Performance analysis of Stochastic Process Algebra models using Stochastic Simulation

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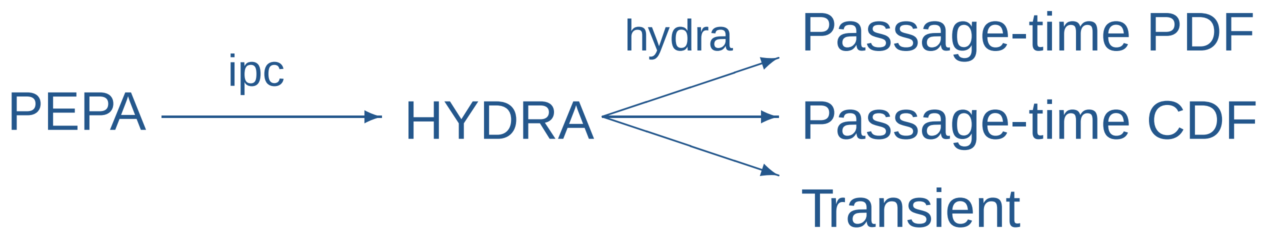
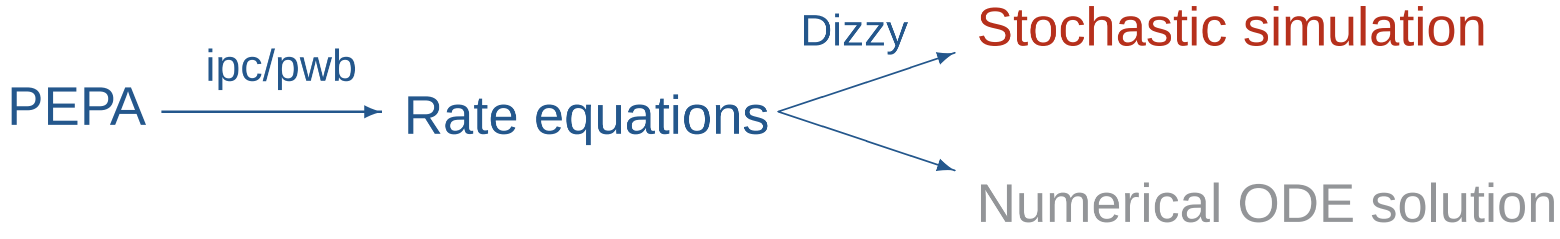
Produced with prosper and LaTeX

The story used to be...



- For state spaces of less than $O(10^9)$
- Very precise probabilistic results

Now the story is...



- For very large state spaces, e.g. $10^{1000}+$ states
- Aggregate deterministic results

Stochastic Process Algebra

PEPA syntax:

$$P \; ::= \; (a, \lambda).P \;\big|\; P + P \;\big|\; P \bowtie_{L} P \;\big|\; P/L \;\big|\; A$$

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- Competitive choice: $P_1 + P_2$

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- Action hiding: P/L

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- Constant label: A

PEPA: Example

$$\mathrm{Sys} \stackrel{\mathrm{def}}{=} (\mathrm{AA} \underset{\{\mathrm{run}\}}{\bowtie} \mathrm{A1}) \underset{\{\mathrm{alert}\}}{\bowtie} (\mathrm{BB} \underset{\{\mathrm{run}\}}{\bowtie} \mathrm{B1})$$

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Types of Analysis

Steady-state and transient analysis in PEPA:

$$
\begin{aligned}
A1 &\stackrel{\text{def}}{=} (start, r_1).A2 + (pause, r_2).A3 \\
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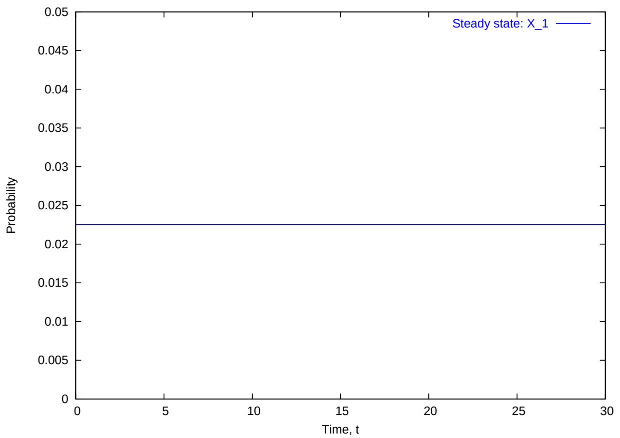
AA &\stackrel{\text{def}}{=} (run, \top).(alert, r_5).AA \\
Sys &\stackrel{\text{def}}{=} AA \underset{\{run\}}{\bowtie} A1
\end{aligned}
$$

$\Rightarrow$



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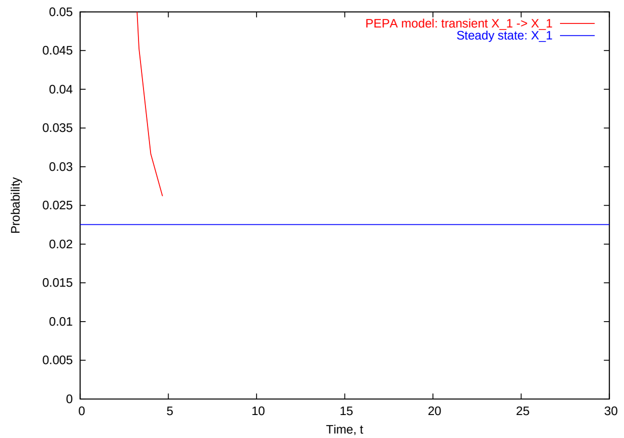

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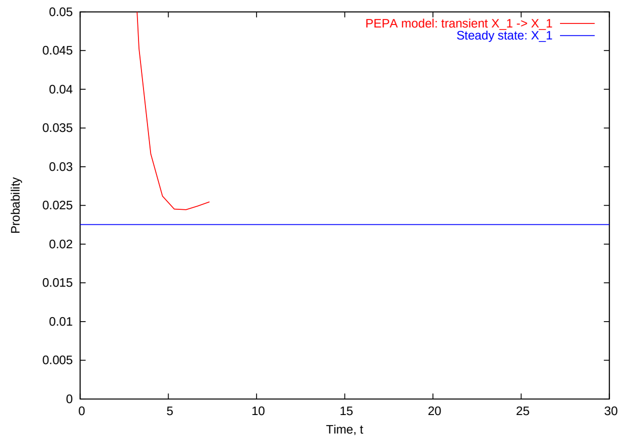

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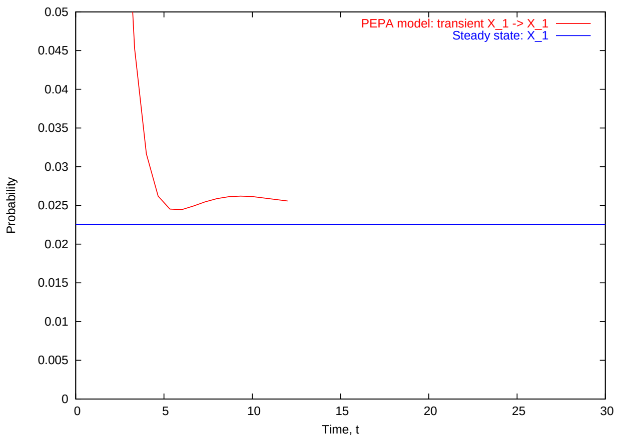

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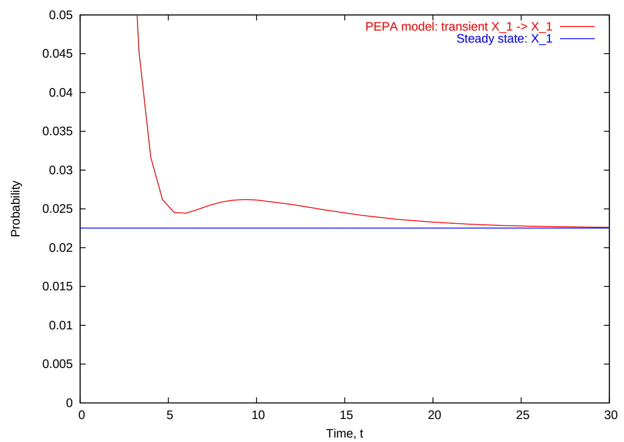

Passage-time Quantiles

Extract a passage-time density from a PEPA model:

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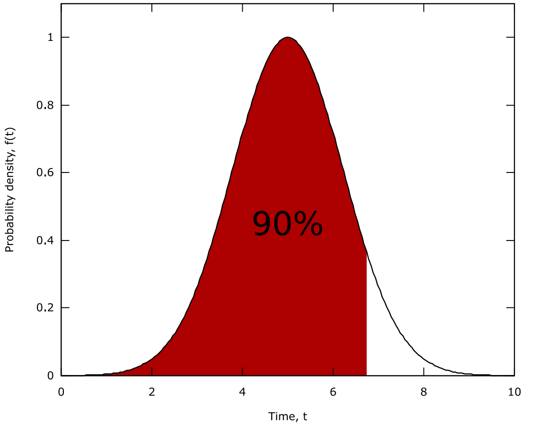

Example of aggregate states

$$
\begin{aligned}
Client &\stackrel{\text{def}}{=} (compute, \top).Client_1 \\
Client_1 &\stackrel{\text{def}}{=} (delay, \mu).Client \\
Server &\stackrel{\text{def}}{=} (compute, \lambda).Server_1 \\
Server_1 &\stackrel{\text{def}}{=} (recover, \nu).Server
\end{aligned}
$$

$$
Sys = (\underbrace{Client \parallel \cdots \parallel Client}_{N}) \underset{\{compute\}}{\bowtie} (\underbrace{Server \parallel \cdots \parallel Server}_{M})
$$

- Cooperating clusters can be represented as tuples

Rate Equation Translation

- Action: $delay$

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$$Client_1 \xrightarrow{n(Client_1)\mu} Client$$

Rate Equation Translation

- Action: $delay$

$$Client_1 \xrightarrow{n(Client_1)\mu} Client$$

- Action: $recover$

$$Server_1 \xrightarrow{n(Server_1)\nu} Server$$

Rate Equation Translation

- Action: $delay$

$$Client_1 \xrightarrow{n(Client_1)\mu} Client$$

- Action: $recover$

$$Server_1 \xrightarrow{n(Server_1)\nu} Server$$

- Action: $compute$

$$Client + Server \xrightarrow{\theta(n(Client))n(Server)\lambda} Client_1 + Server_1$$

where $\theta(x) = 1$ if $x > 0$, else 0.

Why the θ function?

- There are N client cpts enabling a $compute$ action
- There are M server cpts enabling a $compute$ action
- Overall $compute$ rate is:

$$r_{compute}(Sys) = \min(N\top, M\lambda)$$

Why the θ function?

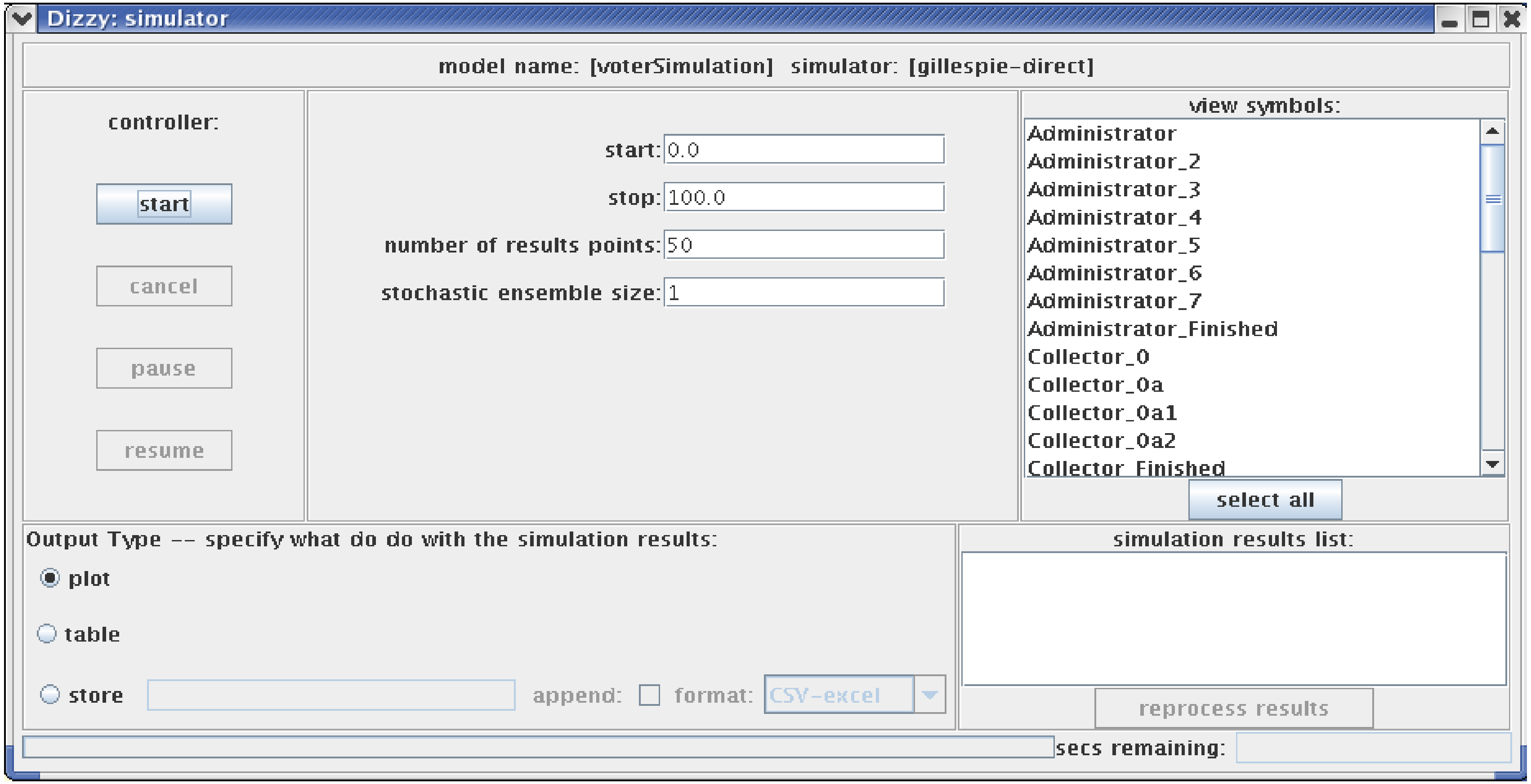
- There are N client cpts enabling a $compute$ action
- There are M server cpts enabling a $compute$ action
- Overall $compute$ rate is:

$$r_{compute}(Sys) = \min(N\top, M\lambda)$$

- If $N = 0$ then overall rate is 0, hence:

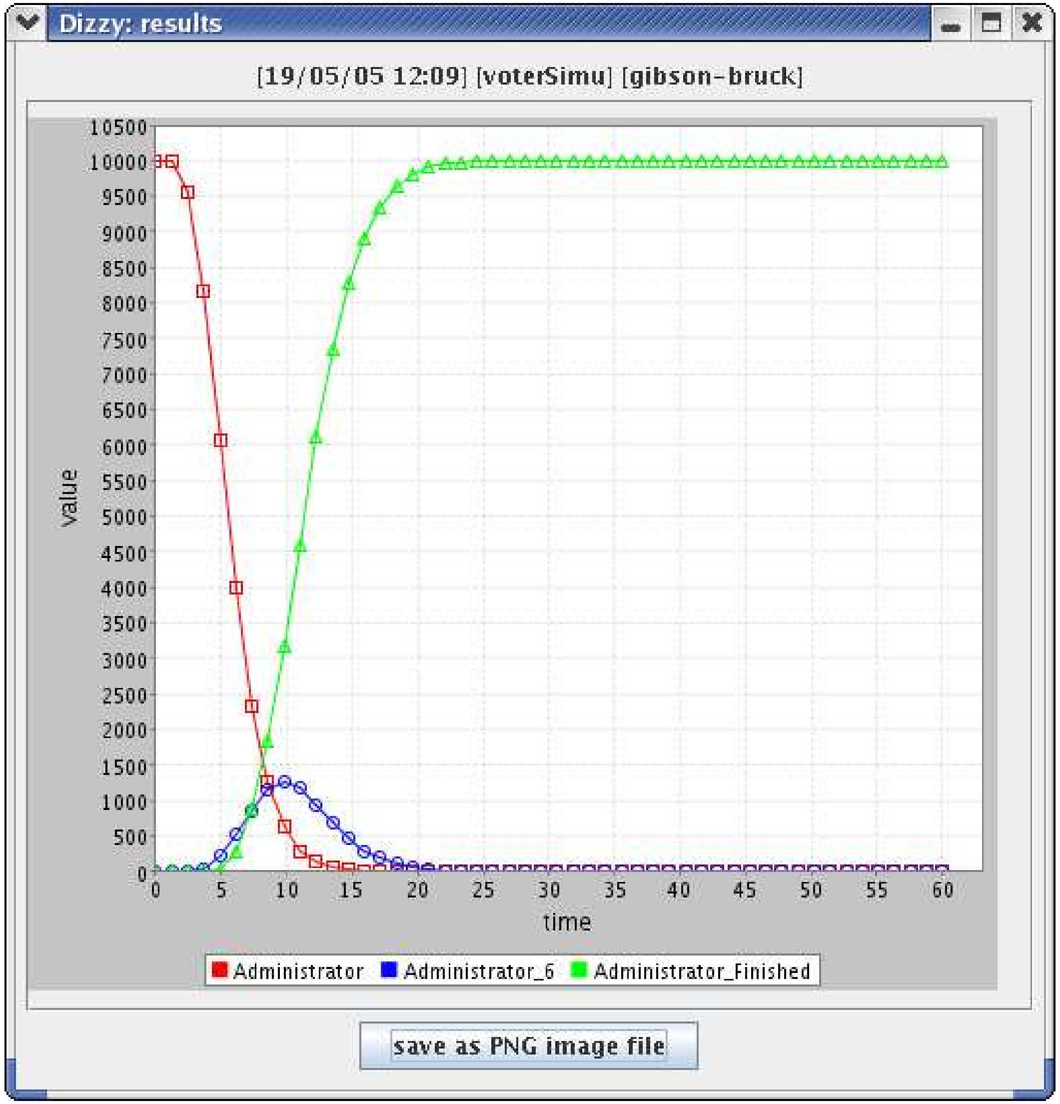
$$r_{compute}(Sys) = \theta(N)\, M\lambda$$

Dizzy setup



Dizzy: simulator
model name: [voterSimulation] simulator: [gillespie-direct]
controller:
start
cancel
pause
resume
start: 0.0
stop: 100.0
number of results points: 50
stochastic ensemble size: 1
view symbols:
Administrator
Administrator_2
Administrator_3
Administrator_4
Administrator_5
Administrator_6
Administrator_7
Administrator_Finished
Collector_0
Collector_0a
Collector_0a1
Collector_0a2
Collector_Finished
select all
Output Type -- specify what do do with the simulation results:
plot
table
store
append:
format:
CSV-excel
simulation results list:
reprocess results
secs remaining:

Dizzy simulation



Voter example

$$Election\_Preparation \underset{\mathcal{L}}{\bowtie} Electoral\_Personae$$

$$Electoral\_Personae \stackrel{\text{def}}{=} Voter0[N] \underset{\mathcal{M}}{\bowtie} Electoral\_App$$

$$Electoral\_App \stackrel{\text{def}}{=} Collector\_0[N] \parallel Counter\_1[N] \parallel Administrator[N]$$

Early voter description

$$Voter0 \stackrel{\text{def}}{=} (choose, c_1).Voter0\_1$$

$$Voter0\_1 \stackrel{\text{def}}{=} (bitcommit, b_1).Voter0\_2$$

$$Voter0\_2 \stackrel{\text{def}}{=} (blind_1, b_2).Voter0\_3$$

$$Voter0\_3 \stackrel{\text{def}}{=} (blind_2, b_3).Voter0\_4$$

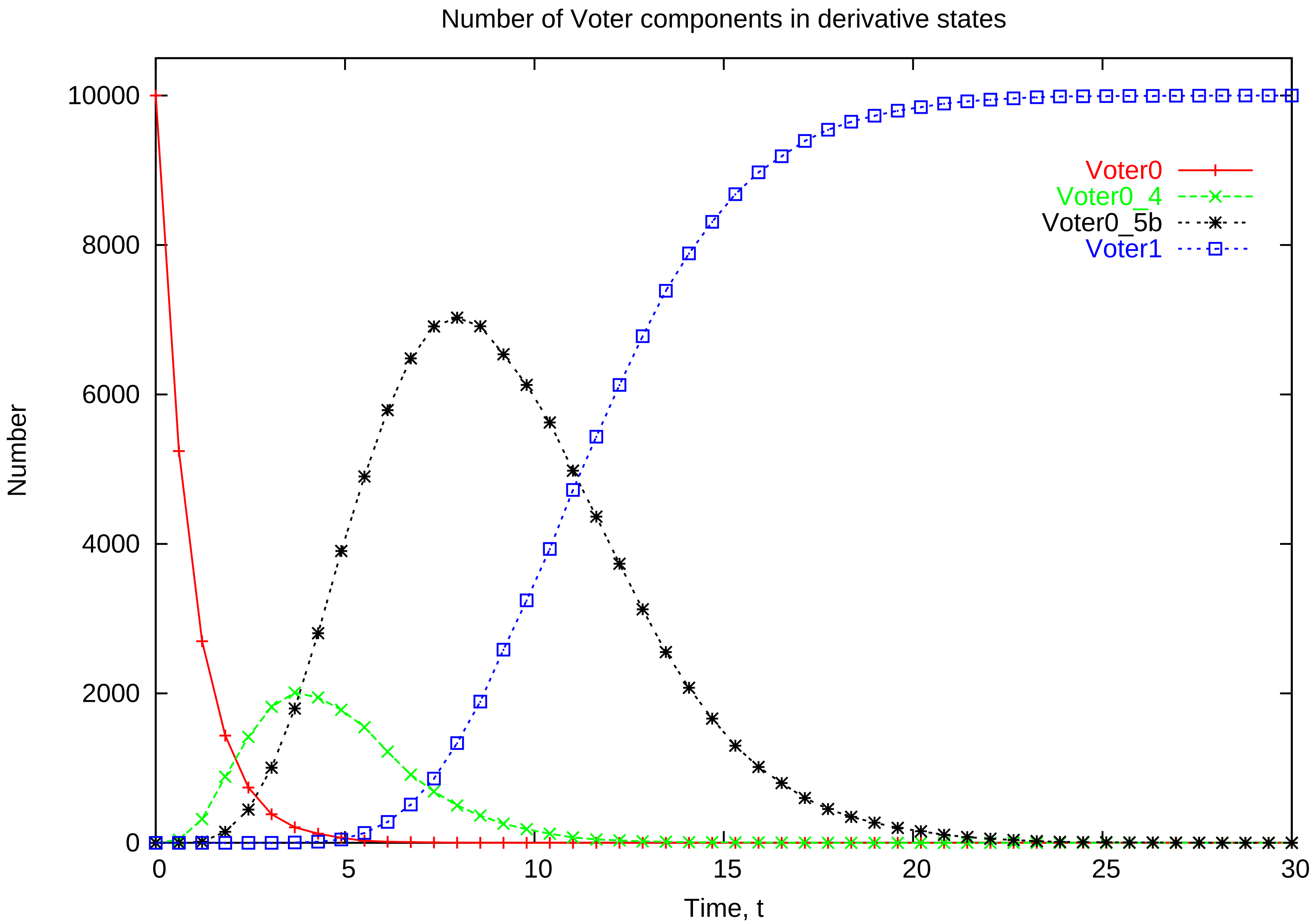
$$Voter0\_4 \stackrel{\text{def}}{=} (voter\_sign, s_1).Voter0\_5$$

$$Voter0\_5 \stackrel{\text{def}}{=} (sendA, s_2).Voter0\_5b$$

$$Voter0\_5b \stackrel{\text{def}}{=} (sendV, \top).Voter1$$

$$Voter1 \stackrel{\text{def}}{=} (unblind_1, u_1).Voter1\_1$$

Voter: early stage



High-level voter description

$$Voter0 \stackrel{\text{def}}{=} (choose, c_1) \ldots (sendV, \top).Voter1$$

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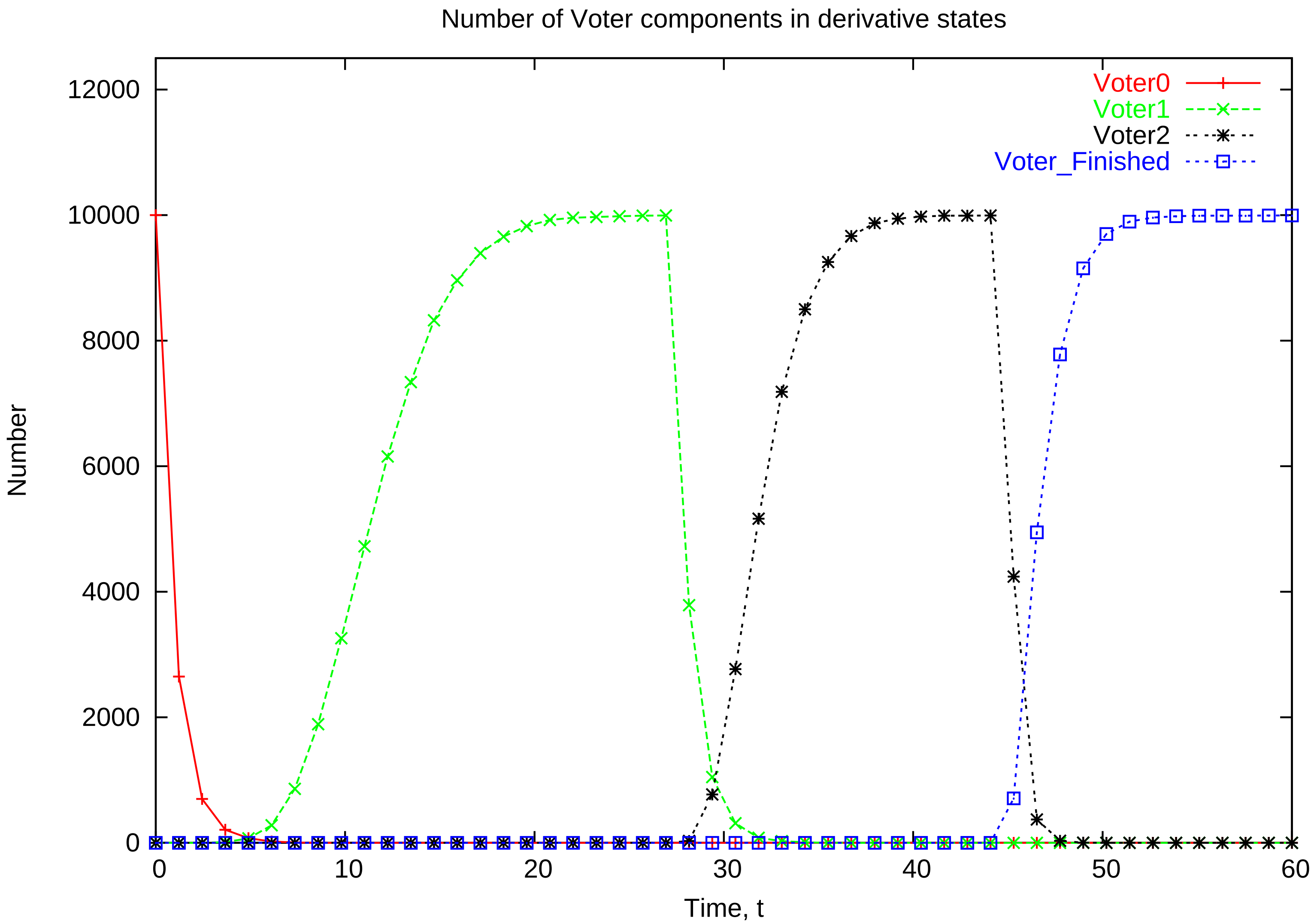
$$Voter2 \stackrel{\text{def}}{=} (check, p \times c_4) \ldots (sendCo, s_7).Voter\_Finished$$

Voter: lifecycle



High-level Election description

$$\mathit{Election\_Preparation}$$

$$\stackrel{\text{def}}{=} (sendV, \top).\mathit{Election\_Preparation} + \cdots$$

$$+ (publishA, er).\mathit{Election\_Voting}$$

High-level Election description

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$$Election\_Voting$$

$$\stackrel{\text{def}}{=} (sendC, \top).Election\_Voting + \cdots$$

$$+ (publishC, er).Election\_Counting$$

High-level Election description

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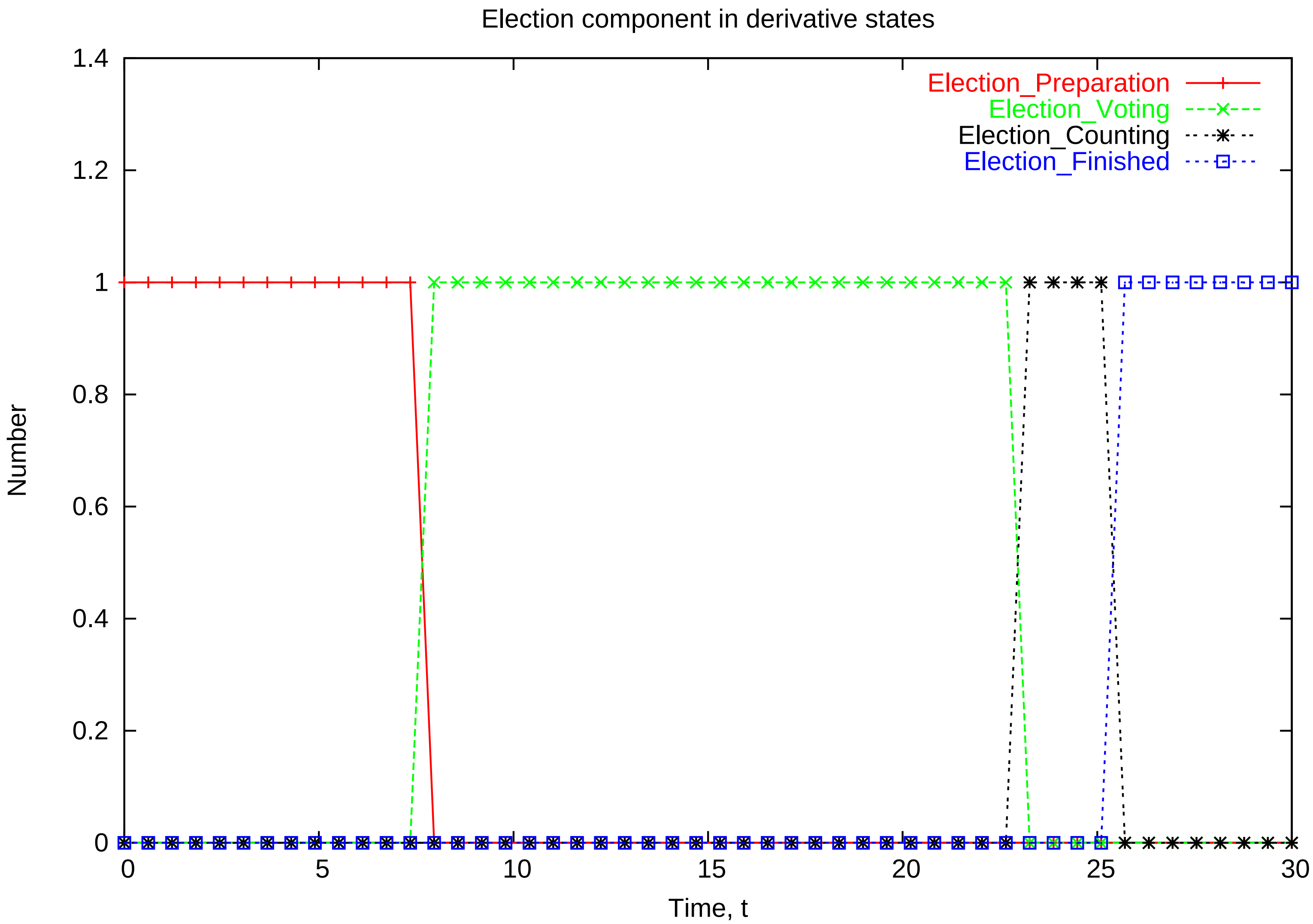
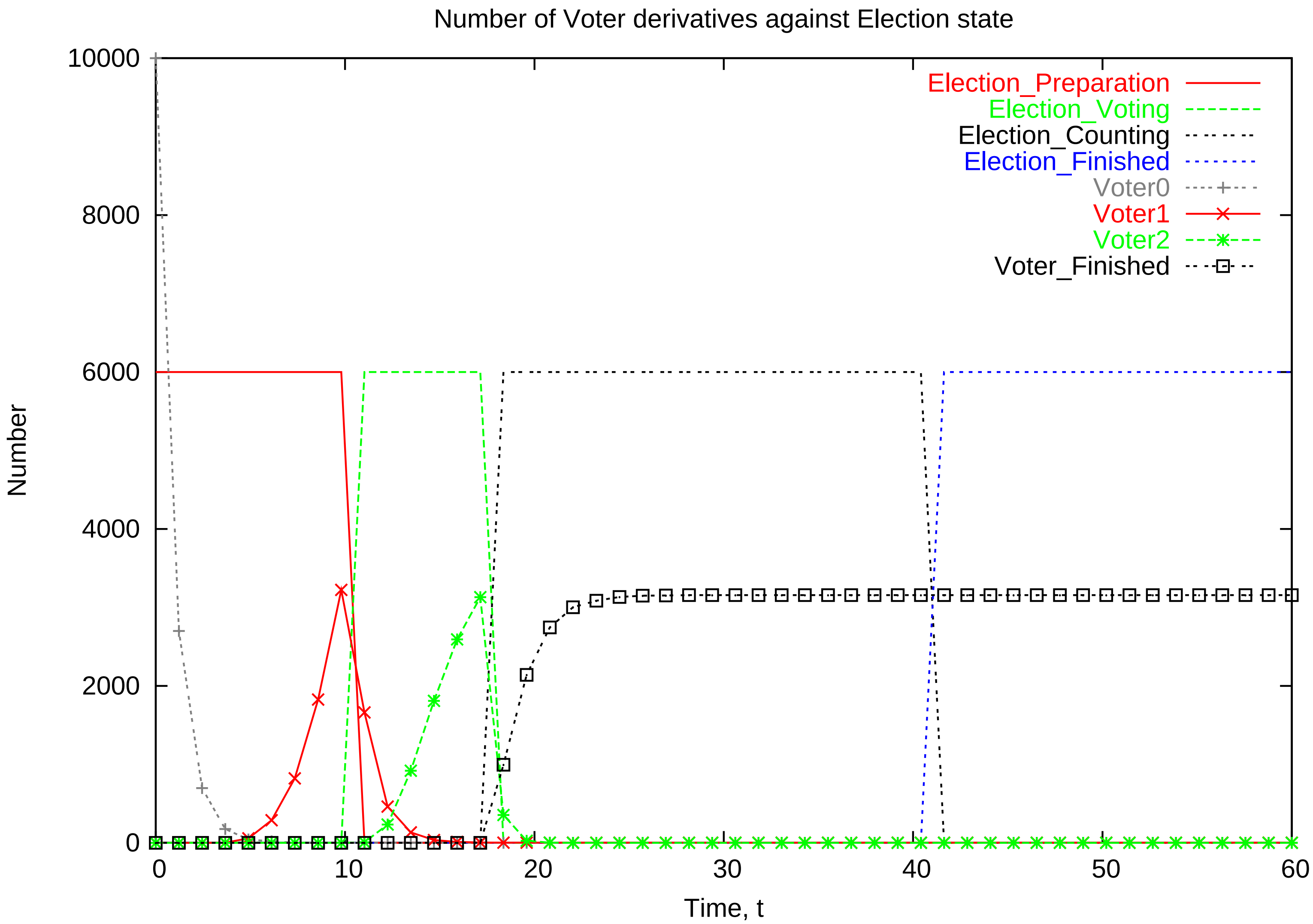
&\mathit{Election\_Counting} \\
&\quad \stackrel{\text{def}}{=} \quad (sendCo, \top).\mathit{Election\_Counting} + \cdots \\
&\qquad\quad + (final\_publish, er).\mathit{Election\_Finished}
\end{aligned}
$$

Election: population of 1



Election + Voter: interaction



Conlcusion

- Novel simulation techniques based on chemical rate equations
- Orders of magnitude larger state spaces can be analysed
- Complexity of simulation method (Gibson-Bruck) is $O(\log n)$ where n is number of rate equations