Imperial College London

The action language $\mathcal{C}+$

'Nonmonotonic causal theories'
(Outline)

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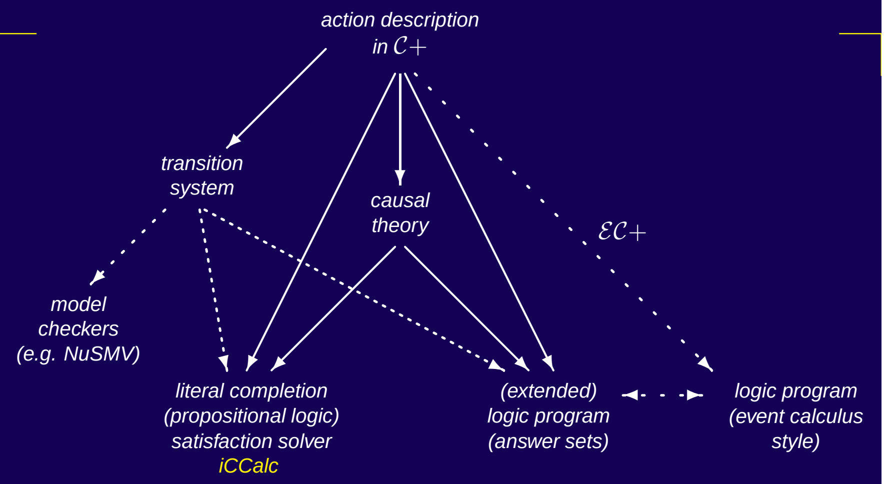
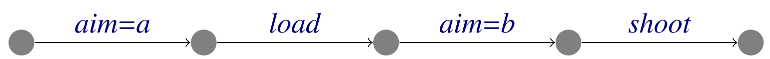
The action language $\mathcal{C}+$



Example



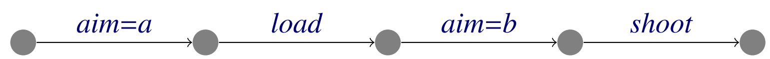
$alive(a)$
$alive(b)$
$\neg loaded$

$$target[i{+}1]{=}X \;\Leftarrow\; aim[i]{=}X$$

$target{=}X$ after $aim{=}X$

$aim{=}X$ causes $target{=}X$

Example



$alive(a)$
$alive(b)$
$\neg loaded$

$target{=}a$ $target{=}b$

$$target[i{+}1]{=}X \;\Leftarrow\; aim[i]{=}X$$

$target{=}X$ after $aim{=}X$

$aim{=}X$ causes $target{=}X$

Example

$$\bullet \xrightarrow{\ aim{=}a\ } \bullet \xrightarrow{\ load\ } \bullet \xrightarrow{\ aim{=}b\ } \bullet \xrightarrow{\ shoot\ } \bullet$$

$alive(a)$
$alive(b)$
$\neg loaded$

$alive(a)[0]$ $\quad$ $alive(b)[0]$ $\quad$ $\neg loaded[0]$

$aim[0]{=}a$ $\quad$ $load[1]$ $\quad$ $aim[2]{=}b$ $\quad$ $shoot[3]$

Example

$$\bullet \xrightarrow{\ aim{=}a\ } \bullet \xrightarrow{\ load\ } \bullet \xrightarrow{\ aim{=}b\ } \bullet \xrightarrow{\ shoot\ } \bullet$$

$alive(a)$
$alive(b)$
$\neg loaded$

$target{=}a$ $\qquad$ $target{=}b$

$$loaded[i{+}1] \ \Leftarrow\ load[i]$$

loaded after *load*

load causes *loaded*

Example

$$\bullet \xrightarrow{\ aim{=}a\ } \bullet \xrightarrow{\ load\ } \bullet \xrightarrow{\ aim{=}b\ } \bullet \xrightarrow{\ shoot\ } \bullet$$

$alive(a)$
$alive(b)$
$\neg loaded$ $\qquad$ $loaded$

$target{=}a$ $\qquad$ $target{=}b$

$$loaded[i{+}1] \ \Leftarrow\ load[i]$$

loaded after *load*

load causes *loaded*

Example

$$\bullet \xrightarrow{\ aim{=}a\ } \bullet \xrightarrow{\ load\ } \bullet \xrightarrow{\ aim{=}b\ } \bullet \xrightarrow{\ shoot\ } \bullet$$

$alive(a)$
$alive(b)$
$\neg loaded$ $\qquad$ $loaded$ $\qquad$ $\neg loaded$

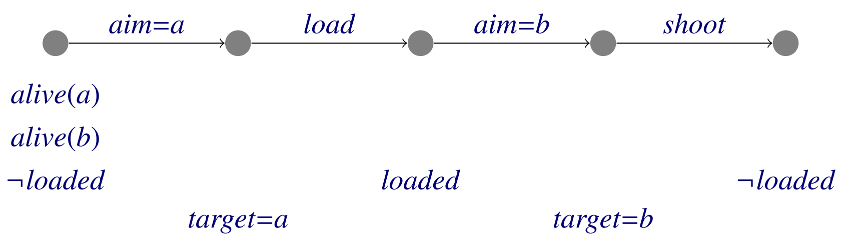
$target{=}a$ $\qquad$ $target{=}b$

$$\neg loaded[i{+}1] \ \Leftarrow\ shoot[i]$$

$\neg$*loaded* after *shoot*

shoot causes $\neg$*loaded*

Example



$$\neg alive(X)[i{+}1] \;\Leftarrow\; shoot[i] \wedge loaded[i] \wedge target[i]{=}X$$

$\neg alive(X)$ after $shoot \wedge loaded \wedge target{=}X$

$shoot$ causes $\neg alive(X)$ if $loaded \wedge target{=}X$

Persistence

$$loaded[i{+}1] \;\leftsquigarrow\; loaded[i]$$

First guess:

$$loaded[i{+}1] \;\leftarrow\; loaded[i],\ \text{not}\,\neg loaded[i{+}1]$$

$$\frac{loaded[i] : loaded[i{+}1]}{loaded[i{+}1]}$$

'Nonmonotonic causal theories':

$$loaded[i{+}1] \;\leftarrow\; \text{not}\,\neg loaded[i],\ \text{not}\,\neg loaded[i{+}1]$$

$$\frac{: loaded[i],\ loaded[i{+}1]}{loaded[i{+}1]}$$

'Nonmonotonic causal theories'

$$\frac{: loaded[i],\ loaded[i{+}1]}{loaded[i{+}1]}$$

$$loaded[i{+}1] \;\leftarrow\; \text{not}\,\neg loaded[i],\ \text{not}\,\neg loaded[i{+}1]$$

$$loaded[i{+}1] \;\Leftarrow\; loaded[i{+}1] \wedge loaded[i]$$

That is *sometimes* equivalent to:

$$\frac{loaded[i] : loaded[i{+}1]}{loaded[i{+}1]}$$

$$loaded[i{+}1] \;\leftarrow\; loaded[i],\ \text{not}\,\neg loaded[i{+}1]$$

'Nonmonotonic causal theories'

$$P \;\Leftarrow\; Q \wedge R \wedge \ldots \wedge S$$

$$\frac{: Q,\ R,\ \ldots,\ S}{P}$$

$$P \;\leftarrow\; \text{not}\,\neg Q,\ \text{not}\,\neg R,\ \ldots,\ \text{not}\,\neg S$$

$$\neg alive(X)[i{+}1] \;\Leftarrow\; shoot[i] \wedge loaded[i] \wedge target[i]{=}X$$

$$\neg alive(X)[i{+}1] \;\leftarrow\; \text{not}\,\neg shoot[i],\ \text{not}\,\neg loaded[i],\ \text{not}\,\neg target[i]{=}X$$

which is *sometimes* equivalent to

$$\neg alive(X)[i{+}1] \;\leftarrow\; shoot[i],\ loaded[i],\ target[i]{=}X$$

‘Fluent dynamic laws’

$$F \text{ if } G \text{ after } H$$
$$F[i{+}1] \;\Leftarrow\; G[i{+}1] \wedge H[i]$$

$$\alpha \text{ causes } F \text{ if } H$$
$$F \text{ if } \top \text{ after } \alpha \wedge H$$
$$F[i{+}1] \;\Leftarrow\; \alpha[i] \wedge H[i]$$

$$\mathit{shoot} \text{ causes } \neg \mathit{alive}(X) \text{ if } \mathit{loaded} \wedge \mathit{target}{=}X$$
$$\neg \mathit{alive}(X) \text{ after } \mathit{shoot} \wedge \mathit{loaded} \wedge \mathit{target}{=}X$$
$$\neg \mathit{alive}(X)[i{+}1] \;\Leftarrow\; \mathit{shoot}[i] \wedge \mathit{loaded}[i] \wedge \mathit{target}[i]{=}X$$

‘Fluent dynamic laws’

$$\text{inertial } \mathit{alive}(X)$$

$$\mathit{alive}(X)[i{+}1] \;\leftsquigarrow\; \mathit{alive}(X)[i]$$
$$\neg \mathit{alive}(X)[i{+}1] \;\leftsquigarrow\; \neg \mathit{alive}(X)[i]$$

$$\mathit{alive}(X)[i{+}1] \;\Leftarrow\; \mathit{alive}(X)[i{+}1] \wedge \mathit{alive}(X)[i]$$
$$\neg \mathit{alive}(X)[i{+}1] \;\Leftarrow\; \neg \mathit{alive}(X)[i{+}1] \wedge \neg \mathit{alive}(X)[i]$$

$$\mathit{alive}(X) \text{ if } \mathit{alive}(X) \text{ after } \mathit{alive}(X) \qquad !!$$
$$\neg \mathit{alive}(X) \text{ if } \neg \mathit{alive}(X) \text{ after } \neg \mathit{alive}(X)$$

‘Static laws’

$$\mathit{happy}(\mathit{mary}) \text{ if } \neg \mathit{alive}(a) \wedge \neg \mathit{alive}(b)$$

$$\mathit{happy}(\mathit{mary})[i] \;\Leftarrow\; \neg \mathit{alive}(a)[i] \wedge \neg \mathit{alive}(b)[i]$$

$$\text{default } \neg \mathit{happy}(X)$$

$$\neg \mathit{happy}(X)[i] \Leftarrow \neg \mathit{happy}(X)[i]$$

$$\neg \mathit{happy}(X) \text{ if } \neg \mathit{happy}(X) \qquad !!$$

How to compute?

Given action description D (and causal theory Γ^D_m).
(m is maximum timestamp: the length of paths/runs/histories of interest)

Assume D (and causal theory Γ^D_m) are definite.
‘Definite’: for every causal rule

$$F \Leftarrow G$$

F is a literal or $\bot$.

How to compute?

Given *definite* action description D (and causal theory Γ^D_m).

Method 1 Translate Γ^D_m to extended logic program and compute its answer sets.
(Detail: not quite. We want *models* not answer sets.
Add $p \leftarrow \mathrm{not}\,\neg p;\ \neg p \leftarrow \mathrm{not}\,p$ for every atom p. A detail.)

How to compute?

Given *definite* action description D (and causal theory Γ^D_m).

Method 2 Construct the (classical) propositional formula

$$comp(\Gamma^D_m)$$

Use a sat-solver to compute the (ordinary, classical) models of $comp(\Gamma^D_m)$.
This is the method used in the 'Causal Calculators' *CCalc* and *iCCalc*.

Literal completion

For a *definite* causal theory Γ, translate to set of (classical) formulas $comp(\Gamma)$:

$$\left.\begin{array}{c} F \Leftarrow G_1 \\ \vdots \\ F \Leftarrow G_n \end{array}\right\} \quad \text{becomes} \quad F \leftrightarrow G_1 \vee \cdots \vee G_n$$

If F is an atom and there are no causal rules with F as the head then $F \leftrightarrow \bot$ (which is logically equivalent to $\neg F$).

A causal rule $\bot \Leftarrow G$ becomes $\neg G$.

Models of causal theory Γ are the (classical) models of the formulas $comp(\Gamma)$.

What to compute: 'Prediction'

'Prediction':
— Initially F.
— Partially specified events of type $\alpha[0], \alpha[1], \ldots, \alpha[k]$ happen.
— Is it possible that G holds in state m? How?

We want to know whether

$$comp(\Gamma^D_m) \cup \{F[0],\ \alpha[0],\ \alpha[1],\ \ldots,\ \alpha[k],\ G[m]\}$$

is satisfiable.

If satisfiable, a propositional sat-solver will return all models.

What to compute: 'Prediction' (2)

'Prediction' (2):
- — Initially F.
- — Partially specified events of type $\alpha[0], \alpha[1], \dots, \alpha[k]$ happen.
- — Does it *follow* that G holds in state m?

We want to know whether

$$comp(\Gamma_m^D) \cup \{F[0],\ \alpha[0],\ \alpha[1],\ \dots,\ \alpha[k]\} \models G[m]$$

In other words,

$$comp(\Gamma_m^D) \cup \{F[0],\ \alpha[0],\ \alpha[1],\ \dots,\ \alpha[k],\ \neg G[m]\} \quad \text{satisfiable ?}$$

What to compute: 'Planning'

'Planning'
- — Initially F.
- — Goal: G.

We try *consecutively* for $k = 0, 1, \dots, m$:

$$comp(\Gamma_k^D) \cup \{F[0] \wedge G[k]\} \quad \text{satisfiable ?}$$

The sat-solver returns all models, and these contain a representation of the 'plan': $e[0], e[1], \dots, e[k{-}1]$.

(This is not *really* planning.)

Transition system

Given an action description D:

Γ_0^D — states

Γ_1^D — (labelled) transitions

Γ_m^D — paths/runs/traces of length m