Synchronisation in PEPA models

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In stochastic models...

Synchronisation can significantly affect performance results

Presentation

- PEPA and analysis
- The PEPA process algebra
- Synchronisation in practice
- Results

Types of Analysis

Steady-state and transient analysis in PEPA:

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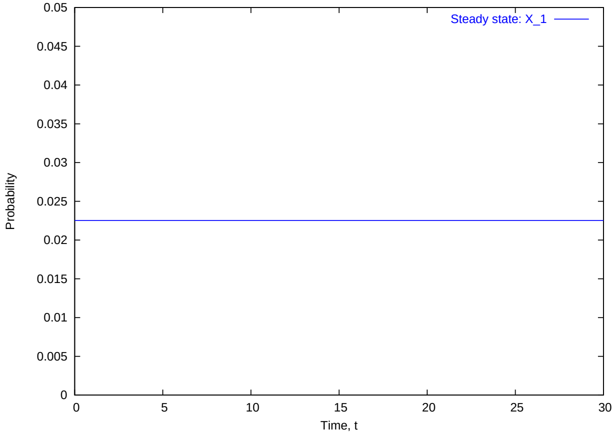
A1 &\stackrel{def}{=} (start, r_1).A2 + (pause, r_2).A3 \\
A2 &\stackrel{def}{=} (run, r_3).A1 + (fail, r_4).A3 \\
A3 &\stackrel{def}{=} (recover, r_1).A1 \\
AA &\stackrel{def}{=} (run, \top).(alert, r_5).AA \\
Sys &\stackrel{def}{=} AA \underset{\{run\}}{\bowtie} A1
\end{aligned}
$$

$\Rightarrow$



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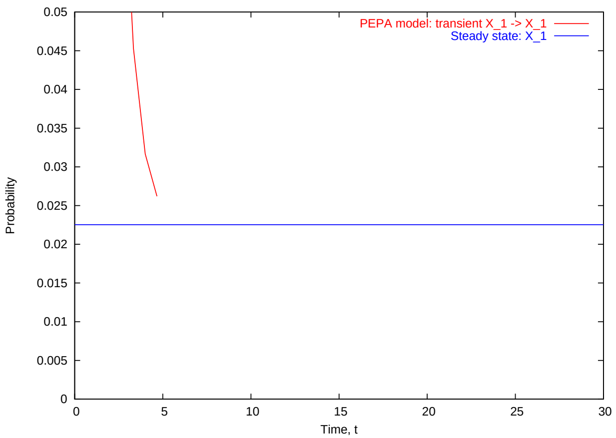

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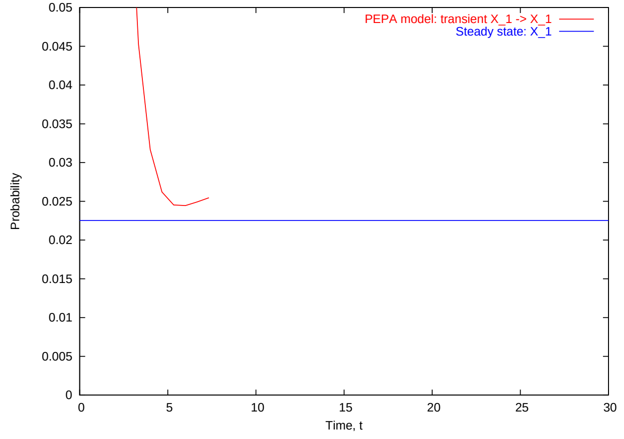

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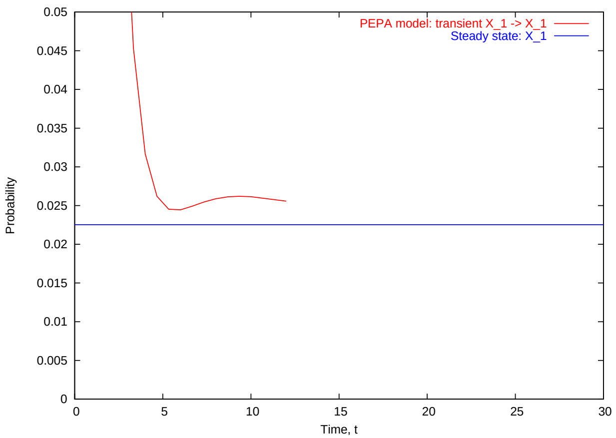

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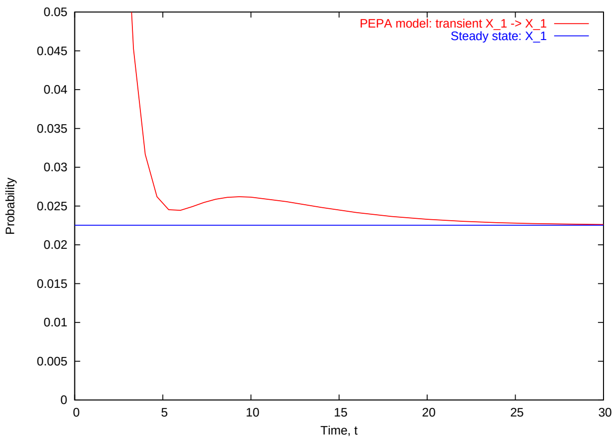

Passage-time Quantiles

Extract a passage-time density from a PEPA model:

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\quad \Longrightarrow
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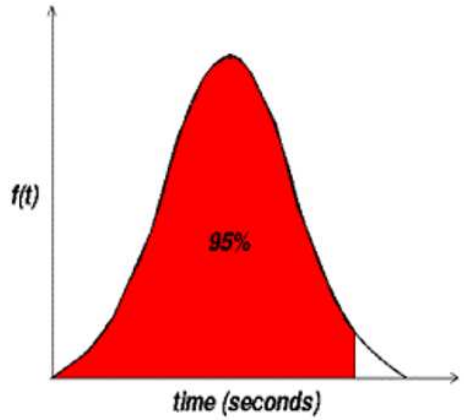

Stochastic Process Algebra

PEPA syntax:

$$P \quad ::= \quad (a, \lambda).P \;\Big|\; P + P \;\Big|\; P \underset{L}{\bowtie} P \;\Big|\; P/L \;\Big|\; A$$

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PEPA syntax:

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- Cooperation: $P_1 \underset{L}{\bowtie} P_2$
- Action hiding: P/L
- Constant label: A

State of the Art

- PEPA model: passage time/transient analysis - $O(10^8)$ states
- Semi-Markov PEPA: passage time/transient analysis - $O(10^7)$ states

PEPA: A Transmitter-Receiver

$$
\begin{aligned}
\text{System} &\stackrel{\text{def}}{=} (\text{Transmitter} \underset{\emptyset}{\bowtie} \text{Receiver}) \underset{\{\mathtt{transmit},\mathtt{receive}\}}{\bowtie} \text{Network} \\[2ex]
\text{Transmitter} &\stackrel{\text{def}}{=} (\text{transmit}, \lambda_1).(\text{t} - \text{recover}, \lambda_2).\text{Transmitter} \\
\text{Receiver} &\stackrel{\text{def}}{=} (\text{receive}, \top).(\text{r} - \text{recover}, \mu).\text{Receiver} \\
\text{Network} &\stackrel{\text{def}}{=} (\text{transmit}, \top).(\text{delay}, \nu_1).(\text{receive}, \nu_2).\text{Network}
\end{aligned}
$$

- A simple transmitter-receiver over a network

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- Apparent rate is given by:

$$r_a(\mathrm{P}) = \sum_{\mathrm{P} \xrightarrow{(a, \lambda_i)}} \lambda_i$$

Apparent Rate Examples

- $r_a(\mathrm{P} \xrightarrow{(\mathrm{a},\lambda)}) = \lambda$

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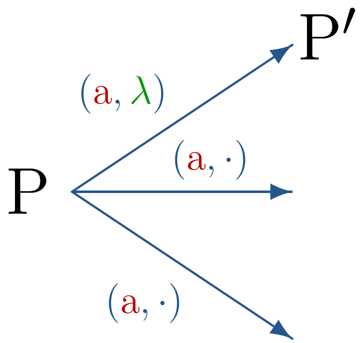
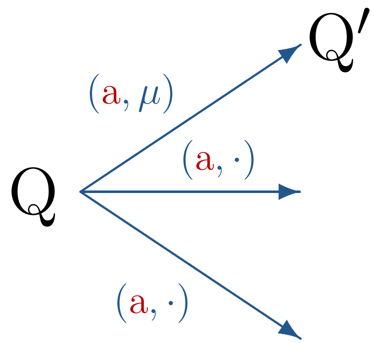
- $$r_a\left(\mathrm{P} \begin{array}{l} \nearrow^{(\mathrm{a},\lambda_1)} \\ \searrow_{(\mathrm{a},\lambda_2)} \end{array} \right) = \lambda_1 + \lambda_2$$

Apparent Rate Examples

- $r_a(P \xrightarrow{(a,\lambda)}) = \lambda$
- $r_a(P \xrightarrow{(a,\top)}) = \top$
- $$r_a\left(\begin{tikzcd}[column sep=huge] & {} \\ P \arrow[ur, "{(a,\lambda_1)}"] \arrow[dr, "{(a,\lambda_2)}"'] & \\ & {} \end{tikzcd} \right) = \lambda_1 + \lambda_2$$
- $$r_a\left(\begin{tikzcd}[column sep=huge] & {} \\ P \arrow[ur, "{(a,\top)}"] \arrow[dr, "{(a,\top)}"'] & \\ & {} \end{tikzcd} \right) = 2\top$$

Synchronisation Rate

- In PEPA, when synchronising two model components, P and Q where both P and Q enable many a-actions:
  - 

and


Synchronisation Rate

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  - $$\begin{tikzcd} & \mathrm{P}' \\ \mathrm{P} \arrow[ur, "(\mathrm{a},\lambda)"] \arrow[r, "(\mathrm{a},\cdot)"] \arrow[dr, "(\mathrm{a},\cdot)"'] & {} \\ & {} \end{tikzcd} \quad \text{and} \quad \begin{tikzcd} & \mathrm{Q}' \\ \mathrm{Q} \arrow[ur, "(\mathrm{a},\mu)"] \arrow[r, "(\mathrm{a},\cdot)"] \arrow[dr, "(\mathrm{a},\cdot)"'] & {} \\ & {} \end{tikzcd}$$

- The synchronised rate for $\mathrm{P} \underset{a}{\bowtie} \mathrm{Q} \xrightarrow{(\mathrm{a},R)} \mathrm{P}' \underset{a}{\bowtie} \mathrm{Q}'$ is:

$$R = \frac{\lambda}{r_a(\mathrm{P})}\frac{\mu}{r_a(\mathrm{Q})}\min(r_a(\mathrm{P}), \mathrm{r_a}(\mathrm{Q}))$$

Approximate Synchronisation

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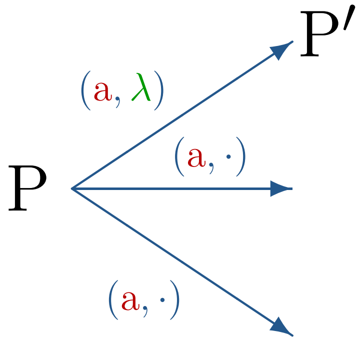
  - 

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    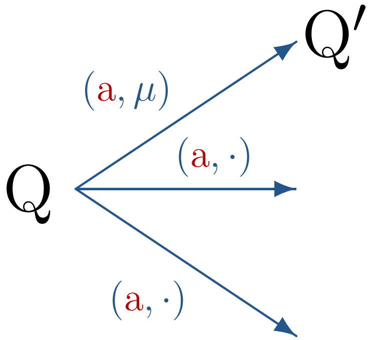

Approximate Synchronisation

- Some tools such as: Möbius, PRISM, PWB use an approximate synchronisation model
- With two model components, P and Q where both P and Q enable many a-actions:
  - $\begin{tikzcd} & & \mathrm{P}' \\ \mathrm{P} \arrow[urr, "(\mathrm{a}, \lambda)"] \arrow[rr, "(\mathrm{a}, \cdot)"] \arrow[drr, "(\mathrm{a}, \cdot)"'] & & {} \\ & & {} \end{tikzcd}$ and $\begin{tikzcd} & & \mathrm{Q}' \\ \mathrm{Q} \arrow[urr, "(\mathrm{a}, \mu)"] \arrow[rr, "(\mathrm{a}, \cdot)"] \arrow[drr, "(\mathrm{a}, \cdot)"'] & & {} \\ & & {} \end{tikzcd}$
- The *approximated* rate for $\mathrm{P} \underset{a}{\bowtie} \mathrm{Q} \xrightarrow{(\mathrm{a}, R)} \mathrm{P}' \underset{a}{\bowtie} \mathrm{Q}'$ is:

$$R = \min(\lambda, \mu)$$

Example

- As an example:
  - $\text{Client} \stackrel{\text{def}}{=} (\text{data}, \lambda).\text{Client}'$
  - $\text{Network} \stackrel{\text{def}}{=} (\text{data}, \top).\text{NetworkGo}$
    $+ (\text{data}, \top).\text{NetworkStall}$

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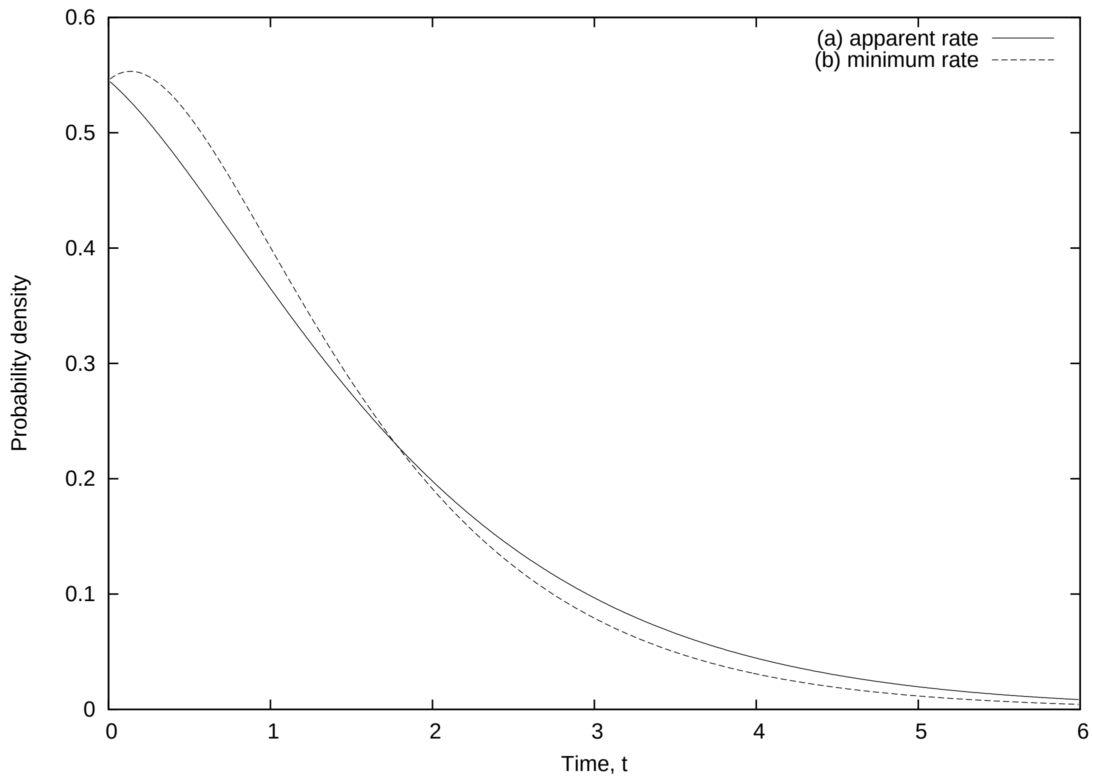
    $+ \; (\mathrm{data}, \top).\mathrm{NetworkStall}$
- The combination $\mathrm{Client} \underset{data}{\bowtie} \mathrm{Network}$ should evolve with an overall data rate parameter of λ
- Under the tool approximation the overall synchronised rate becomes 2λ

Results: Multiple Passive

$$\begin{aligned} A &\stackrel{\text{def}}{=} (\text{run}, \lambda_1).(\text{stop}, \lambda_2).A \\ B &\stackrel{\text{def}}{=} (\text{run}, \top).(\text{pause}, \lambda_3).B \\ \text{Sys}_A &\stackrel{\text{def}}{=} A \underset{\{run\}}{\bowtie} (B \mid\mid B) \end{aligned}$$

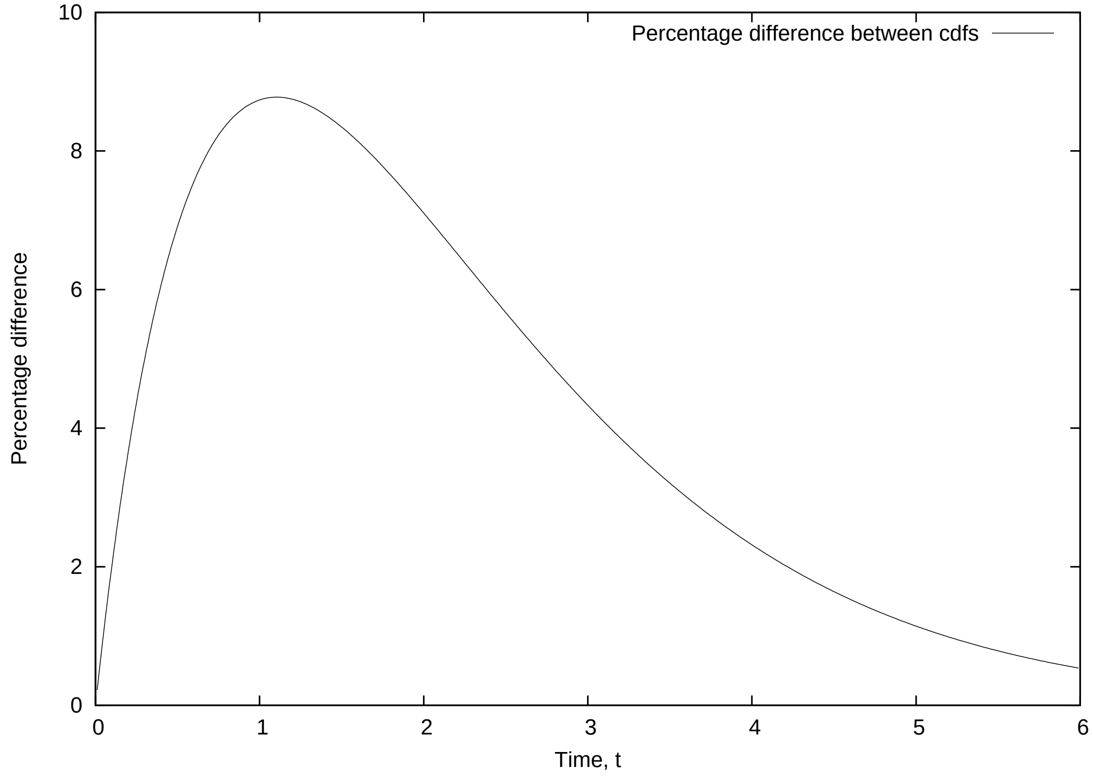
- Multiple passive ($\top$-rate) actions are enabled against a single real rate

Results: Multiple Passive



- Passage time density between consecutive stop actions

Results: Multiple Passive



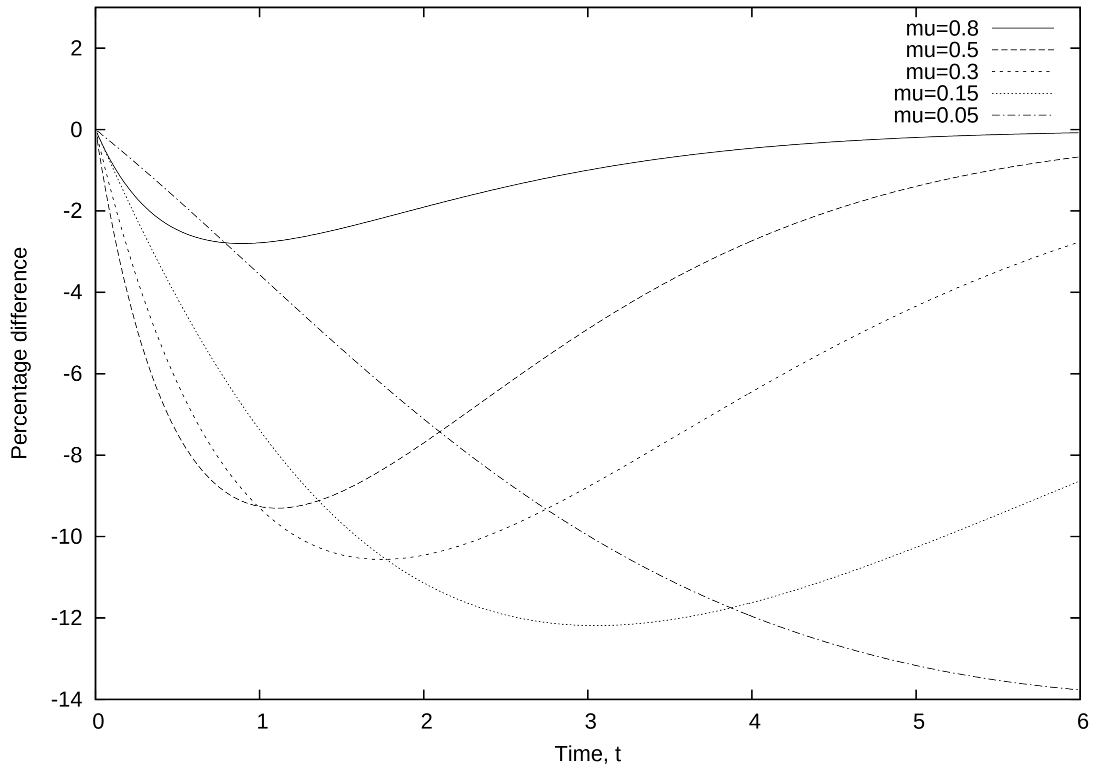
- Percentage difference in CDF functions over passage time between consecutive stop actions

Results: Multiple Active

$$\begin{aligned} \mathrm{A} &\stackrel{\text{def}}{=} (\mathrm{run}, \lambda_1).(\mathrm{stop}, \lambda_2).\mathrm{A} \\ \mathrm{B} &\stackrel{\text{def}}{=} (\mathrm{run}, \mu_1).(\mathrm{pause}, \lambda_3).\mathrm{B} \\ \mathrm{Sys_C} &\stackrel{\text{def}}{=} \mathrm{A} \underset{\{run\}}{\bowtie} (\mathrm{B} \,||\, \mathrm{B}) \end{aligned}$$

- Multiple real-rate actions (in $(\mathrm{B} \,||\, \mathrm{B})$) are synchronised against a single real-rate action (in A)

Results: Multiple Active



- Percentage difference in CDF functions over passage time between consecutive stop actions (for decreasing μ)

Isn't this really unusual?

- Q: How common is this kind of modelling problem? Isn't this bizarre non-determinism to see in a component?
- A: Having an explicit individual component with either:
  - $P \stackrel{\text{def}}{=} (a, \lambda).P' + (a, \mu).P''$ (multiple active)
  - $Q \stackrel{\text{def}}{=} (a, \top).Q' + (a, \top).Q''$ (multiple passive)
- ...might be unusal, but simple multi-agent synchronisation of $S \underset{\{a\}}{\bowtie} (R \mid\mid R \mid\mid \cdots \mid\mid R)$ for some S where $R \stackrel{\text{def}}{=} (a, \top).(b, \mu).R'$ causes just this problem
- This is a very common client–server architecture

Conclusion

- Synchronisation style makes a big difference to performance results!
- To summarise, using the tool approximation:
  - with multiple passive actions – sees an overestimation of passage-time results
  - with multiple active actions – sees an underestimation of passage-time results – why?