How to find $\sum_{r=1}^{n} r^2$

$$\sum_{r=1}^{n} r = \frac{n}{2}(n+1) \quad \text{but} \quad \sum_{r=1}^{n} r^2 = ?$$

- Consider $\sum_{r=1}^{n}((r+1)^3 - r^3)$

$$= \quad ((n+1)^3 - n^3) + (n^3 - (n-1)^3) + \cdots + (3^3 - 2^3) + (2^3 - 1^3)$$

$$= \quad (n+1)^3 - 1^3 \quad (\dagger)$$

- Also $\sum_{r=1}^{n}((r+1)^3 - r^3) = \sum_{r=1}^{n}(3r^2 + 3r + 1)$

$$= \quad 3\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r + n = 3\sum_{r=1}^{n} r^2 + \frac{3n}{2}(n+1) + n \quad (*)$$

- Combining $(\dagger)$ and $(*)$: $\sum_{r=1}^{n} r^2 = \frac{n}{6}(2n+1)(n+1)$
- Can the same trick be applied to find $\sum_{r=1}^{n} r^3$?