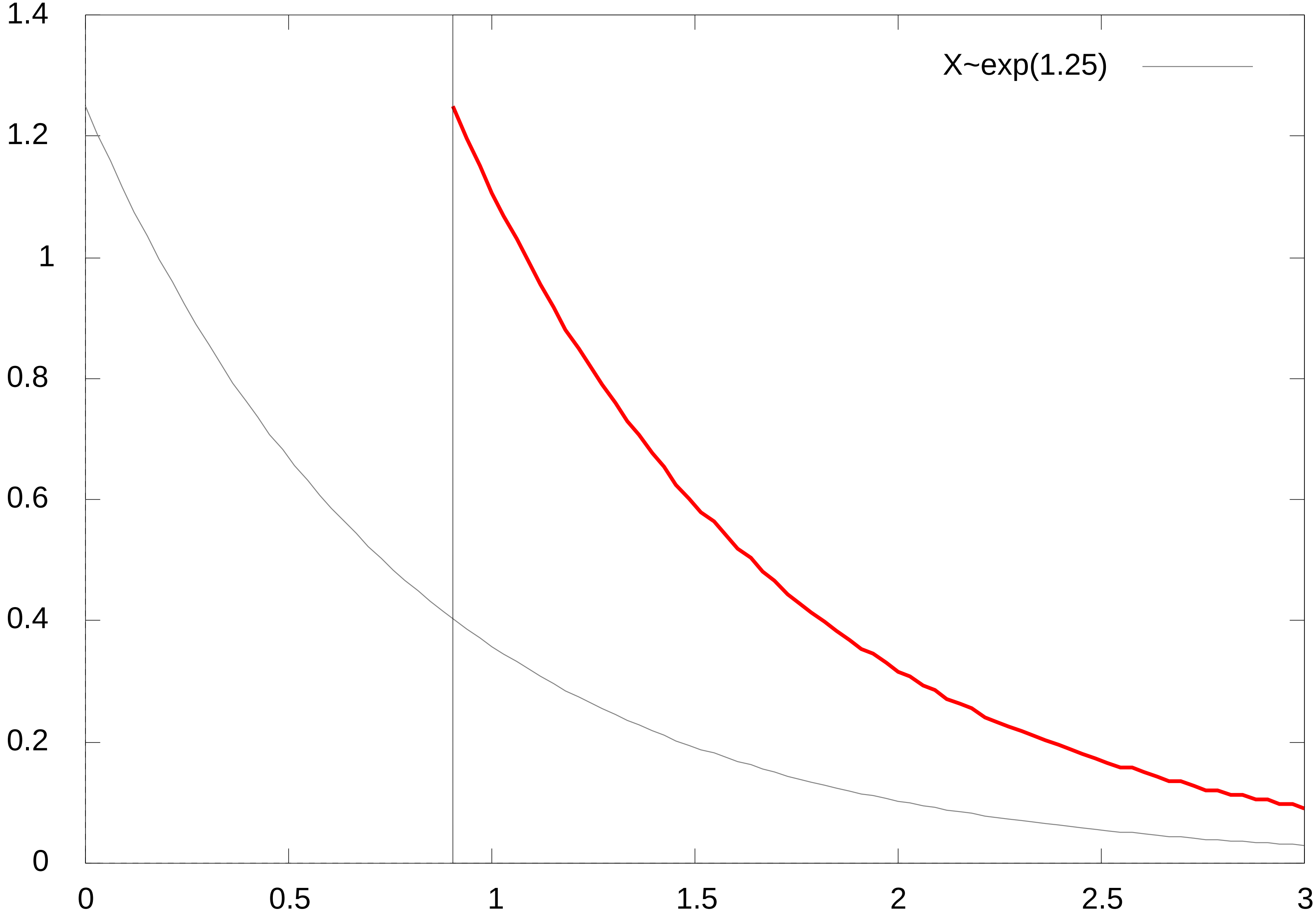

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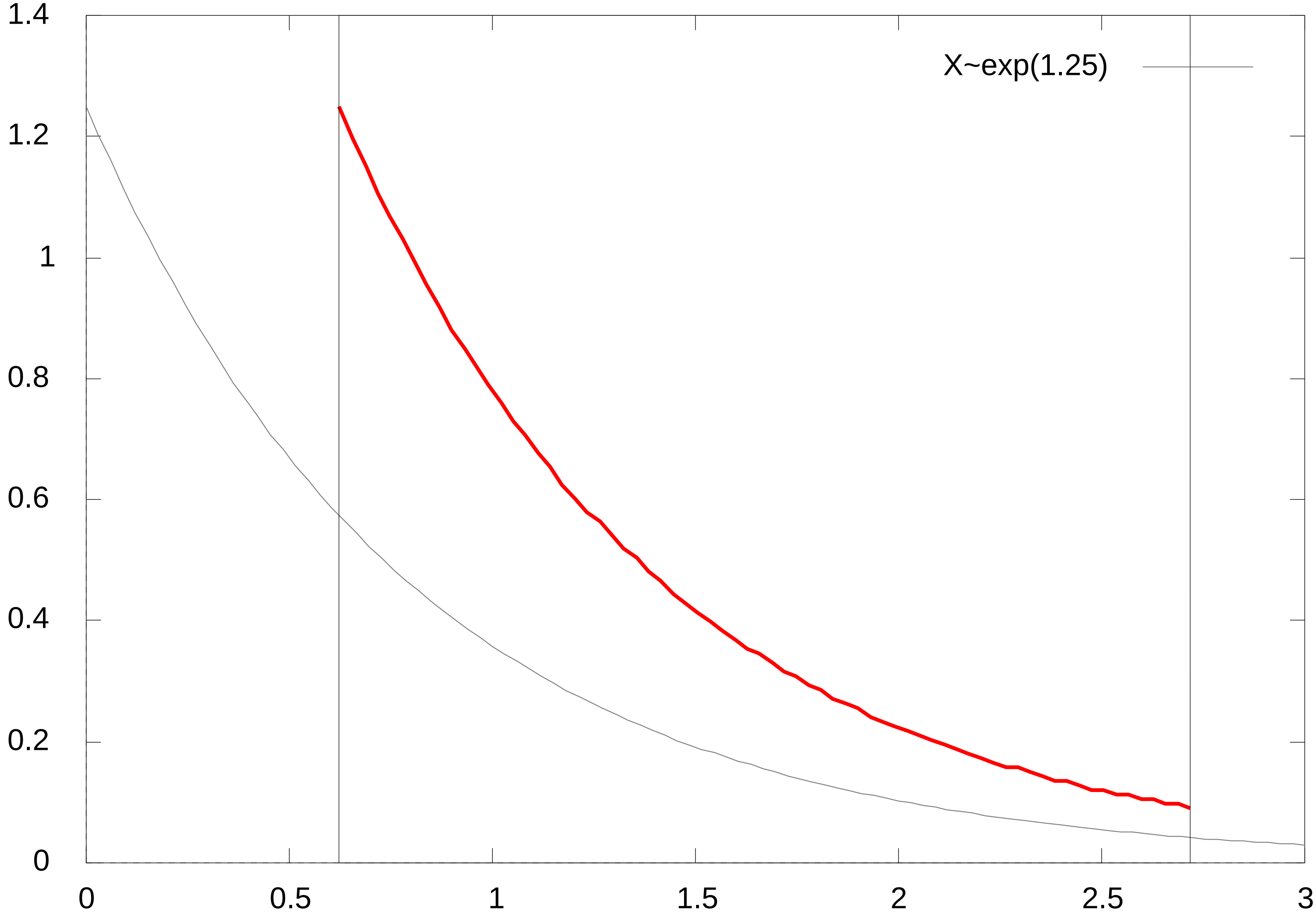

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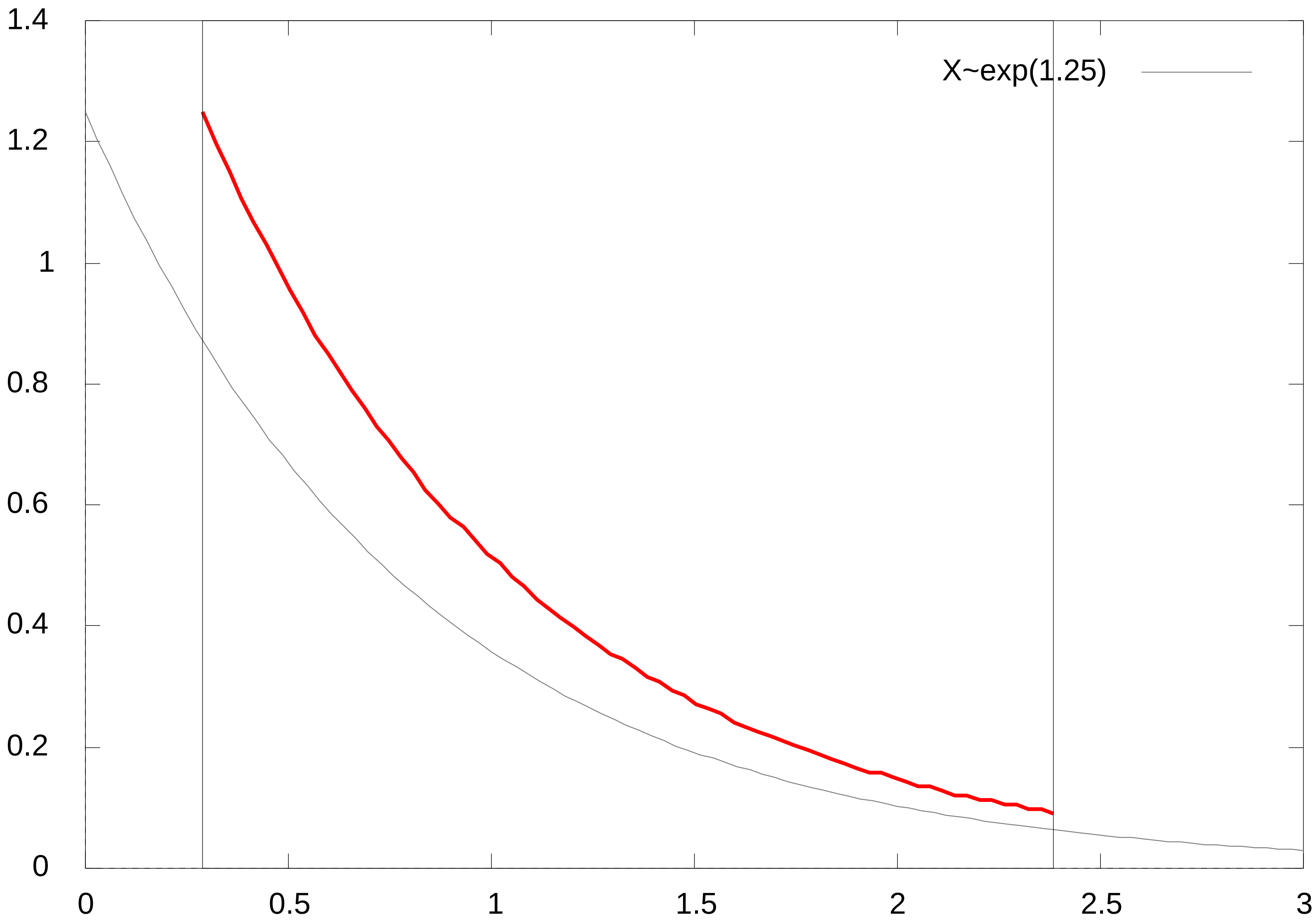

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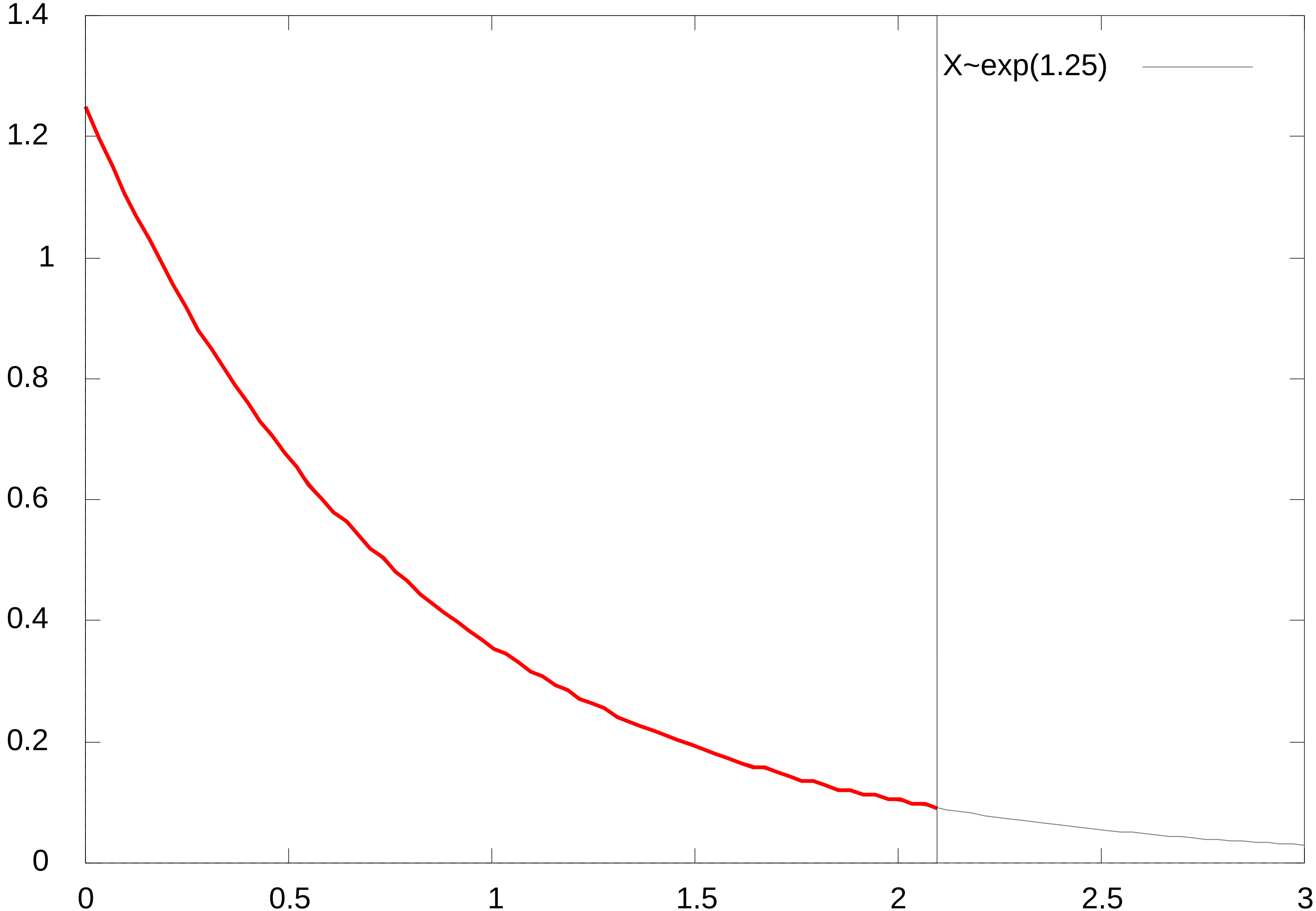


Results?

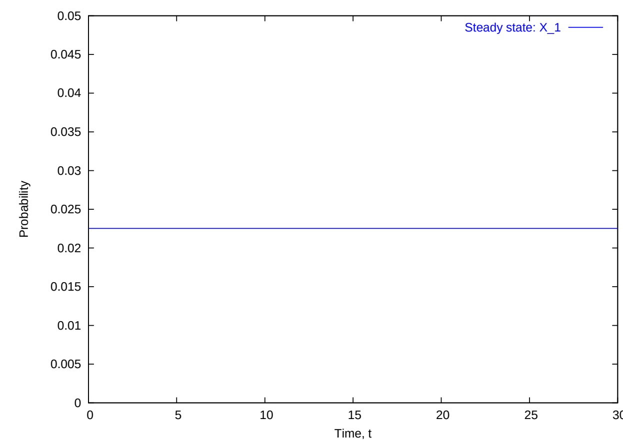
- So what can we do with a PEPA model?

Types of Analysis

Steady-state and transient analysis in PEPA:

$$
\begin{aligned}
A1 &\stackrel{\text{def}}{=} (\text{start}, r_1).A2 + (\text{pause}, r_2).A3 \\
A2 &\stackrel{\text{def}}{=} (\text{run}, r_3).A1 + (\text{fail}, r_4).A3 \\
A3 &\stackrel{\text{def}}{=} (\text{recover}, r_1).A1 \\
AA &\stackrel{\text{def}}{=} (\text{run}, \top).(\text{alert}, r_5).AA \\
Sys &\stackrel{\text{def}}{=} AA \underset{\{run\}}{\bowtie} A1
\end{aligned}
$$

$\Rightarrow$



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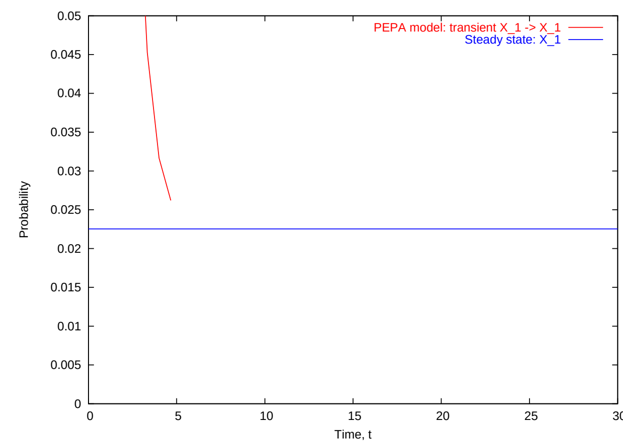

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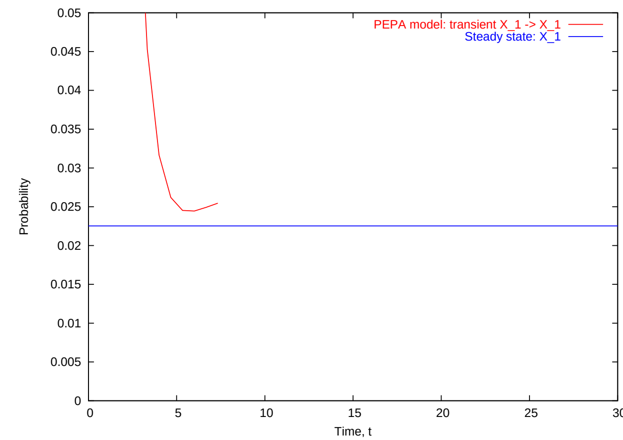

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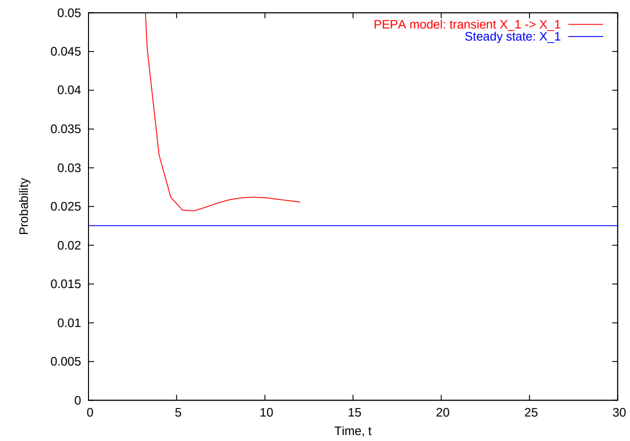

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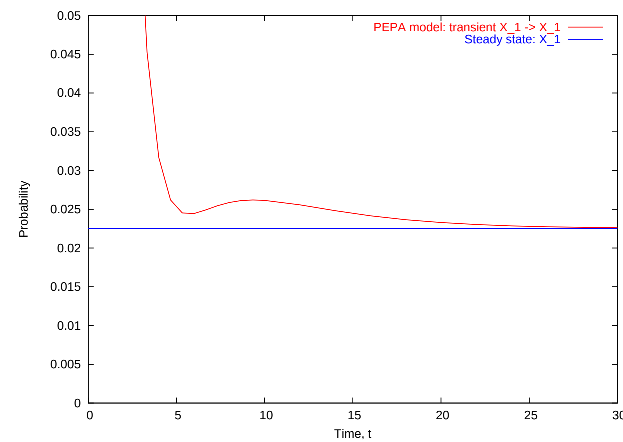

Passage-time Quantiles

Extract a passage-time density from a PEPA model:

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\end{aligned}
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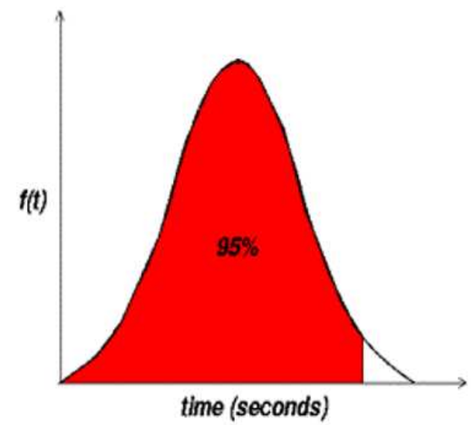
$\Rightarrow$



Brief PEPA Syntax

Syntax:

$$P \; ::= \; (a, \lambda).P \;\Big|\; P + P \;\Big|\; P \underset{L}{\bowtie} P \;\Big|\; P/L \;\Big|\; A$$

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Syntax:

$$P \;::=\; (a,\lambda).P \;\Big|\; P+P \;\Big|\; P \underset{L}{\bowtie} P \;\Big|\; P/L \;\Big|\; A$$

- Action prefix: $(a,\lambda).P$

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- Cooperation: $P_1 \bowtie_{L} P_2$

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- Action hiding: P/L
- Constant label: A

Semi-Markov PEPA Syntax

Syntax:

$$P \;::=\; (a, D).P \;\Big|\; P + P \;\Big|\; P \underset{L}{\bowtie} P \;\Big|\; P/L \;\Big|\; A$$

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Generic delay parameter:

$$D \;::=\; \lambda \;\Big|\; S$$

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Generic delay parameter:

$$D \;::=\; \lambda \;\Big|\; S$$

Semi-Markov parameter:

$$S \;::=\; \top \;\Big|\; \omega : L(s)$$

Voting Example I

$$\text{System} \stackrel{\text{def}}{=} (\text{Voter} \mid\mid \text{Voter} \mid\mid \text{Voter}) \bowtie_{\{\text{vote}\}} ((\text{Poler} \bowtie_{L} \text{Poler}) \bowtie_{L'} \text{Poler\_group\_0})$$

where

- $L = \{\text{recover\_all}^*\}$
- $L' = \{\text{recover}, \text{break}, \text{recover\_all}^*\}$

Voting Example II

$$\text{Voter} \stackrel{\text{def}}{=} (\text{vote}, \lambda).(\text{pause}, \mu).\text{Voter}$$

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\text{Voter} &\stackrel{\text{def}}{=} (\text{vote}, \lambda).(\text{pause}, \mu).\text{Voter} \\
\text{Poler} &\stackrel{\text{def}}{=} (\text{vote}, \top).(\text{register}, \gamma).\text{Poler} \\
&\quad + (\text{break}, \nu).\text{Poler\_broken}
\end{aligned}
$$

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\text{Poler} &\stackrel{\text{def}}{=} (\text{vote}, \top).(\text{register}, \gamma).\text{Poler} \\
&\quad + (\text{break}, \nu).\text{Poler\_broken} \\
\text{Poler\_broken} &\stackrel{\text{def}}{=} (\text{recover}, \tau).\text{Poler} \\
&\quad + (\text{recover\_all}^*, \top).\text{Poler}
\end{aligned}
$$

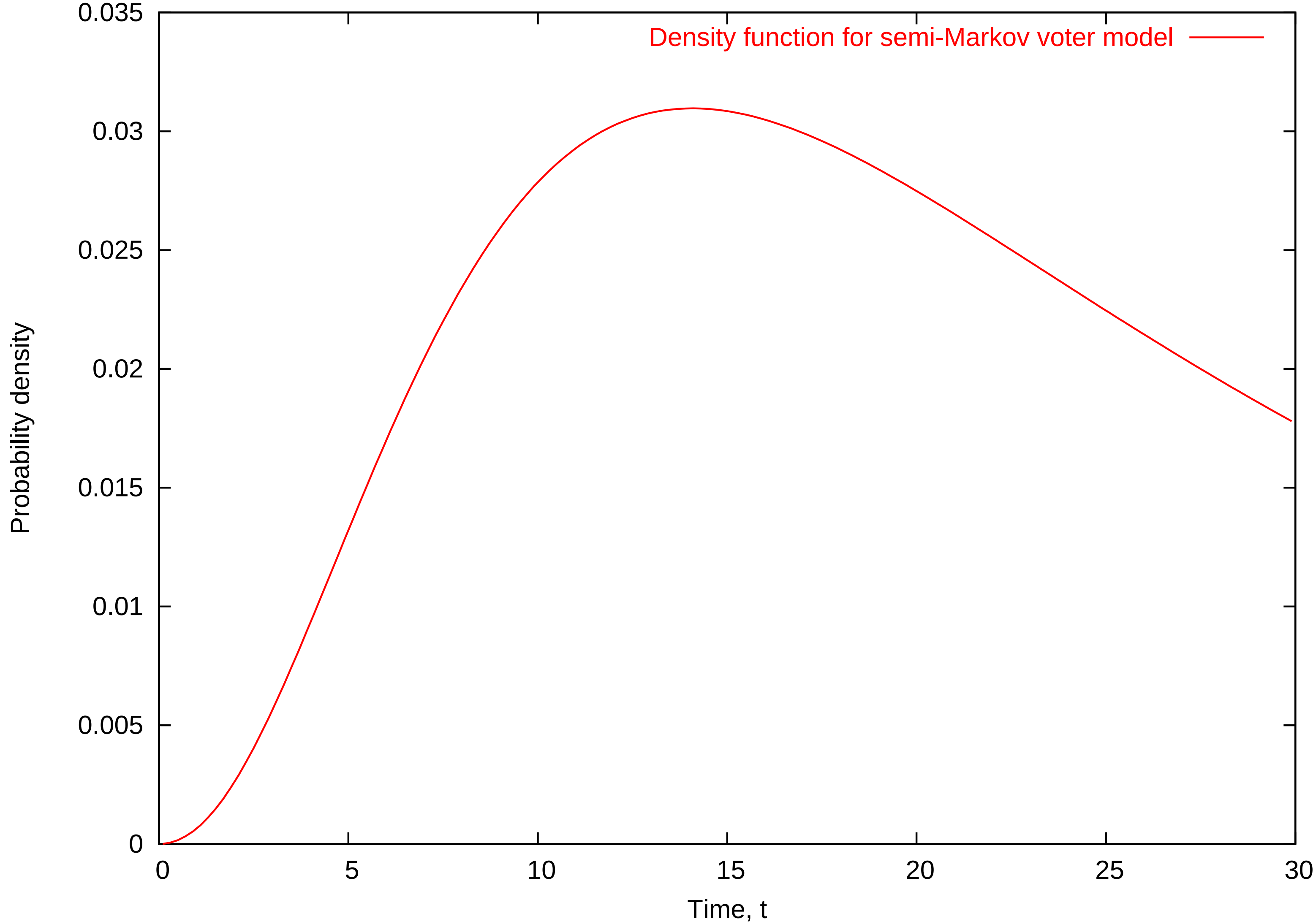
Voting Example III

$$
\begin{aligned}
\text{Poler\_group\_0} &\stackrel{\text{def}}{=} (\text{break}, \top).\text{Poler\_group\_1} \\
\text{Poler\_group\_1} &\stackrel{\text{def}}{=} (\text{break}, \top).\text{Poler\_group\_2} \\
&\quad + (\text{recover}, \top).\text{Poler\_group\_0} \\
\text{Poler\_group\_2} &\stackrel{\text{def}}{=} (\text{recover\_all}^*, \mathit{1} : \texttt{gamma}(\delta, \texttt{k}, \texttt{s})) \\
&\quad .\text{Poler\_group\_0}
\end{aligned}
$$

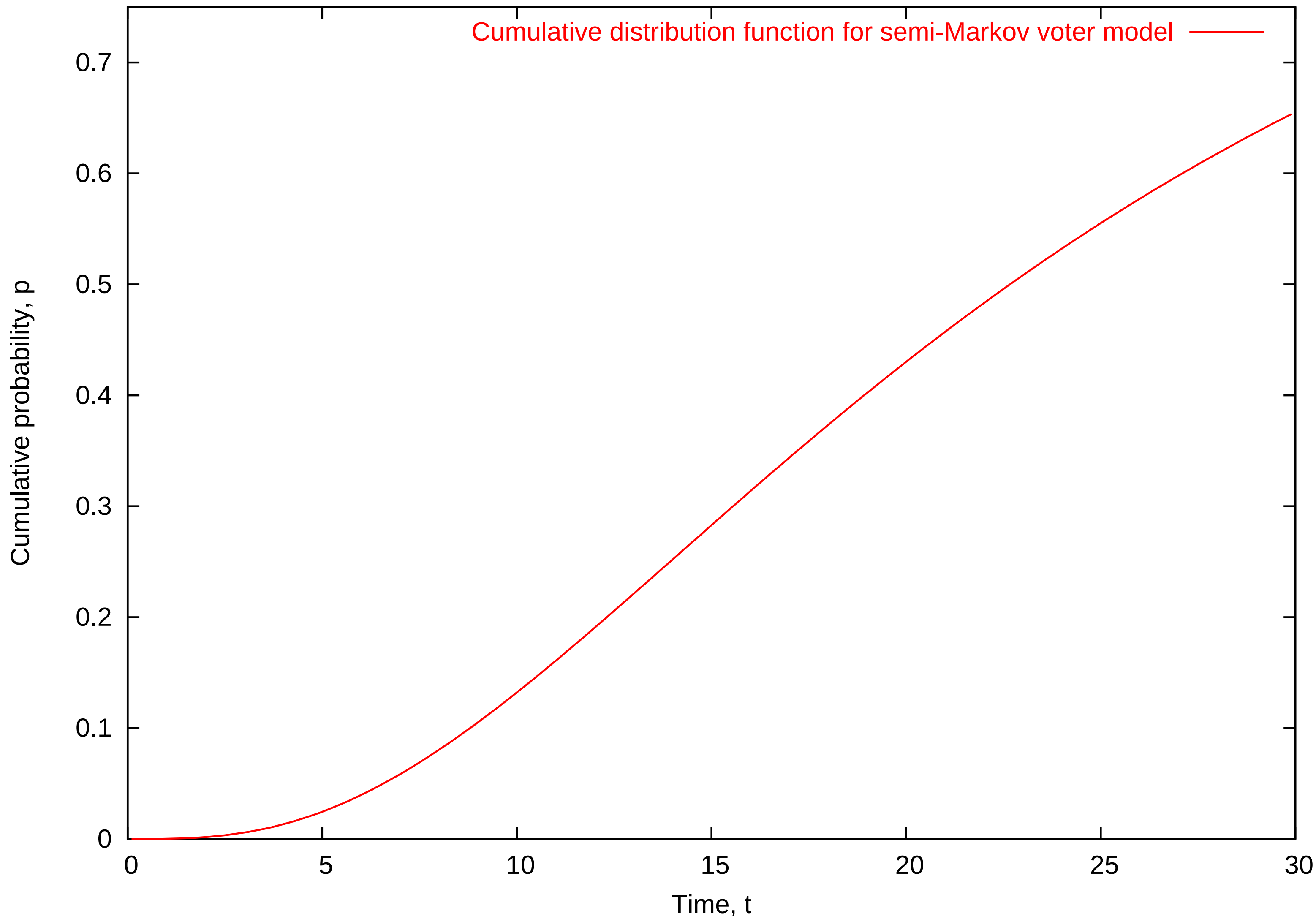
Analysis Question

- How long before the first full failure recovery?
  - a passage time question...

Passage-time density



Passage-time cumulative distn.



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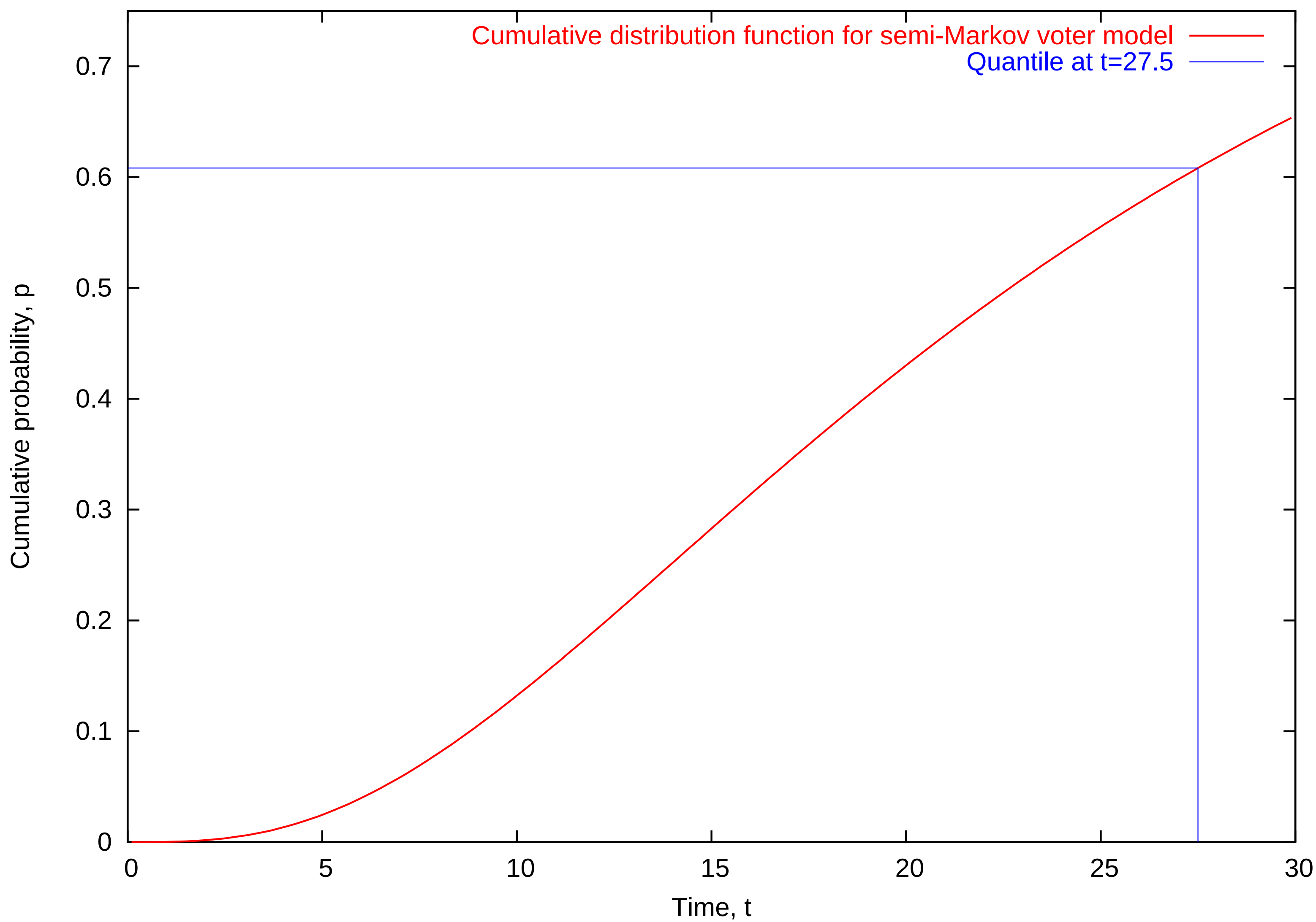

Conclusion

- Added ability to handle general distributions to PEPA
- Uses probabilistic selection to choose between general distributions
- Preserve normal PEPA behaviour if only Markovian actions are active
- Can do both passage-time and transient analysis of semi-Markov models

What are SMPs good for?

- There is no concurrent activation of general distributions
- SMPs have two significant application areas:
  - Mutually exlcusive operation e.g. failure, recovery
  - Scheduling policy on a 1-processor system