Knowledge Representation

Tutorial Exercise

Fixpoint semantics for definite programs

Question 1

(a) Let $IDB = \{ p \leftarrow q, r ; q \leftarrow s \}$ and $EDB = \{ r, s \}$.
Check that $Cl_{IDB}(EDB) = T_{IDB}^{\uparrow \omega} (EDB) \neq T_{IDB}^{\uparrow \omega} (EDB)$.
(b) Same again but for $IDB = \{ p \leftarrow q, r ; q \leftarrow s ; r \}$ and $EDB = \{ s \}$.

Question 2 (Optional) Following on from the previous question. Let $P$ be a set of definite clauses and $X$ any set of atoms of $P$.

(a) Show (by induction on $n$) that for all $n \geq 0$:  
\[ T_P^{\uparrow n}(X) \subseteq T_P^{\uparrow n}(X) \]
and hence that $T_P^{\uparrow n}(X) \subseteq T_P^{\uparrow n}(X)$.
(b) We know (e.g., from the example in Question 1) that in general $T_P^{\uparrow \omega}(X) \neq T_P^{\uparrow \omega}(X)$.
But show (by induction on $n$) that for all $n \geq 0$, $T_P^{\uparrow n}(\emptyset) \subseteq T_P^{\uparrow n+1}(\emptyset)$, and hence 
\[ T_P^{\uparrow n}(\emptyset) = T_P^{\uparrow n}(\emptyset) \]
from which follows $T_P^{\uparrow n}(\emptyset) = T_P^{\uparrow n}(\emptyset)$.

Question 3

Suppose $IDB$ is a set of definite clauses and $EDB$ is a set of atoms. Show that $Cl_{IDB}(EDB)$ is the least Herbrand model of $IDB \cup EDB$.

Hint: if $EDB$ is a set of atoms, then $T_{EDB}(X) = EDB$. And clearly $T_{P \cup P}(X) = T_{P}(X) \cup T_{P}(X)$.

Question 4

$Cl_{IDB}$ maps sets of atoms to sets of atoms. Show that if $IDB$ is definite then $Cl_{IDB}$ is a classical consequence operator (Tarski):

- $X \subseteq Cl_{IDB}(X)$
- $Cl_{IDB}(Cl_{IDB}(X)) \subseteq Cl_{IDB}(X)$
- if $X_1 \subseteq X_2$ then $Cl_{IDB}(X_1) \subseteq Cl_{IDB}(X_2)$

Question 5

Consider the following set of (normal, not definite) clauses $IDB$:

\[ p \leftarrow r, \quad q \leftarrow r, \quad \text{not } p \]

What is $Cl_{IDB}\{r\}$? (Hint: there isn’t one.)