ipc/HYDRA: Passage-time and transient analysis of PEPA models

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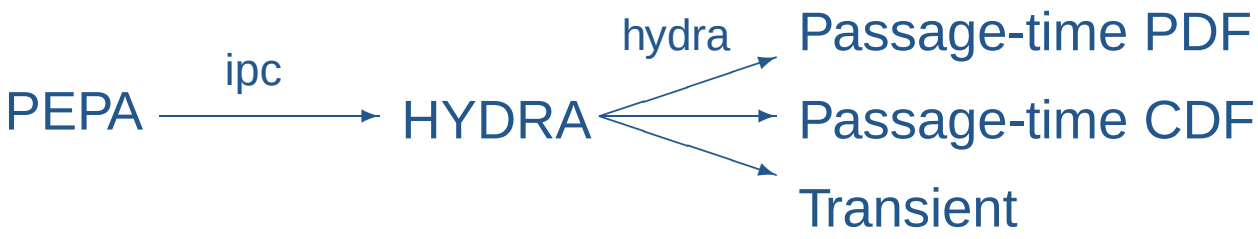
Produced with prosper and LaTeX

Tool Pipeline



1. ipc compiles (Markovian) process algebra PEPA model with probe performance requirement to HYDRA Petri net
2. HYDRA performs disk-based analysis of Petri net to perform:
   - (steady-state analysis)
   - passage-time PDF or CDF
   - transient analysis

Types of Analysis

Steady-state and transient analysis in PEPA:

$$
\begin{aligned}
\mathrm{A1} &\stackrel{\mathrm{def}}{=} (\mathrm{start}, r_1).\mathrm{A2} + (\mathrm{pause}, r_2).\mathrm{A3} \\
\mathrm{A2} &\stackrel{\mathrm{def}}{=} (\mathrm{run}, r_3).\mathrm{A1} + (\mathrm{fail}, r_4).\mathrm{A3} \\
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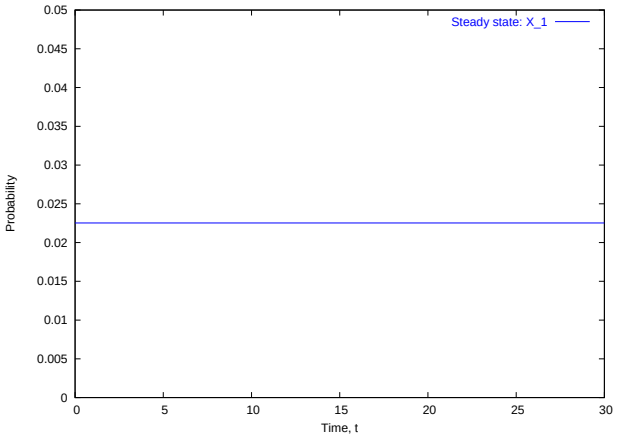
\mathrm{Sys} &\stackrel{\mathrm{def}}{=} \mathrm{AA} \underset{\{run\}}{\bowtie} \mathrm{A1}
\end{aligned}
$$

$\Longrightarrow$



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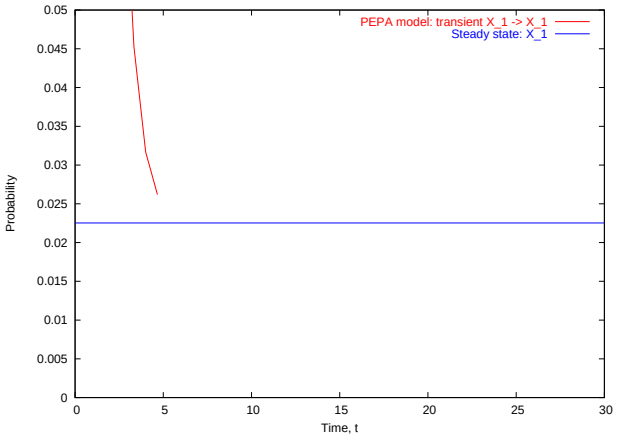

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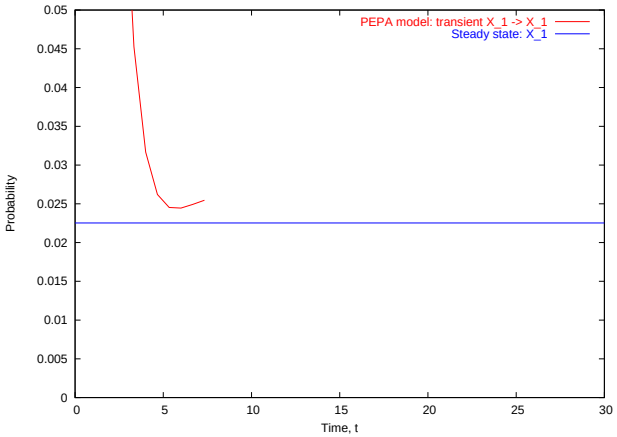

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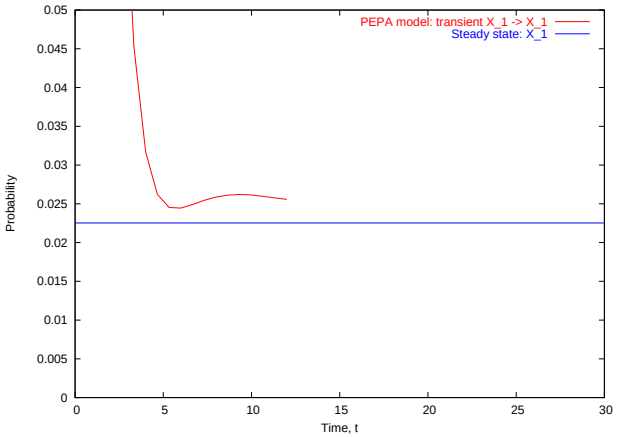

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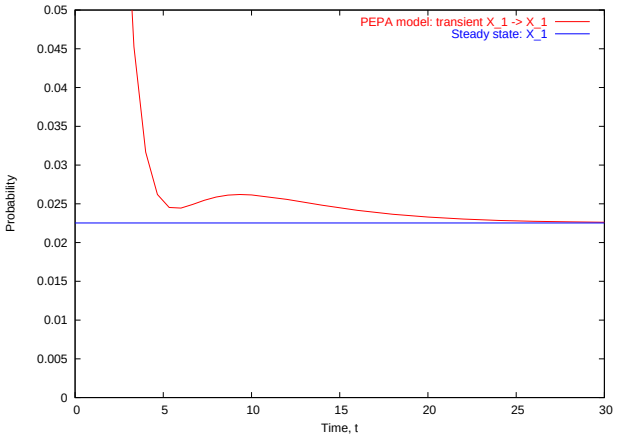

Passage-time Quantiles

Extract a passage-time density from a PEPA model:

$$
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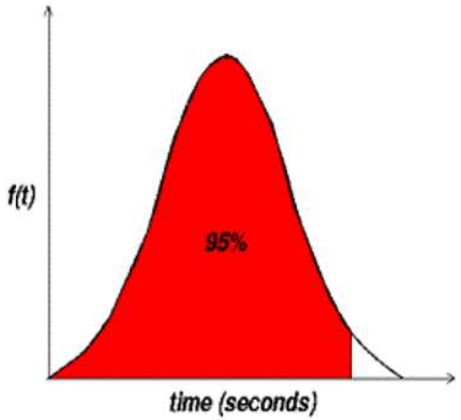
\end{aligned}
\quad \Longrightarrow
$$



Stochastic Process Algebra

PEPA syntax:

$$\mathrm{P} \quad ::= \quad (\mathrm{a}, \lambda).\mathrm{P} \;\Big|\; \mathrm{P} + \mathrm{P} \;\Big|\; \mathrm{P} \underset{L}{\bowtie} \mathrm{P} \;\Big|\; \mathrm{P}/\mathrm{L} \;\Big|\; \mathrm{A}$$

- Action prefix: $(\mathrm{a}, \lambda).\mathrm{P}$
- Competitive choice: $\mathrm{P}_1 + \mathrm{P}_2$
- Cooperation: $\mathrm{P}_1 \underset{L}{\bowtie} \mathrm{P}_2$
- Action hiding: P/L
- Constant label: A

PEPA: Example

$$\begin{aligned}
\text{Sys} &\stackrel{\text{def}}{=} (\text{AA} \underset{\text{run}}{\bowtie} \text{A1}) \underset{\text{alert}}{\bowtie} (\text{BB} \underset{\text{run}}{\bowtie} \text{B1}) \\
\\
\text{A1} &\stackrel{\text{def}}{=} (\text{start}, r_1).\text{A2} + (\text{pause}, r_2).\text{A3} \\
\text{A2} &\stackrel{\text{def}}{=} (\text{run}, r_3).\text{A1} + (\text{fail}, r_4).\text{A3} \\
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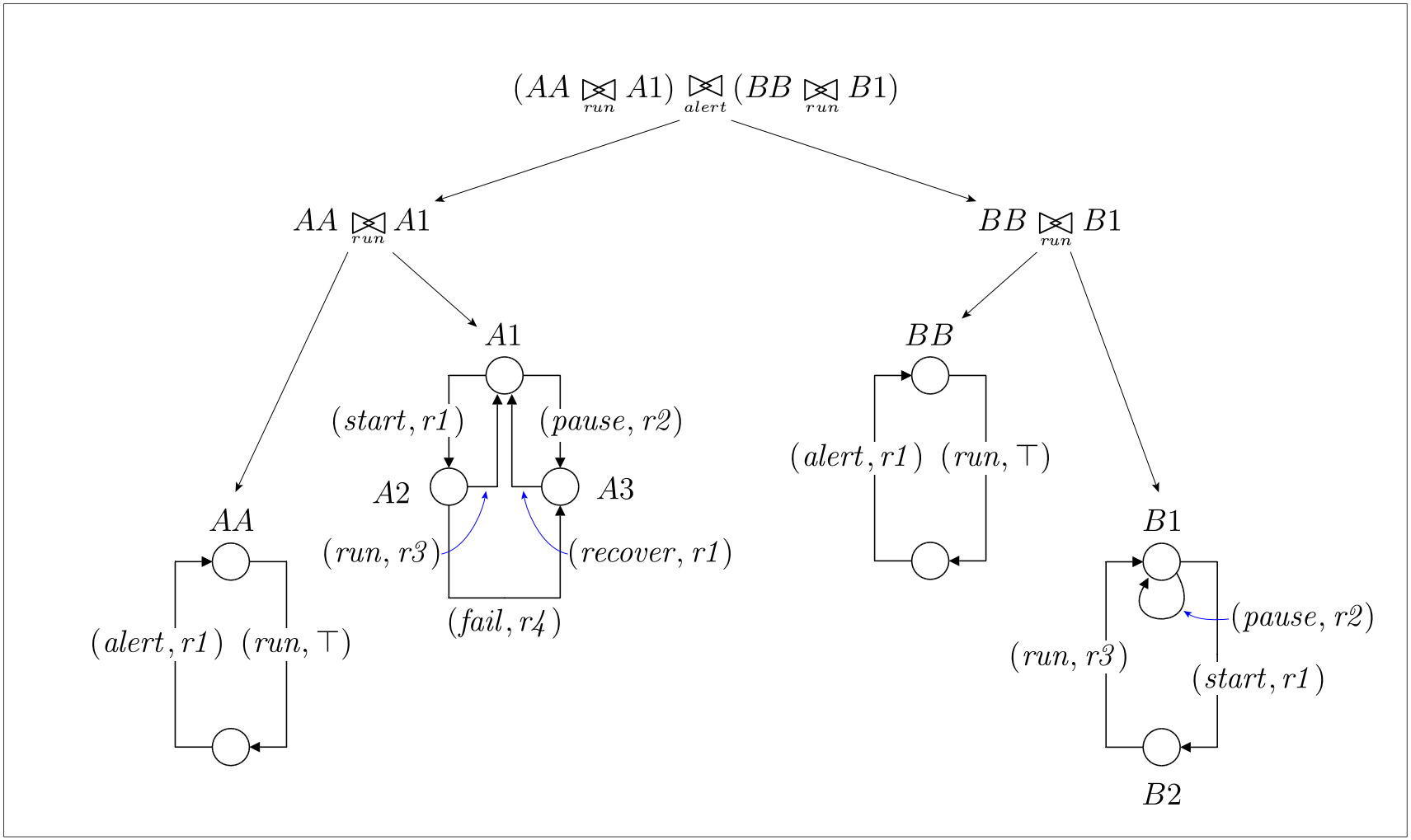
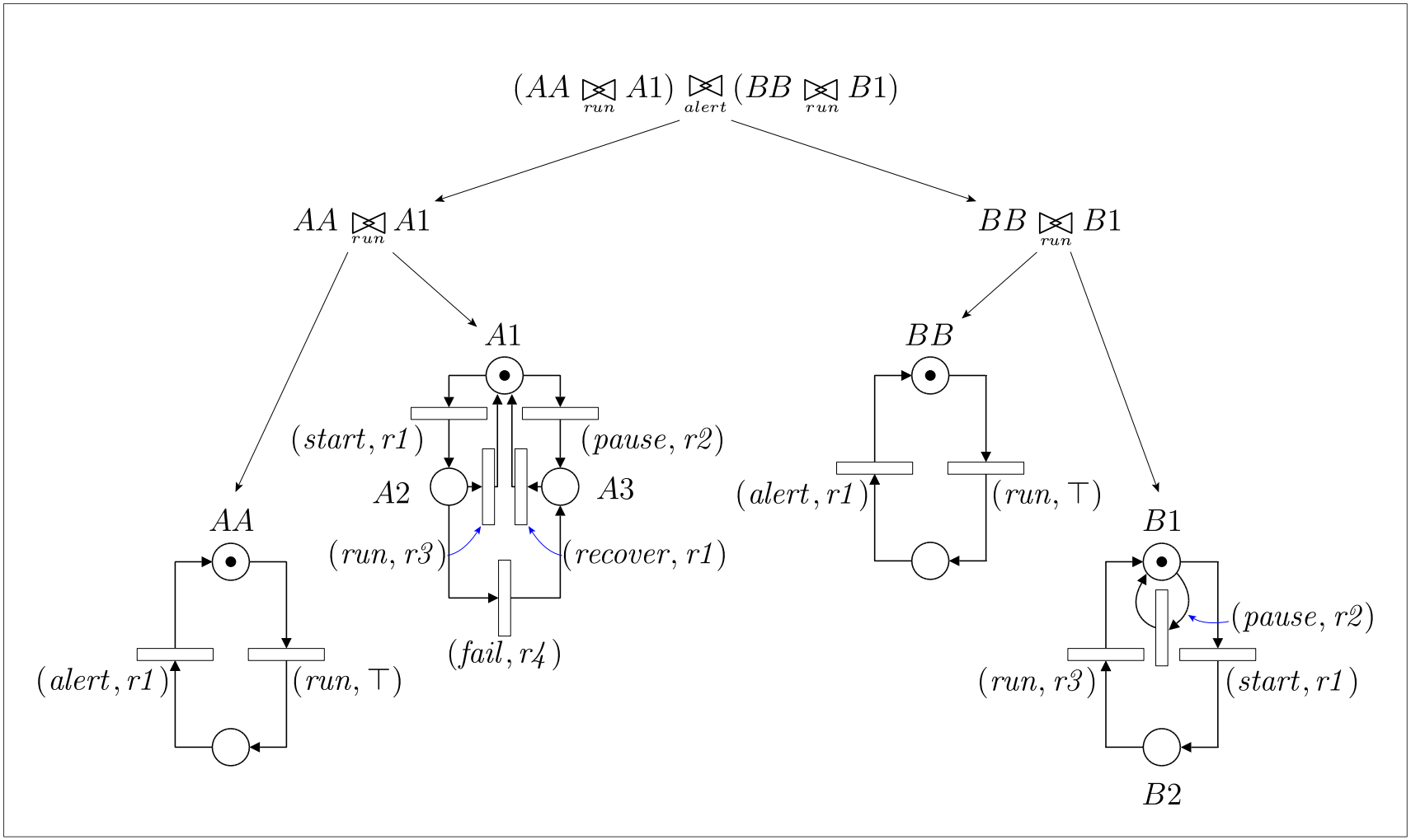
\text{AA} &\stackrel{\text{def}}{=} (\text{run}, \top).(\text{alert}, r_5).\text{AA} \\
\\
\text{B1} &\stackrel{\text{def}}{=} (\text{start}, r_1).\text{B2} + (\text{pause}, r_2).\text{B1} \\
\text{B2} &\stackrel{\text{def}}{=} (\text{run}, r_3).\text{B1} \\
\text{BB} &\stackrel{\text{def}}{=} (\text{run}, \top).(\text{alert}, r_5).\text{BB}
\end{aligned}$$

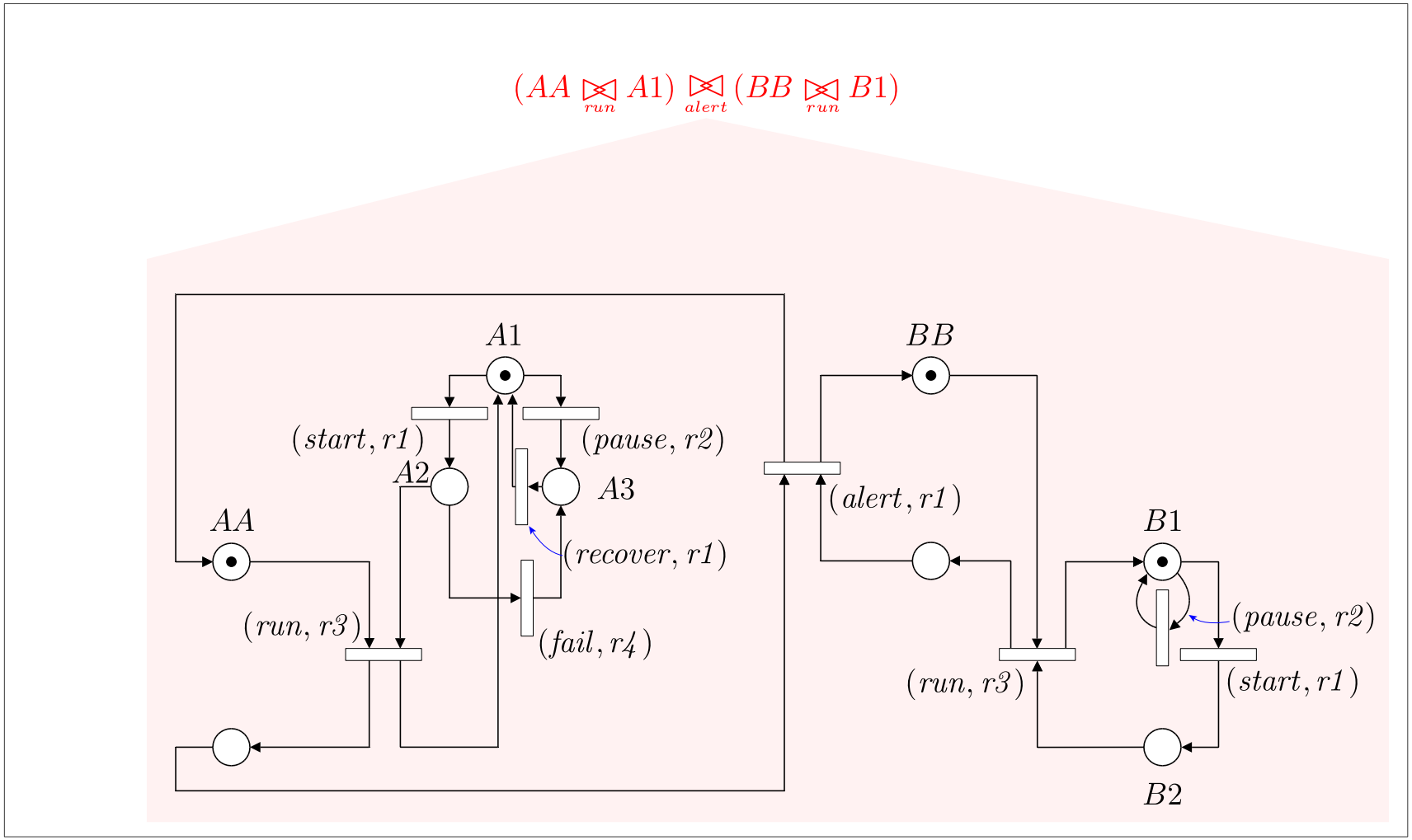
Example ipc Compilation



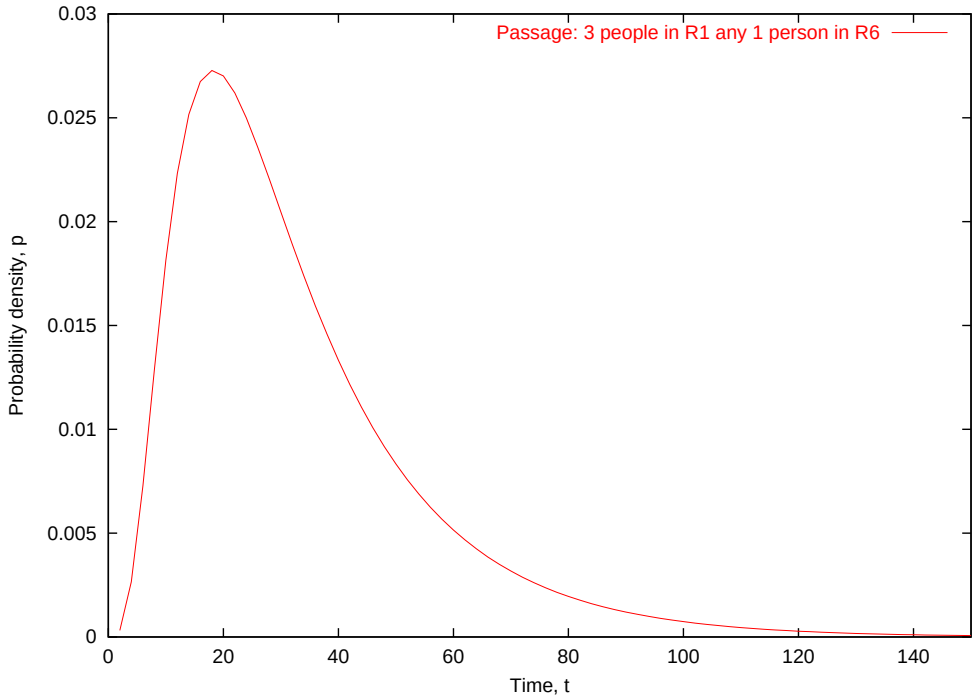
Example ipc Compilation



Example ipc Compilation

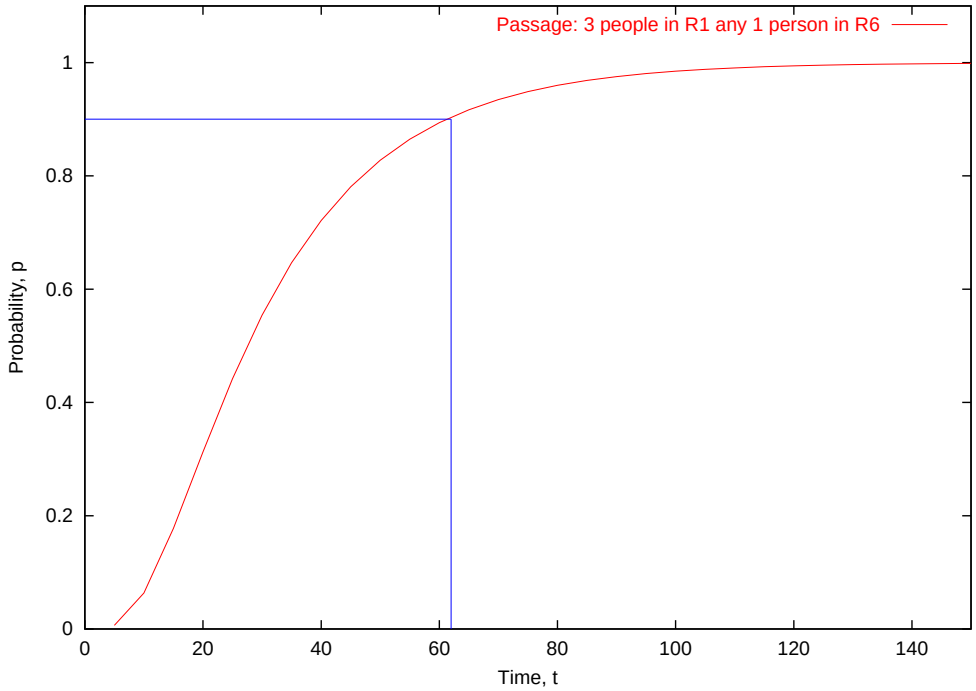


Passage-time PDF Result



- Passage-time distribution from a *start action* to *stop-action*

Passage-time CDF Result



- CDF of passage-time distribution from a *start action* to *stop-action*
  - CDF quantiles $\longrightarrow$ service level agreements

State Space Figures

- PEPA model: passage time/transient analysis – $O(10^8)$ states
- Semi-Markov PEPA: passage time/transient analysis – $O(10^7)$ states

Conclusions

- Future developments will see:
  - improved stochastic probe performance measure specification
  - semi-Markov PEPA support
  - PEPA nets support
  - distributed HYDRA release
  - PEPA rate parameter space sweeping