Proof that $\sqrt{2}$ is irrational

- An irrational number is one that cannot be expressed as $\frac{m}{n}$ where $m \in \mathbb{Z}, n \in \mathbb{Z}^+$ and share no factors
- Proof by contradiction:
  - Assume that $\sqrt{2} = \frac{m}{n}$ where m and n share no factors

  $\Rightarrow$ $m = \sqrt{2}n$, thus $m^2 = 2n^2$

  $\Rightarrow$ m^2 is even and so m must be even too
  - As m is even, let $m = 2p$, we get $4p^2 = 2n^2$, thus $n^2 = 2p^2$ implies that n^2 and n are even also
  - Thus m and n do share a common factor of 2. This contradicts our initial assumption and proves that $\sqrt{2}$ is irrational