Performance Analysis 11: Open queueing networks

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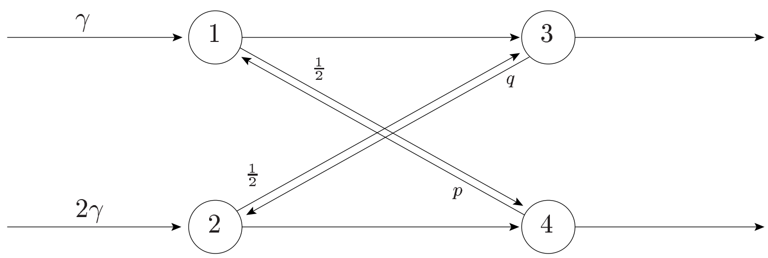

Figure 1: Open queueing network with 4 nodes.

1. (a) Construct a matrix equation for the aggregate arrival rates (the traffic equations) in Fig. 1
   (b) Solve the traffic equations from part (a)
2. Given that the service rate of each node in Fig. 1 is $\mu_i = \frac{3\gamma}{2-p+q}$, $1 \leq i \leq 4$:
   (a) Use Jackson's theorem to write down an expression for the steady state probability for each state in the queueing network in Fig. 1
   (b) Hence find the steady-state probability that there are no customers in the network
   (c) Using Little's law, find an expression for the total response time in the system
   (d) Find the average waiting time from entering node 4 to leaving the system completely

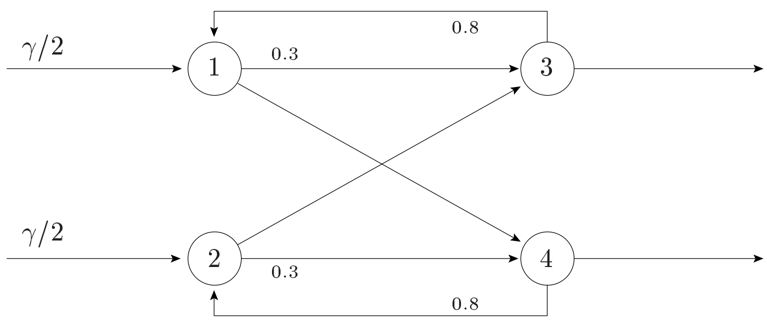

Figure 2: A symmetric open queueing network with 4 nodes.

3. In Fig. 2, the service rates for the queueing nodes are, $\mu_1 = \mu_2 = 1$ and $\mu_3 = \mu_4 = 0.5$

   (a) Using conditions for stability in the network, find the largest lower bound on the total average response time, W, through the system?

   (b) Find a condition on the arrival rate γ for the response time, $W < 20$.