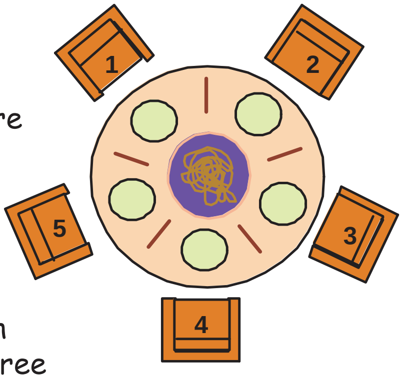
Resource Allocation - Dining Philosophers

Five philosophers sit around a circular table. Each philosopher spends his life alternately thinking and eating. In the centre of the table is a large bowl of spaghetti. A philosopher needs two forks to eat a helping of spaghetti.

One fork is placed between each pair of philosophers and they agree that each will only use the fork to his immediate right and left.



Lynch - Chapter 11

Dining Philosophers - **Properties**

Safety:

Freedom from deadlock

Mutual exclusion

> A philosopher may not eat until he has exclusive use of the two forks adjacent to him.

```
assert EXCLUSION = forall [i:1..N]
                   []!(EATING[i] &&
                       EATING[(i%N)+1])
```

OK?

Liveness:

Freedom from starvation - for individual and all

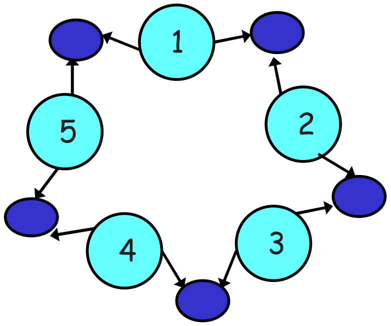
```
assert SOMEEAT = exists [i:1..N] []<> EATING[i]
assert NoSTARVATION = forall [i:1..N] [] <> EATING[i]
```

Lynch - Chapter 10

Naïve algorithm

```
Philospher (i): Loop
    think; sitdown;
        snd get to right fork;
        rcv ok;
        snd get to left fork;
        rcv ok;
                eat;
        snd put to right fork;
        snd put to left fork;
    arise; ...
```

```
Fork: Loop
        {rcv get from right phil;
        snd ok to right phil}
      or
        {rcv get from left phil;
        snd ok to left phil}
```



Properties?
Safety?
Liveness?

LTSA demo

Impossibility Result for **Symmetric** Algorithm

Theorem: *There is no deterministic, distributed and symmetric solution to the Dining Philosophers Problem.*

Informal Proof:

Assume there is a system **A** which solves the problem for n processes.

Consider an execution of A that begins with all processes in the *same initial state*. Each process proceeds "round-robin" by executing a step at a time.

By induction on the number r of round-robin rounds, all processes are in identical states after r rounds. Therefore if **any** process is able to eat (liveness property), then **all** process will be able to eat. This violates the exclusion property.

Impossibility Result for **Symmetric** Algorithm

How do we overcome this?

Algorithms must have the following basic properties:

1. **Distinguishability**

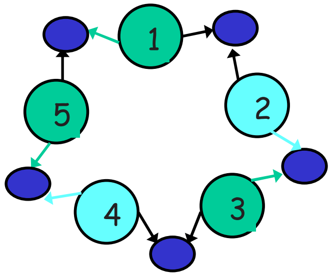
   In every state of the system, at least one process in every set of conflicting (competing) processes must be distinguishable from the others in the set (asymmetry).

2. **Fairness**

   Conflicts should be resolved without detriment to a particular process.

Asymmetric algorithm - using IDs

```
Philospher (i):
…
   Even(i): snd get to left fork first,
   then right;
  or
   Odd(i): snd get to right fork first,
   then left;
…
```



Distinguishability?

id (odd and even)

Fairness?

can impose different conditions

Asymmetric algorithm - using IDs

Properties?

Safety:

Freedom from deadlock

EXCLUSION

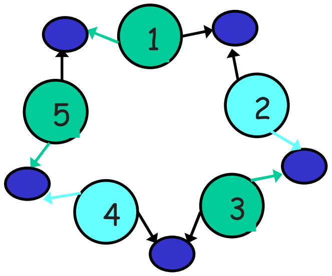
Liveness:

STARVATION-FREEDOM

Strong fairness?

Weak fairness?

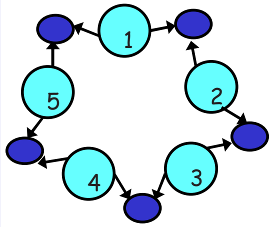
No fairness?



demo

Probabilistic algorithm -

```
Philospher (i):
   Loop {…
      gotforks:=False;
      While !gotforks
         {Random choice:
               getforks(left, right)
            or getforks(right,left)}
      eat;
      …}

getforks(first,second):
      {snd wait to first fork: rcv ok;
       snd get to second fork; rcv m;
      if m!=ok snd put to first fork:
      else gotforks:=True
      }
```

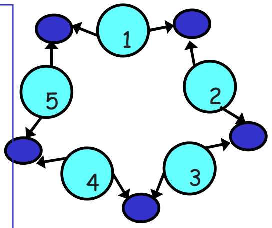


Identical Philosophers, but randomly choose which fork to take first, and replace it if unable to also take the second fork.

Lehmann and Rabin

Probabilistic algorithm

```
Fork:
    Loop {
        rcv get or wait from first phil;
        snd ok;
        loop {   rcv put; break
              or rcv get from second phil;
                   snd !ok
        }
    }
```



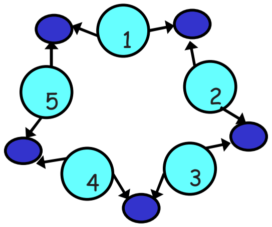
Forks refuse requests if the fork is already taken.

Distinguishability?
identical yet probabilistic to break the symmetry.

Fairness?
different conditions

Probabilistic algorithm

Properties?



Safety:
Freedom from deadlock
EXCLUSION

Liveness:

STARVATION-FREEDOM

Strong fairness?
Weak fairness?
No fairness?

- *What if philosophers don't replace forks, but retain them, as before?*
-
- *Can we improve fairness of allocation? (eg. cf. Peterson)*

demo

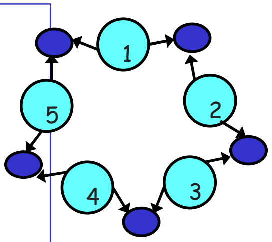
Probabilistic algorithm

```
Violation of LTL property: @WEAK_NoSTARVATION
Trace to terminal set of states:
        phil.1.think
        phil.1.sitdown
        tau
        phil.1.left.wait
        phil.1.right.get.1
        phil.1.eat          EATING.1
        phil.2.think        EATING.1
        phil.3.think        EATING.1
        phil.4.think        EATING.1
        phil.4.sitdown      EATING.1
        tau                 EATING.1
        phil.4.right.wait  EATING.1
Cycle in terminal set:
        phil.1.left.put
        phil.1.right.put
        phil.1.arise
        phil.1.think
        phil.1.sitdown
        tau
        phil.2.sitdown
        tau
        phil.2.left.wait
```

```
    phil.2.right.get.1
    phil.2.eat          EATING.2
    phil.2.left.put
    phil.2.right.put
    phil.1.left.wait
    phil.1.right.get.1
    phil.1.eat          EATING.1
    phil.2.arise        EATING.1
    phil.2.think        EATING.1
    phil.3.sitdown      EATING.1
    tau                 EATING.1
    phil.4.left.get.0      EATING.1
    phil.4.right.put       EATING.1
    tau                 EATING.1
    phil.3.left.wait       EATING.1
    phil.3.right.get.1    EATING.1
    phil.3.eat          EATING.1 && EATING.3
    phil.3.left.put     EATING.1
    phil.3.right.put    EATING.1
    phil.3.arise        EATING.1
    phil.3.think        EATING.1
    phil.4.right.wait      EATING.1
LTL Property Check in: 2516ms
```

Probabilistic (**courteous**) algorithm

```
Philospher (i):
Loop {...
    Set flags in left and right forks;
        gotforks:=False;
        While !gotforks
            {Random choice:
                    getforks(left, right)
                or getforks(right,left)}
        eat;
        ...}

getforks(first,second):
        {snd wait to first fork; rcv ok;
        snd get to second fork; rcv m;
        if m!=ok snd replace to first fork
        else gotforks:=True
        }
```



Philosophers set flags to indicate hunger, and behave as probabilistic philosophers.

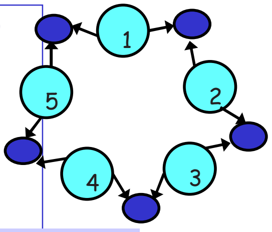
Lehmann and Rabin

Probabilistic (**courteous**) algorithm

```
Fork: (initially flags unset and turn=neutral)
    Loop
        set {left/right} flag whenever rcv
        setflag from phil;

        Snd ok to wait req iff available and
            (only one flag set or
             turn=neutral       or
             turn=philosopher side).
        Snd ok to get req iff available
            else snd !ok.

        reset {left/right} flag and turn to
        other side when rcv put.

        {null} when rcv replace;
```

-> ! available

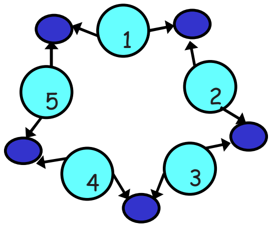
-> available



Lehmann and Rabin

Probabilistic (**courteous**) algorithm

Properties?



Safety:

Freedom from deadlock
EXCLUSION

Liveness:

STARVATION-FREEDOM

Strong fairness?
Weak fairness?
No fairness?

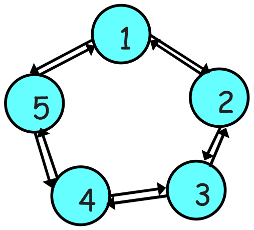
Probability Vs Absolute certainty?
(practice Vs theory?)

demo

Hygienic Philosophers algorithm

Philosophers communicate directly with one another, passing forks and request tokens between them.

- the algorithm maintains an ***acyclic precedence graph*** which ensures freedom from deadlock, exclusion and starvation.



Chandy and Misra

Hygienic Philosophers algorithm

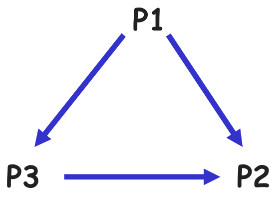
Clean forks are passed between philosophers

- A fork is either **clean** or **dirty**.
  A fork being used to eat with is dirty and remains dirty until it is cleaned. A clean fork remains clean until it is used for eating. A philosopher cleans a fork when passing it (he is hygienic).
- An **eating** philosopher does not satisfy requests for forks until he has finished eating.
- When **not eating**, philosophers defer requests for forks that are clean and satisfy requests for forks that are dirty.

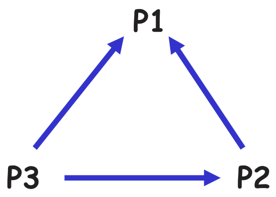
Chandy and Misra

Hygienic Philosophers algorithm

Preserve a precedence graph, where an edge from P1 to P2 indicates that P1 has precedence over P2.



P1 hungry and has precedence



P1 eating

Depth for each Pi?

- Pi has **precedence** over Pj iff
  (i) Pi has the fork and it is clean
  (ii) Pj has the fork and it is dirty.
  (iii) the fork is in transit from Pi to Pj
- **Depth**
  Maximum number of edges from a process with no predecessors, which has depth 0.

Hygienic Philosophers algorithm

Distinguishability is provided by acyclicity. It has been proven that…

- ***An acyclic graph ensures no starvation or deadlock.***
  At least one philosopher has precedence over both his neighbours. He eventually receives each (clean) fork and retains it until he eats, since (by precedence) his requests are eventually satisfied by a finishing or thinking philosopher yielding to his request.
- ***if initially all forks are dirty and the graph is acyclic, then it remains acyclic.***
  The direction of an arc only changes when a philosopher starts eating, which results in both edges being simultaneously directed towards him.

Fairness:

- ***A process in conflict will rise to the top (to zero depth).***
  Each philosopher with precedence - at zero depth - redirects both arcs so as to yield precedence to its neighbours.

Hygienic Philosophers algorithm

messages:

forktoken$_f$:	passes fork f to neighbour which shares f (f can take the value left or right)
reqtoken$_f$:	passes request token for fork f to neighbour

boolean variables:

fork(f):	philosopher holds fork f
reqf(f):	philosopher holds request token for fork f
dirty(f):	fork f is at philosopher and is dirty
thinking/hungry/eating:	state of philosopher

Initialisation:

1) all forks are dirty
2) forks distributed among philosophers such that the precedence graph is acyclic.
3) if u and v are neighbours then either u holds the fork and v the request token or vice versa.

Hygienic Philosophers algorithm

The algorithm for each philosopher is described as a set of rules guard=>action which form a single guarded command.

1. *Requesting a fork f:*
   hungry,reqf(f),~fork(f) => SEND(reqtoken$_f$); reqf(f):=false
2. *Releasing a fork f:*
   ~eating,reqf(f),dirty(f) => SEND(forktoken$_f$)
   dirty(f):=false; fork(f):=false
3. *Receiving a request token for f:*
   receive(reqtoken$_f$) => reqf(f):=true
4. *Receiving a fork token for f:*
   receive(forktoken$_f$) => fork(f):=true {~dirty(f)}
5. *Philosopher hungry to eating transition:*
   hungry,fork(left),fork(right), (~reqf(f) or ~dirty(f)) =>
   eating:=true; hungry:=false; dirty(left):=true; dirty(right):=true;
6. *Philosopher eating to thinking transition:*
   eating,eating time expired => thinking:=true; eating:=false
7. *Philosopher thinking to hungry transition*
   thinking,thinking time expired => hungry:=true; thinking:=false

Hygienic Philosophers

Properties?

Safety:

Freedom from deadlock

EXCLUSION

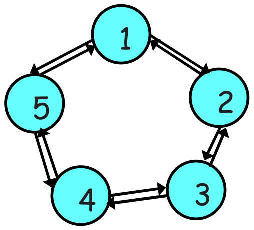
Liveness:

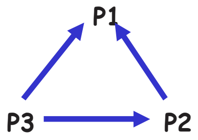
STARVATION-FREEDOM

Strong fairness?

Weak fairness?

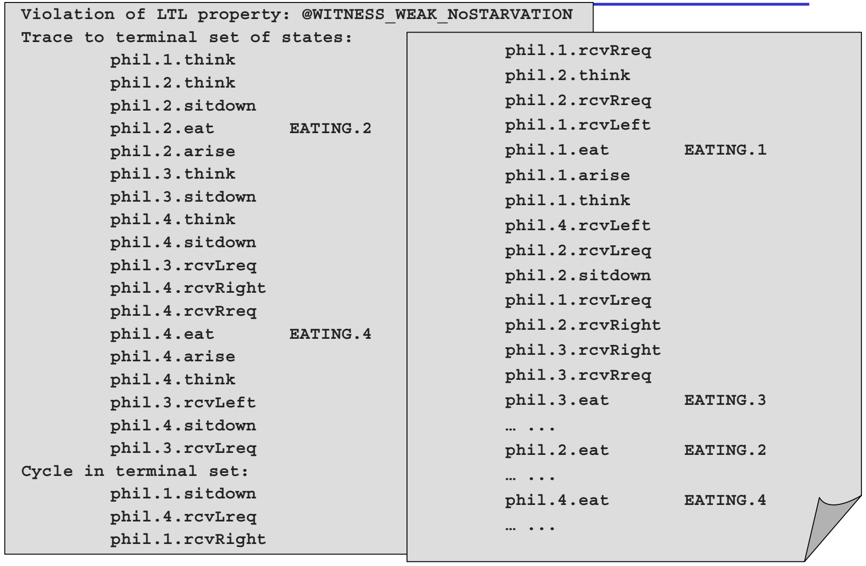
No fairness?





demo

Hygienic Philosophers



```
Violation of LTL property: @WITNESS_WEAK_NoSTARVATION
Trace to terminal set of states:
        phil.1.think
        phil.2.think
        phil.2.sitdown
        phil.2.eat          EATING.2
        phil.2.arise
        phil.3.think
        phil.3.sitdown
        phil.4.think
        phil.4.sitdown
        phil.3.rcvLreq
        phil.4.rcvRight
        phil.4.rcvRreq
        phil.4.eat          EATING.4
        phil.4.arise
        phil.4.think
        phil.3.rcvLeft
        phil.4.sitdown
        phil.3.rcvLreq
Cycle in terminal set:
        phil.1.sitdown
        phil.4.rcvLreq
        phil.1.rcvRight
```

```
        phil.1.rcvRreq
        phil.2.think
        phil.2.rcvRreq
        phil.1.rcvLeft
        phil.1.eat          EATING.1
        phil.1.arise
        phil.1.think
        phil.4.rcvLeft
        phil.2.rcvLreq
        phil.2.sitdown
        phil.1.rcvLreq
        phil.2.rcvRight
        phil.3.rcvRight
        phil.3.rcvRreq
        phil.3.eat          EATING.3
        … ...
        phil.2.eat          EATING.2
        … ...
        phil.4.eat          EATING.4
        … ...
```

Notes

This section has introduced asynchronous resource algorithms which must avoid deadlock, provide exclusion and prevent starvation.

Symmetry, Distinguishability and Fairness are important properties.

Probabilistic algorithms can provide a sound practical means for the avoiding deadlock and starvation, with probability 1.

Distributed precedence provides an asymmetric state with symmetric code, distinguishability and fairness.