Question 1 Look again at the earlier exercise sheet Consequence relations. Solutions to those exercises were handed out. Many of those questions did not assume that the consequence relations referred to were monotonic.

Question 2 Show that the following hold for any supraclassical cumulative operator Cn.

\( Cn \) is cumulative when \( A \subseteq B \subseteq Cn(A) \) implies \( Cn(B) = Cn(A) \).

i) \( Th(Cn(A)) = Cn(A) \)

ii) If \( B \subseteq Cn(A) \) then \( Th(B) \subseteq Cn(A) \).

(Or equivalently, if \( B \subseteq Cn(A) \) and \( C \subseteq Th(B) \) then \( C \subseteq Cn(A) \).)

iii) If \{ canary, \neg(yellow \land blue) \} \models yellow then \{ canary, \neg(yellow \land blue) \} \models \neg blue

(where \( A \models \alpha \) iff \( \alpha \in Cn(A) \)).

PS: Make sure you understand why part (ii) can say ‘Or equivalently ...’.

Question 3 Check that the following set of default rules

\[
\begin{align*}
& a \leftarrow \neg b \\
& c \leftarrow a \\
& b \leftarrow c, \neg a
\end{align*}
\]

provides an example to show that cautious monotony does not hold for (sceptical, cautious) Reiter default logic.

In other words: suppose \((D, W)\) is a Reiter default theory.

Let \( Cn_D \) be the consequence operator corresponding to the ‘sceptical’ or ‘cautious’ consequences of \( W \) under the default rules \( D \); that is, \( \alpha \) is in \( Cn_D(W) \) iff \( \alpha \) is in the intersection of all the extensions of the default theory \((D, W)\).

Let \( W \models_D \alpha \) be shorthand for \( \alpha \in Cn_D(W) \).

Now suppose \( D \) is the set of default rules above. Find formulas \( \alpha \) and \( \beta \) (and a set of formulas \( W \)) such that \( W \models_D \alpha \) and \( W \models_D \beta \) but \( W \cup \{ \beta \} \not\models_D \alpha \).

(The Construct the extensions.)

Because of the relationship between logic programs and Reiter default theories, you can, if you prefer, consider the following logic program instead:

\[
\begin{align*}
& a \leftarrow \text{not } b \\
& c \leftarrow a \\
& b \leftarrow c, \text{not } a
\end{align*}
\]
Knowledge Representation

Tutorial Exercise

Nonmonotonic consequence relations

SOLUTIONS

Question 1 (Handed out ... Th({yellow , ¬(yellow ∧blue)})
So:
{¬blue} ⊆ Cn({canary, ¬(yellow ∧blue)})
{canary,¬(yellow ∧blue)}|∼¬blue.

Question 2

Let D = \{a:
c , : ¬b
a , c: ¬a
b }, W = ∅.
The default theory (D, ∅) has one extension: Th( {a,c}).
(And since the extension is unique, ∅ |∼D c.)
(D, {c}) has two extensions: Th( {a,c}) and Th( {b,c}).

Question 3

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Question 4

Part (i) is easy. A is a subset of every extension of (D, A) by definition, so A ∈ \( \bigcap_{ \text{ext}(D, A) } \).

Part (ii). (Actually the essence of the argument is in Question 6 of the exercise sheet on default logic. Anyway, here it is again.)

Suppose A ⊆ B ⊆ \( \bigcap_{ \text{ext}(D, A) } \). Suppose E ∈ \( \text{ext}(D, A) \). Then by definition E = Cn_{D,E}(A).

We need to show E ∈ \( \text{ext}(D, B) \), i.e., E = Cn_{D,E}(B). Equivalently Cn_{D,E}(A) = Cn_{D,E}(B).
We do it in two separate parts.

Cn_{D,E} is monotonic so A ⊆ B implies Cn_{D,E}(A) ⊆ Cn_{D,E}(B).

First, notice that B ⊆ E (because B ⊆ Cn_{D,E}(A) (assumed) means that B is a subset of all extensions of (D, A), and E is one such).

Cn_{D,E} is monotonic so we have:
Cn_{D,E}(B) ⊆ Cn_{D,E}(E) = Cn_{D,E}(Cn_{D,E}(A)) ⊆ Cn_{D,E}(A).

Given part (ii), part (iii) is also easy. Part (ii) is equivalently stated as
A ⊆ B ⊆ Cn_{D,A} ⇒ \( \text{ext}(D, A) \subseteq \text{ext}(D, B) \)

So when A ⊆ B ⊆ Cn_{D,A}, \( \bigcap_{ \text{ext}(D, B) } \bigcap_{ \text{ext}(D, A) } \).

Or in words: Suppose A ⊆ B ⊆ Cn_{D,A}. Suppose α ∈ Cn_{D,B}. Then α is in every extension of (D, B).

Now suppose E is some extension of (D, A). Then E is also an extension of (D, B) by part(ii). But α is in all extensions of (D, B), so α is also in E.