491 Knowledge Representation

Tutorial Exercise

Stratified databases
SOLUTIONS

Question 1 Stratify:

$$
\begin{aligned}
P_2 &= \{\, tb(X) \leftarrow d(X), \text{not } h(X) \,\} \cup \\
P_1 &= \{\, h(X) \leftarrow p(X), \text{not } r(X) \,\} \cup \\
P_0 &= \{\, p(X) \leftarrow d(X), \\
&\quad p(e), \\
&\quad d(f), \\
&\quad r(f) \,\}
\end{aligned}
$$

$$
\begin{aligned}
M_0 &= T'_{P_0}{\uparrow}^{\omega}(\emptyset) = \{p(e), p(f), d(f), r(f)\} \\
M_1 &= T'_{P_1}{\uparrow}^{\omega}(M_0) = \{h(e)\} \cup M_0 \\
M_2 &= T'_{P_2}{\uparrow}^{\omega}(M_1) = \{tb(f)\} \cup M_1
\end{aligned}
$$

So the ABW model is $M_{\text{DB}} = \{tb(f), h(e), p(e), p(f), d(f), r(f)\}$.

We know that M_{DB} must be a minimal supported model of DB. Let's check.
$T_{\text{DB}}(M_{\text{DB}}) = T_{\text{DB}}(\{tb(f), h(e), p(e), p(f), d(f), r(f)\}) = \{tb(f), h(e), p(e), p(f), d(f), r(f)\}$.
$T_{\text{DB}}(M_{\text{DB}}) \subseteq M_{\text{DB}}$ means that M_{DB} is a model of DB, and $M_{\text{DB}} \subseteq T_{\text{DB}}(M_{\text{DB}})$ that it is supported.
To show that M_{DB} is a minimal model of DB, check that no proper subset of M_{DB} is also a model of DB. Clearly every model of DB must contain $\{p(e), p(f), d(f), r(f)\}$. So we only need to check that $\{p(e), p(f), d(f), r(f)\}$, $\{h(e), p(e), p(f), d(f), r(f)\}$, and $\{tb(f), p(e), p(f), d(f), r(f)\}$ are not models of DB. Very easy.

Question 2

(a) Stratify DB1:

$$
\begin{aligned}
P_2 &= \{\, \mathsf{strong}(x) \leftarrow \mathsf{big}(x), \text{not } \mathsf{weak}(x), \\
&\quad \mathsf{strong}(x) \leftarrow \mathsf{small}(x), \mathsf{muscular}(x), \text{not } \mathsf{weak}(x) \,\} \\
P_1 &= \{\, \mathsf{weak}(x) \leftarrow \text{not } \mathsf{muscular}(x) \,\} \\
P_0 &= \{\, \mathsf{small}(\mathsf{Bill}), \\
&\quad \mathsf{muscular}(\mathsf{Bill}), \\
&\quad \mathsf{big}(\mathsf{Mary}) \,\}
\end{aligned}
$$

$$
\begin{aligned}
M_0 &= T'_{P_0}{\uparrow}^{\omega}(\emptyset) = \{\mathsf{small}(\mathsf{Bill}), \mathsf{muscular}(\mathsf{Bill}), \mathsf{big}(\mathsf{Mary})\} \\
M_1 &= T'_{P_1}{\uparrow}^{\omega}(M_0) = \{\mathsf{weak}(\mathsf{Mary})\} \cup M_0 \\
M_2 &= T'_{P_2}{\uparrow}^{\omega}(M_1) = \{\mathsf{strong}(\mathsf{Bill})\} \cup M_1
\end{aligned}
$$

The ABW model of DB1 is M_2.
(Now check $T_{\text{DB1}}(M_2) = M_2$.)

(b) DB2 cannot be stratified.
The ABW semantics is not defined.

Question 3 Stratify: $P_0 = \{r\}$, $P_1 = \{q \leftarrow r, \text{ not } s\}$, $P_2 = \{p \leftarrow r, \text{ not } q\}$.

$$
\begin{aligned}
M_0 &= T'_{P_0}{\uparrow}^{\omega}(\emptyset) = \{r\} \\
M_1 &= T'_{P_1}{\uparrow}^{\omega}(M_0) = \{q\} \cup M_0 = \{q, r\} \\
M_2 &= T'_{P_2}{\uparrow}^{\omega}(M_1) = \emptyset \cup M_1 = \{q, r\}
\end{aligned}
$$

Compare:

$$
\begin{aligned}
T'_P(\emptyset) &= \{r\} \\
T'_P(\{r\}) &= \{p, q, r\} \cup \{r\} = \{p, q, r\} \\
T'_P(\{p, q, r\}) &= \{q, r\} \cup \{p, q, r\} = \{p, q, r\}
\end{aligned}
$$