MIT EECS
ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

CSAIL

Feature allocations, probability functions, and paintboxes

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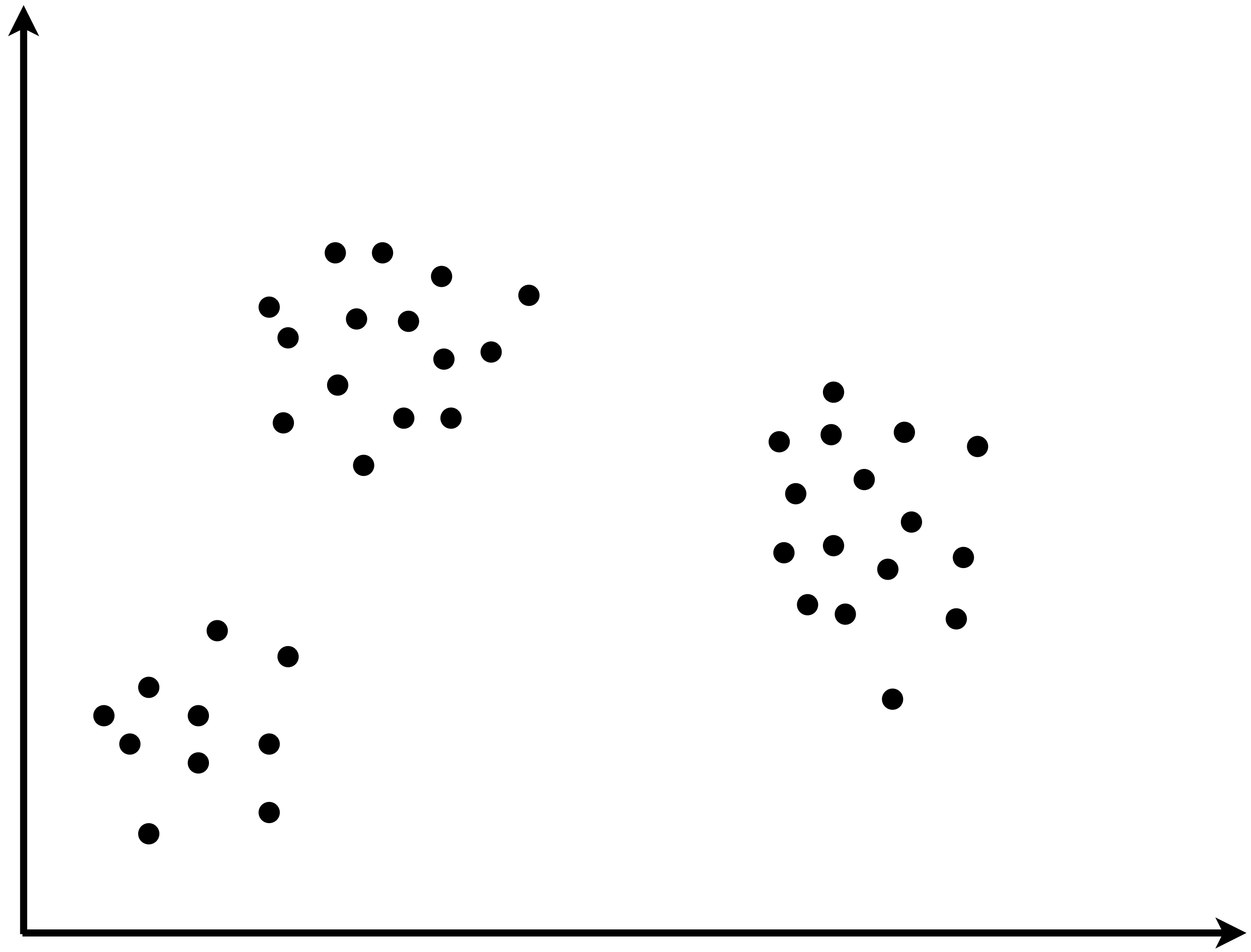
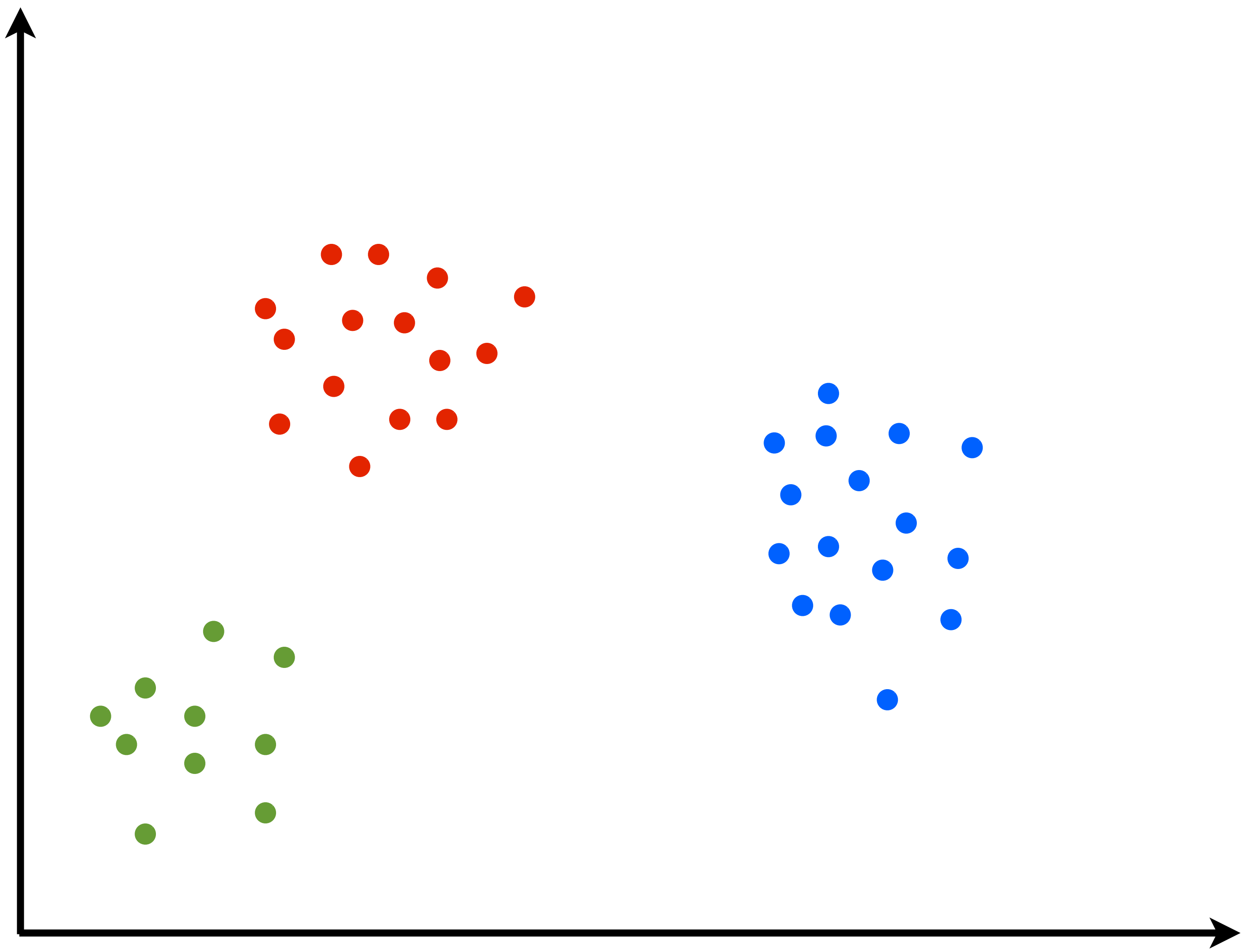
ITT Career Development
Assistant Professor,
MIT

IDSS
MIT INSTITUTE FOR DATA, SYSTEMS, AND SOCIETY

Clustering/Partition

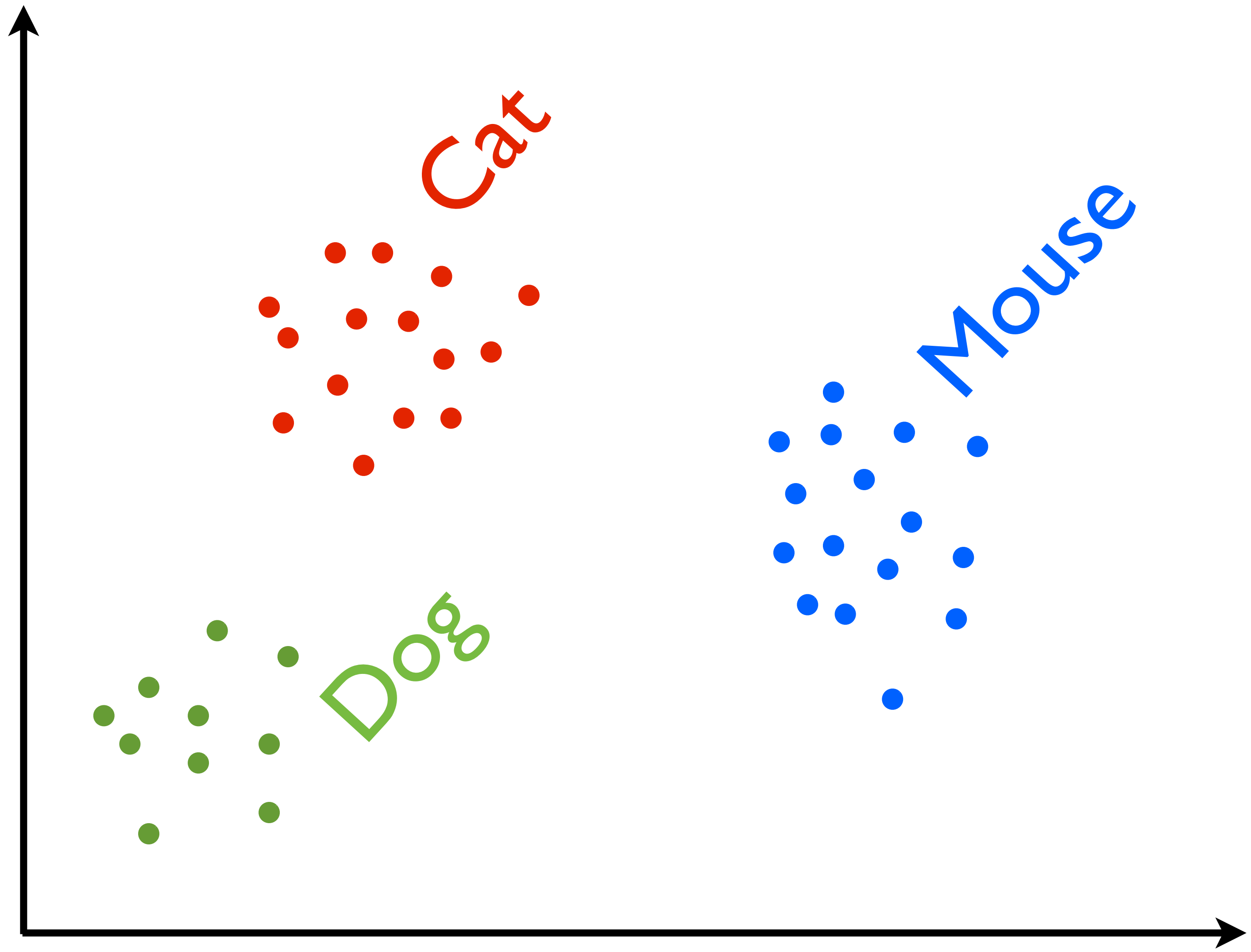


Clustering/Partition



"clusters",
"classes",
"blocks (of a partition)"

Clustering/Partition



“clusters”,
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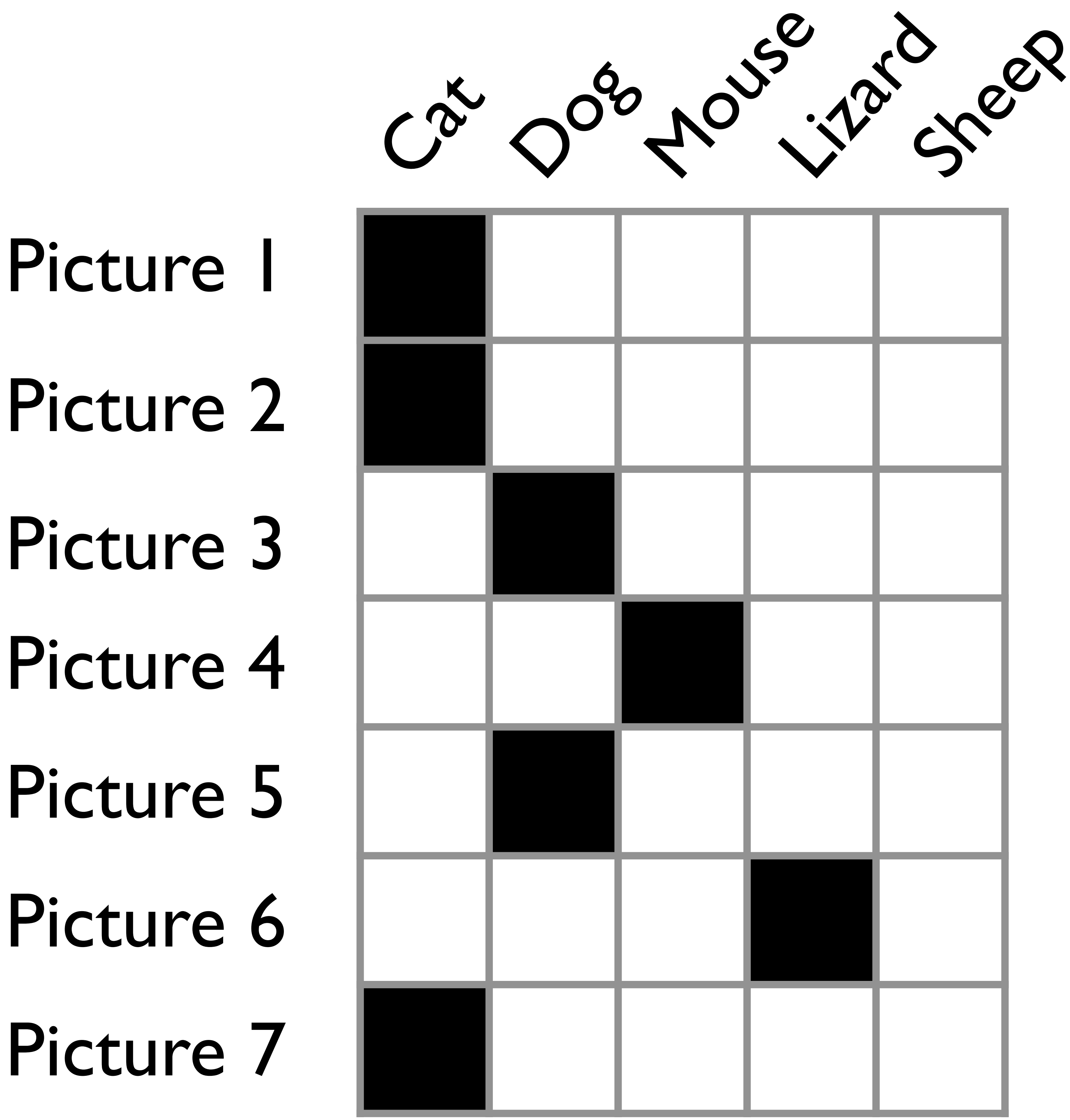
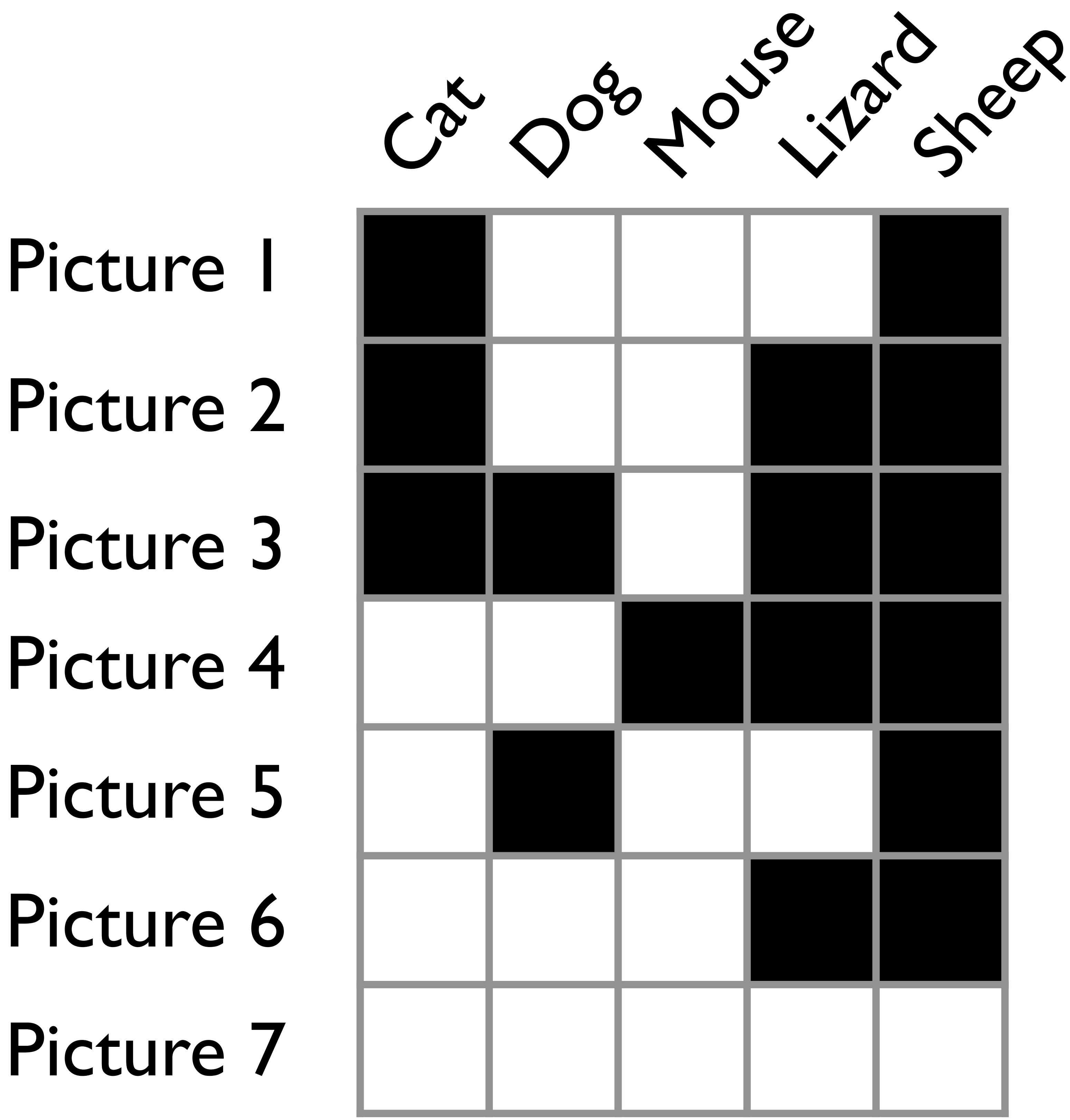
Clustering/Partition

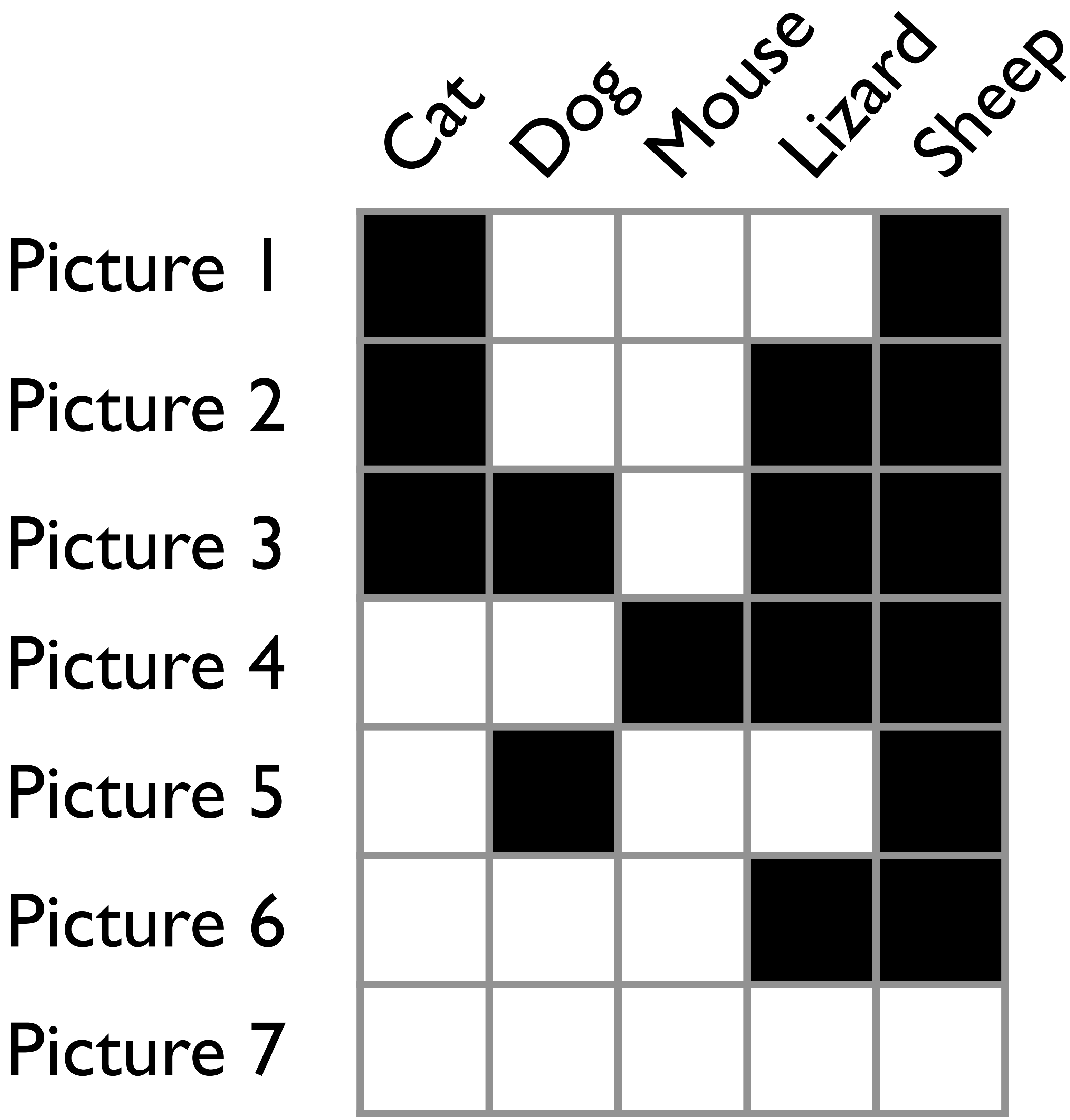


Latent feature allocation



"features",
"topics"

Latent feature allocation



"features",
"topics"

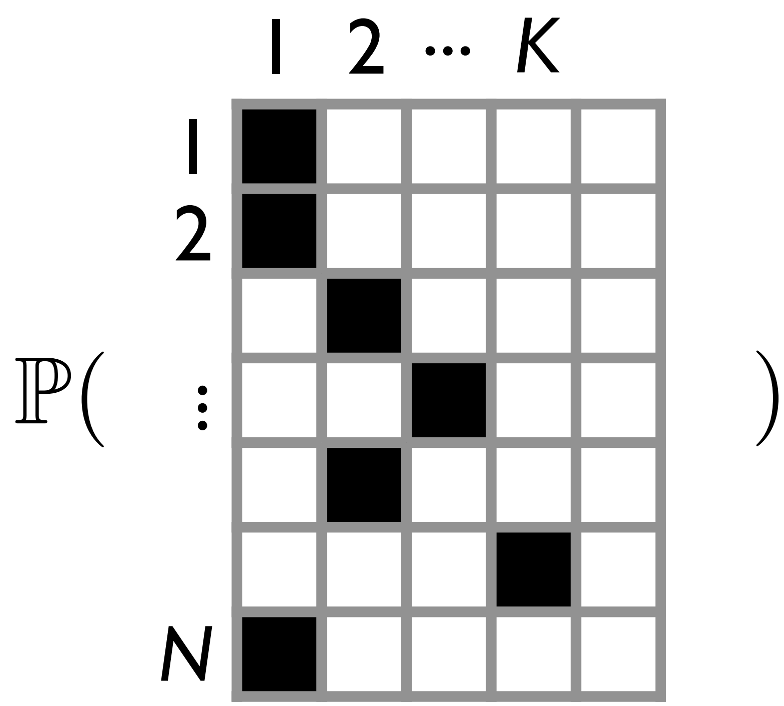
- Exchangeable
- Finite # of features per data point

Characterizations

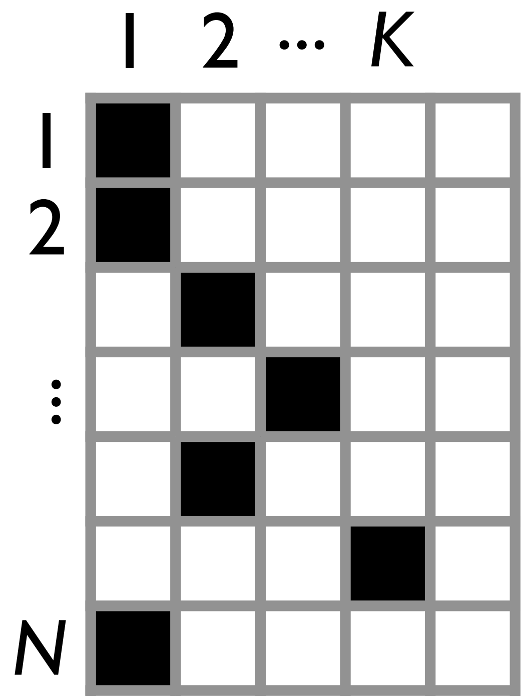
- Exchangeable cluster distributions are characterized

- What about exchangeable feature distributions?

Exchangeable probability functions

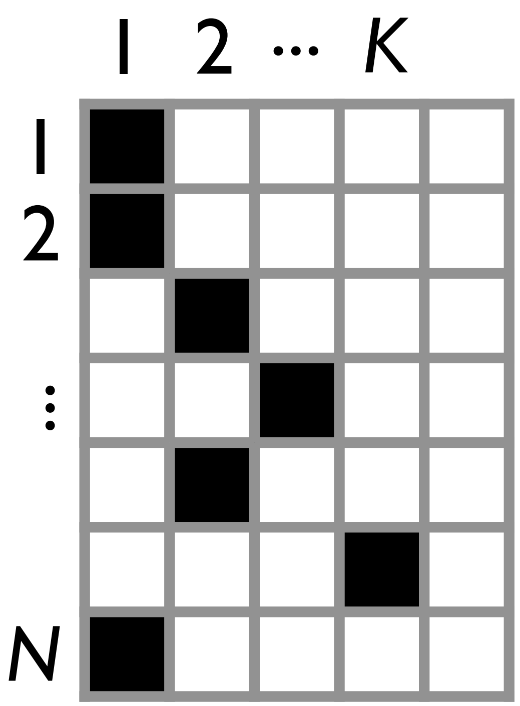


Exchangeable probability functions

$\mathbb{P}($  $) = p\,(S_{N,1},\ldots,S_{N,K})$

Exchangeable probability functions

Size of *K*th cluster

$\mathbb{P}($ 

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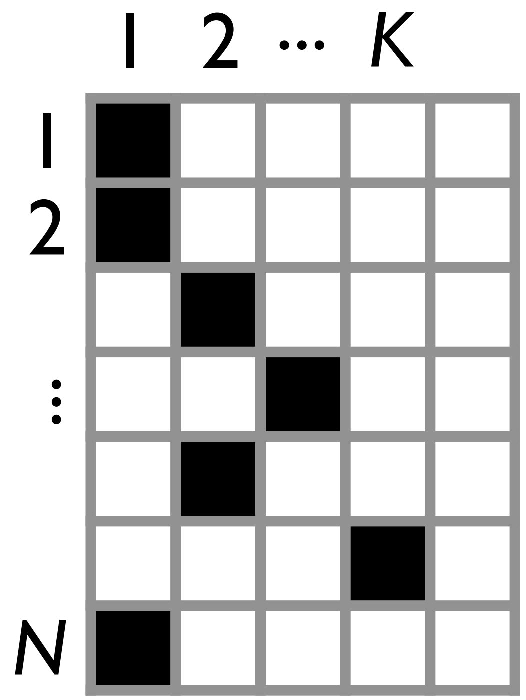
Exchangeable probability functions

Exchangeable partition probability function (EPPF)

$\mathbb{P}($ 

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[Pitman 1995]

Exchangeable probability functions

“Exchangeable **feature** probability function” (EFPF)?

Example: Indian buffet process

[Griffiths, Ghahramani 2006]

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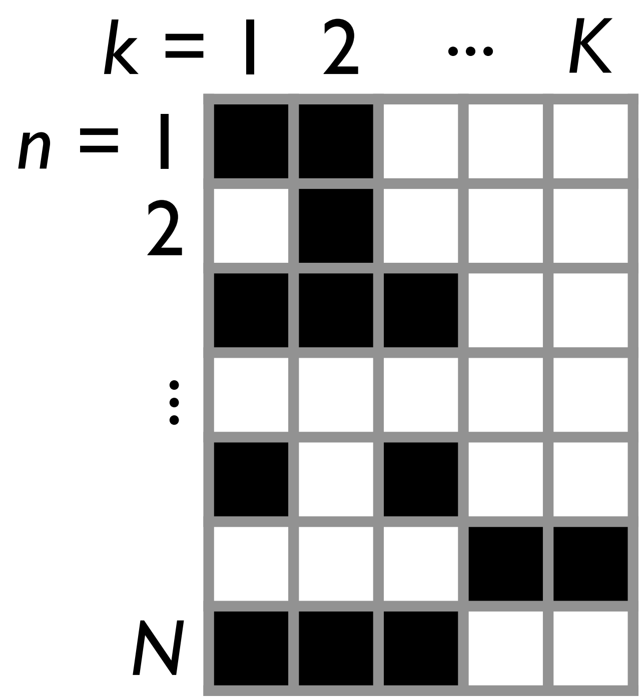

[Griffiths, Ghahramani 2006]

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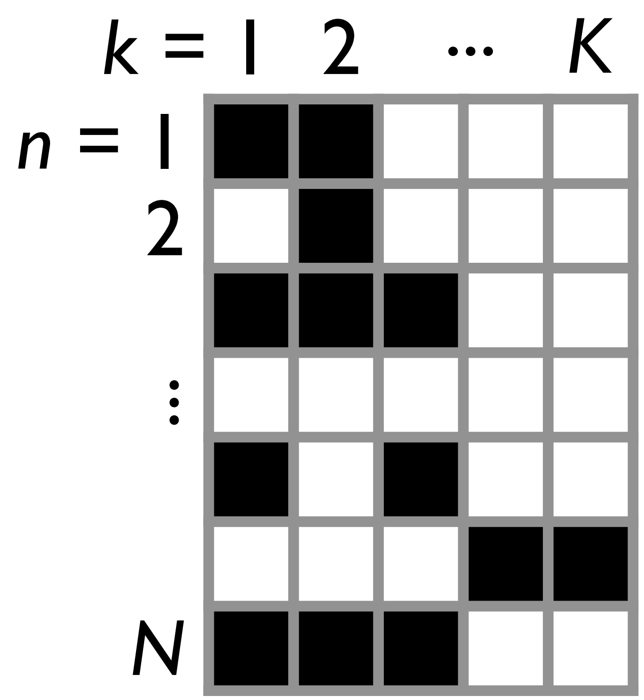

For n = 1, 2, ..., N

[Griffiths, Ghahramani 2006]

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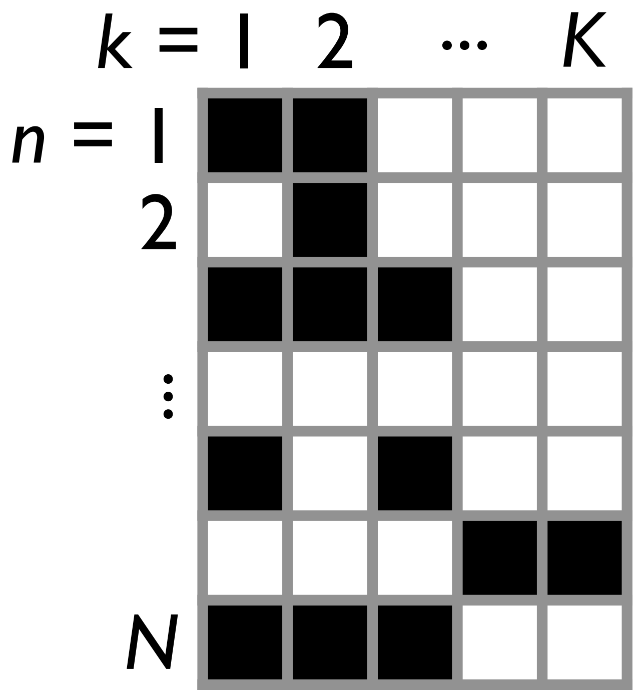

For n = 1, 2, ..., N

1. Data point n has an existing feature k that has already occurred $S_{n-1,k}$ times with probability $\frac{S_{n-1,k}}{\theta + n - 1}$

[Griffiths, Ghahramani 2006]

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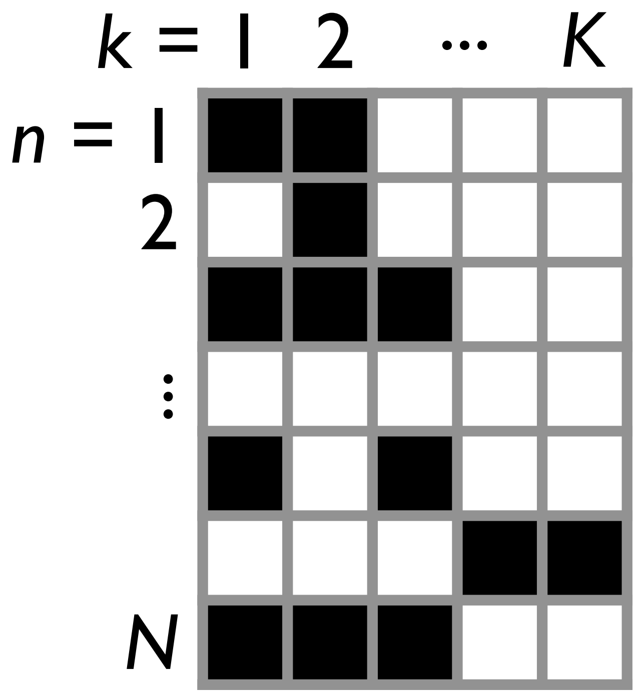

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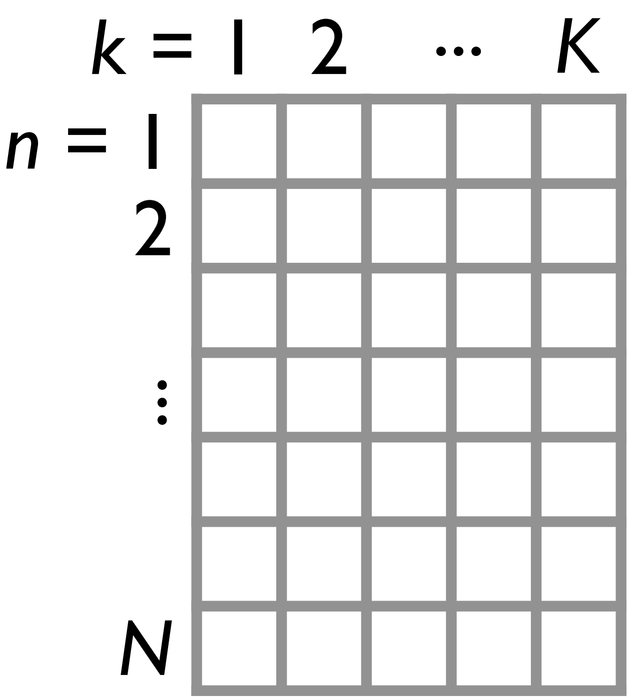

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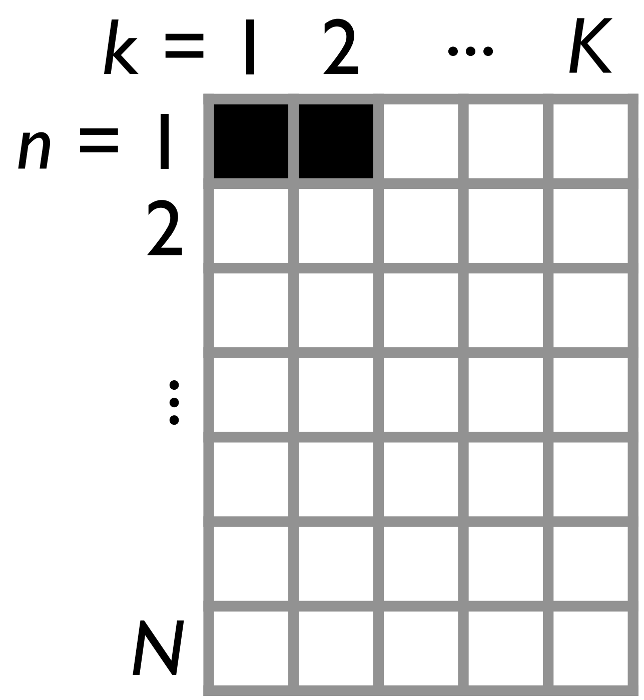

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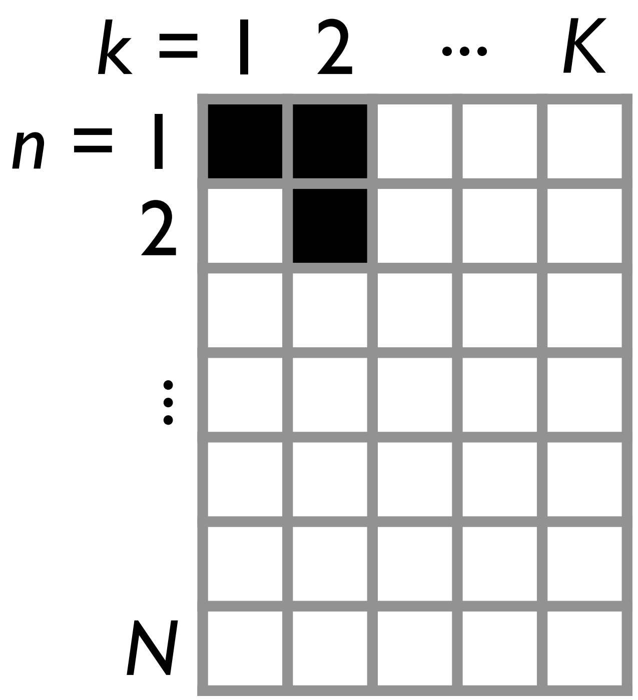

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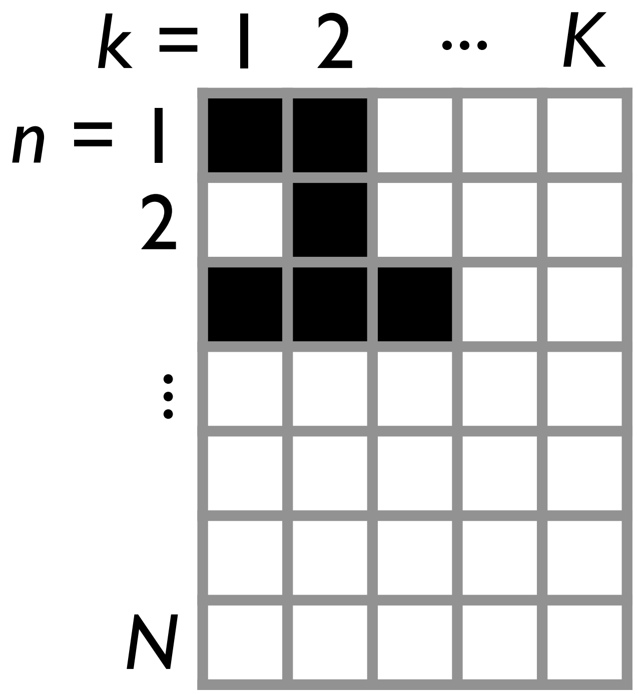

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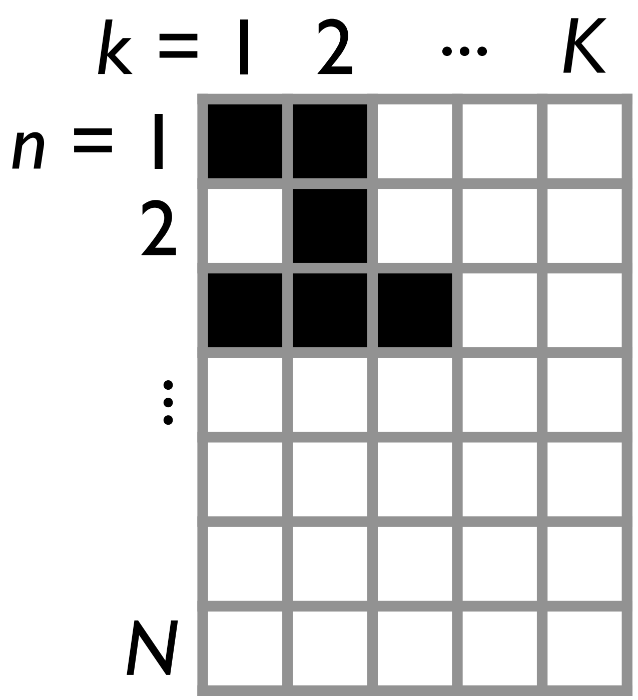

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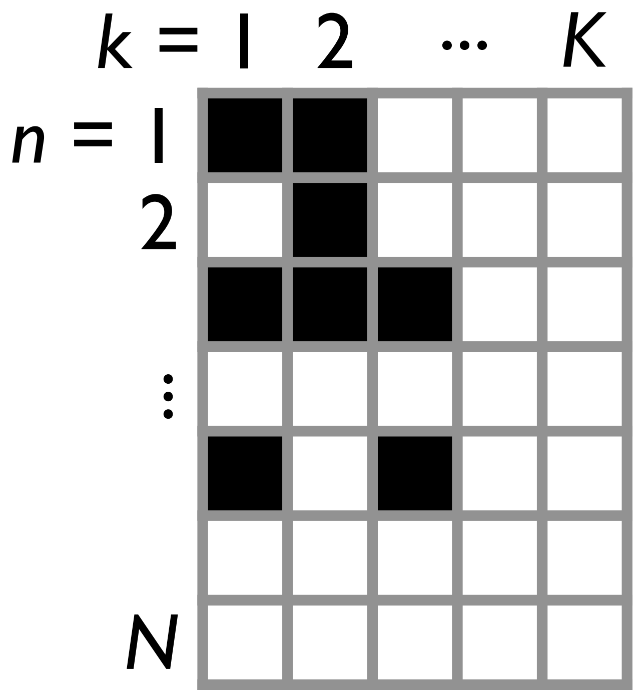

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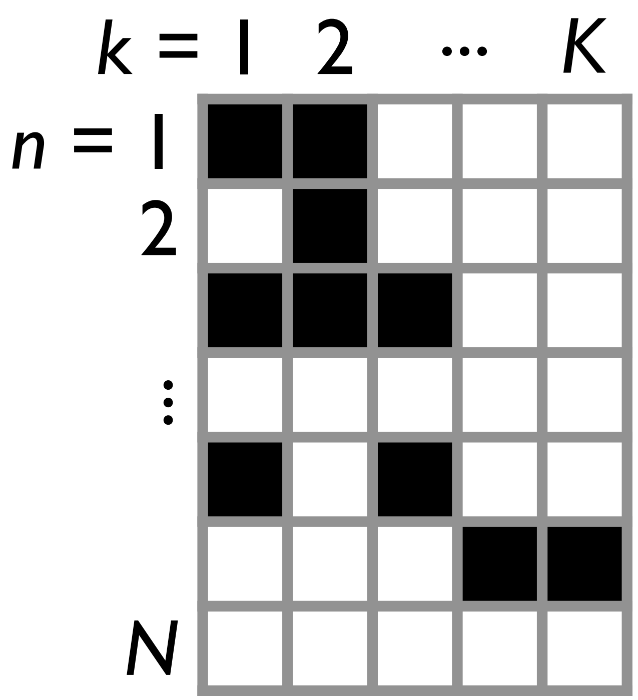

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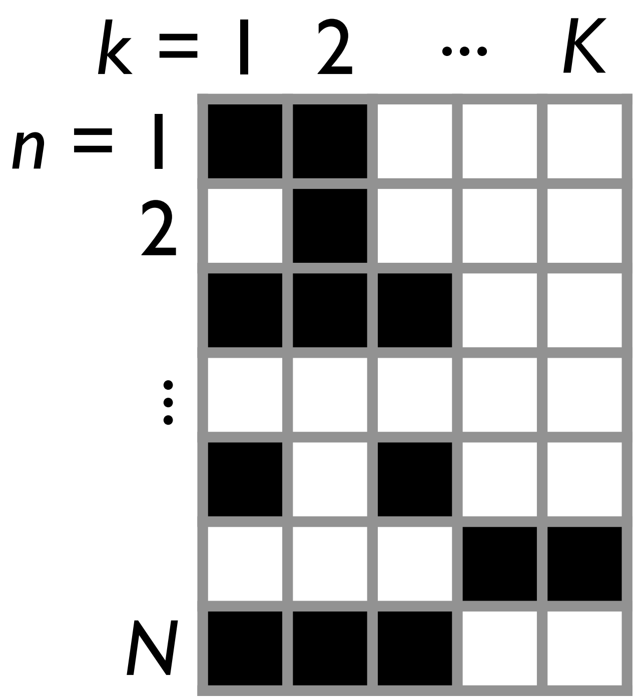

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Exchangeable probability functions

"Exchangeable feature probability function" (EFPF)?

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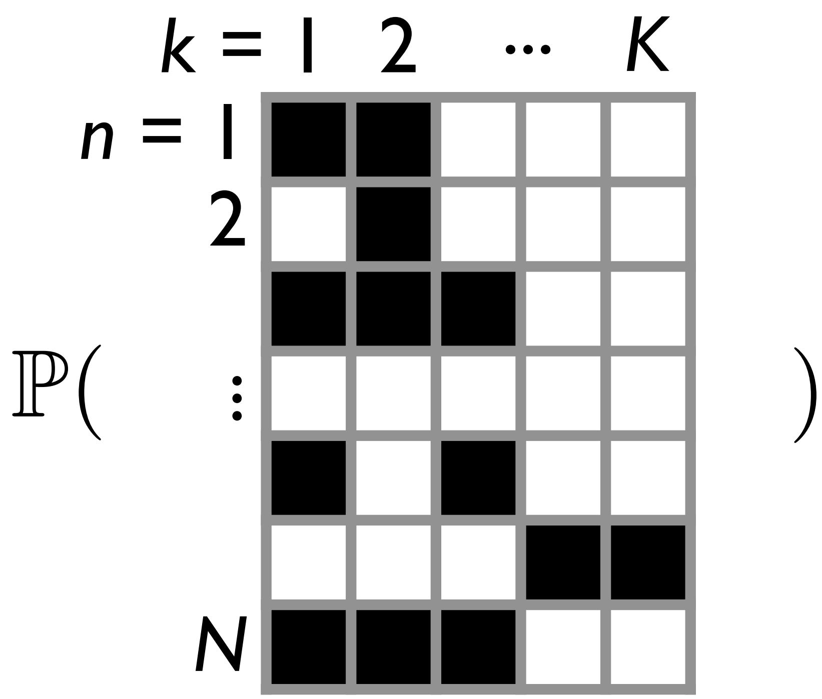
Example: Indian buffet process (IBP)



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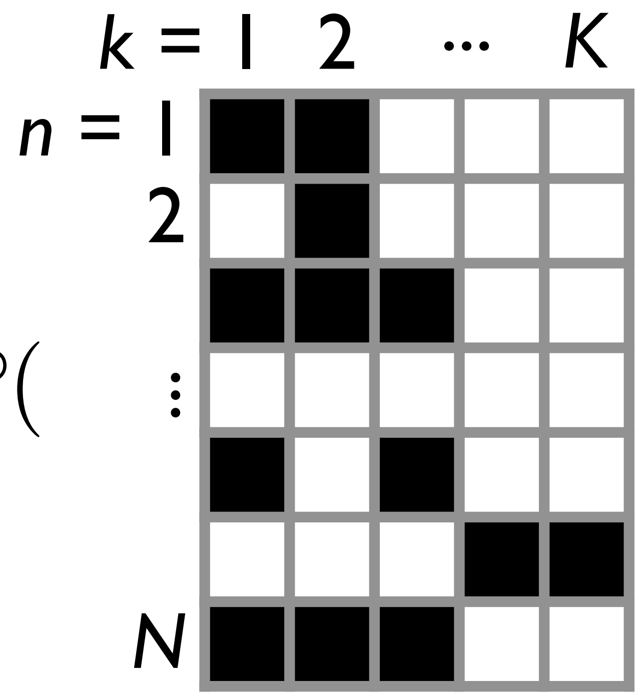
$\mathbb{P}($ 

$)$

$$= \frac{1}{K_N!}(\theta\gamma)^{K_N} \exp\left(-\theta\gamma \sum_{n=1}^{N} (\theta + n - 1)^{-1}\right) \prod_{k=1}^{K_N} \frac{\Gamma(S_{N,k})\Gamma(N - S_{N,k} + \theta)}{\Gamma(N + \theta)}$$

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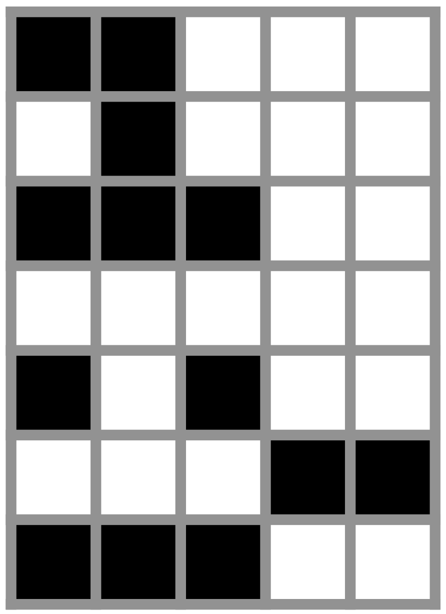
$\mathbb{P}($  $)$

Size of kth feature

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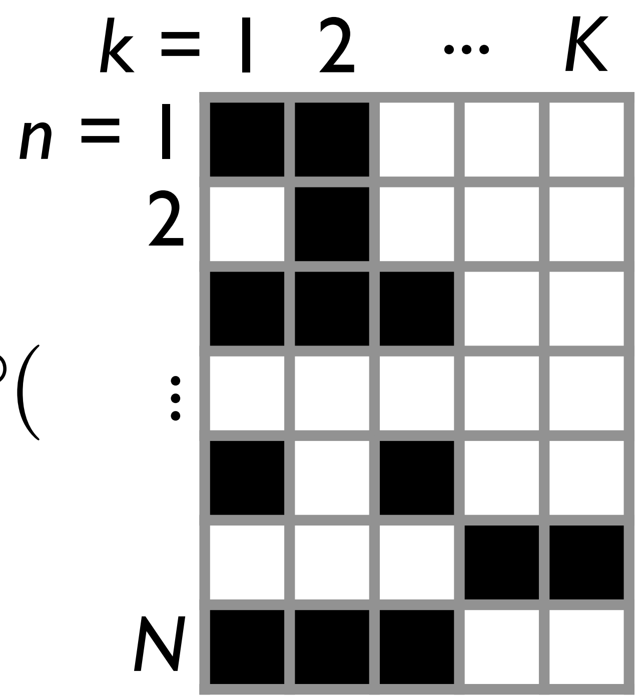
$\mathbb{P}($  $)$

Size of *k*th feature

Number of features

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Example: Indian buffet process (IBP)

$$\mathbb{P}\left(\begin{array}{c|ccccc} & k=1 & 2 & & \cdots & K \\ \hline n=1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ & 1 & 1 & 1 & 0 & 0 \\ \vdots & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & 1 & 1 \\ N & 1 & 1 & 1 & 0 & 0 \end{array} \right)$$

Number of data points

Size of kth feature

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$$= p(N; S_{N,1}, S_{N,2}, \ldots, S_{N,K})$$

[Broderick, Jordan, Pitman 2013]

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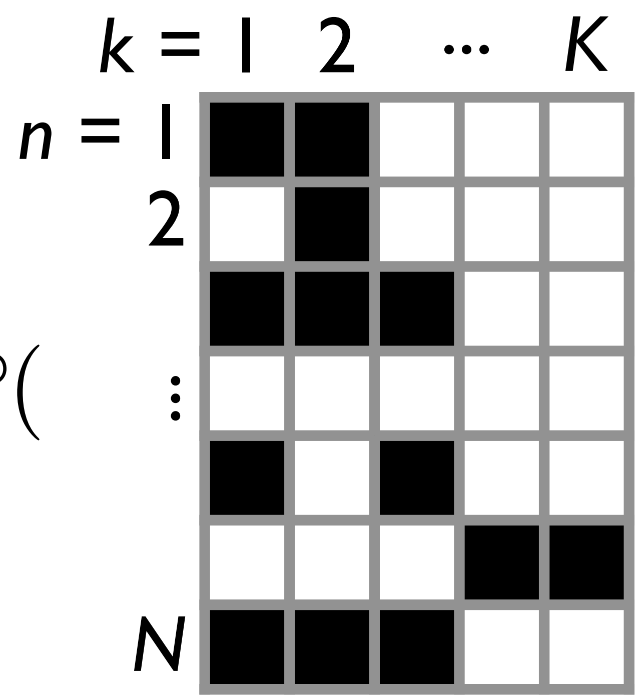
$\mathbb{P}\Big($



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"EFPF"

[Broderick, Jordan, Pitman 2013]

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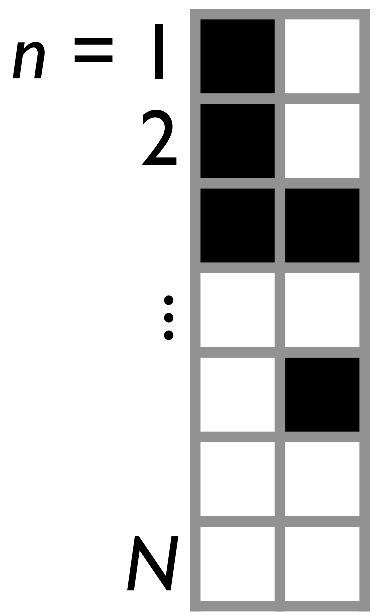
Counterexample



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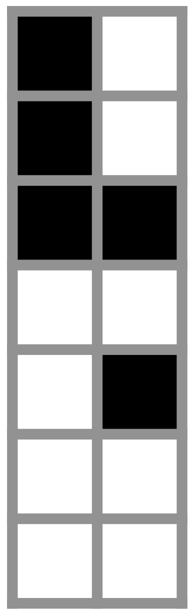

$\mathbb{P}(\text{row} =$  $) = p_1$

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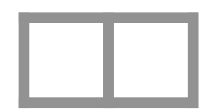
$\mathbb{P}(\text{row} =$  $) = p_3$

$\mathbb{P}(\text{row} =$  $) = p_4$

[Broderick, Jordan, Pitman 2013]

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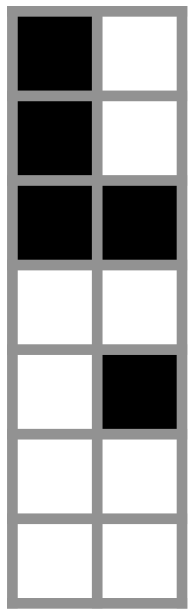
Counterexample

n = 1
2
⋮
N



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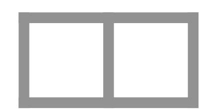
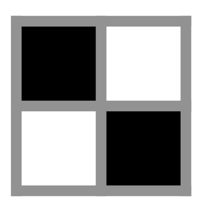
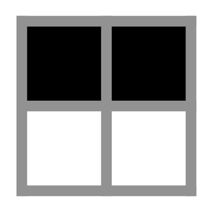
$\mathbb{P}(\text{row} =$  $) = p_4$





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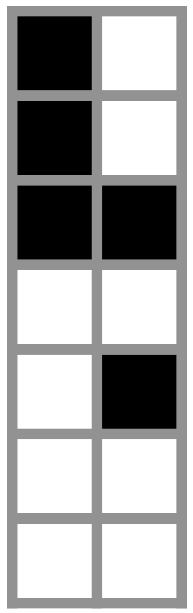

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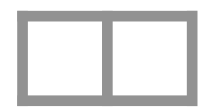
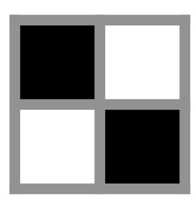
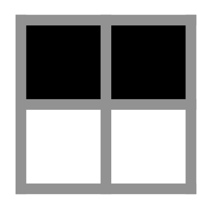
$\mathbb{P}(\text{row} =$  $) = p_4$

$\mathbb{P}($  $)$ $\mathbb{P}($  $)$

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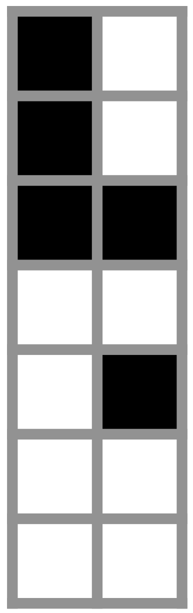

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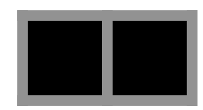
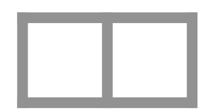
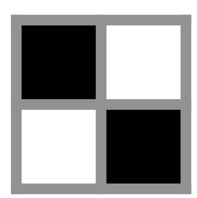
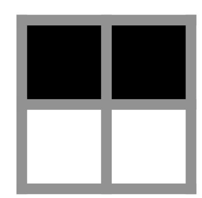
$\mathbb{P}(\text{row} =$  $) = p_3$

$\mathbb{P}(\text{row} =$  $) = p_4$

$\mathbb{P}($  $)$ $\mathbb{P}($  $)$

$p_1 p_2$ $p_3 p_4$

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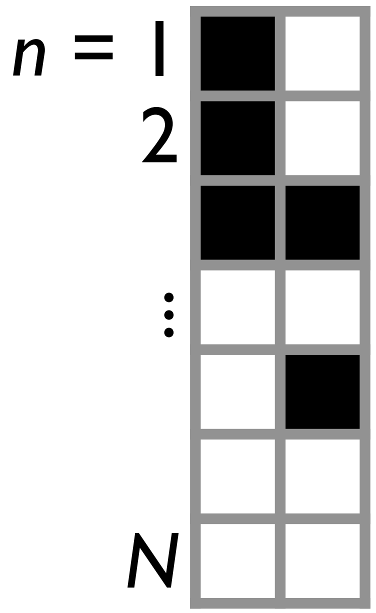

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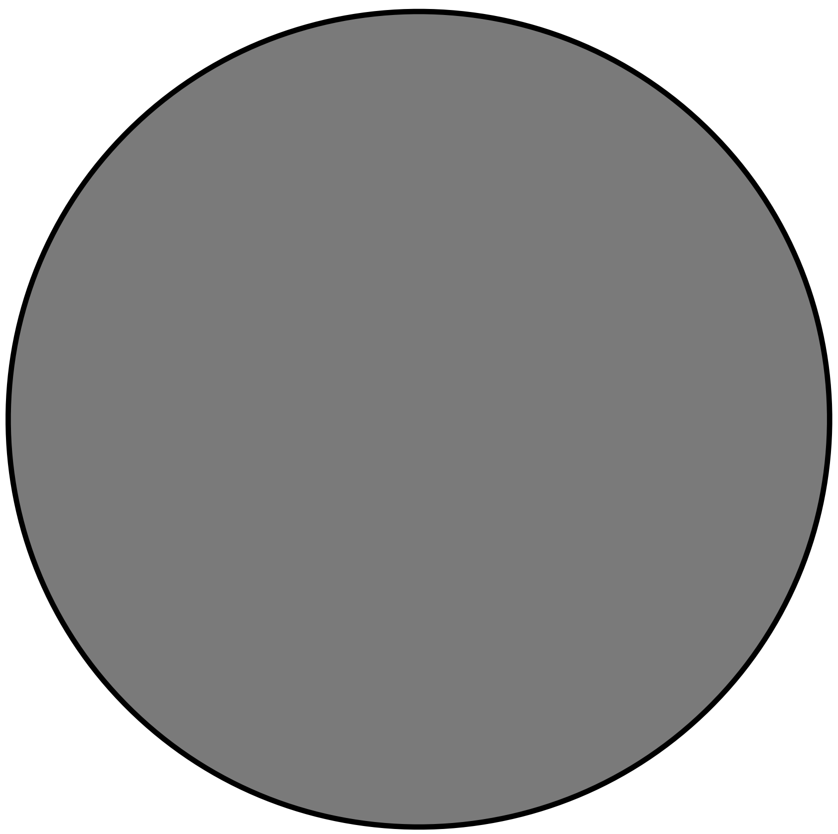
$$\mathbb{P}\left(\begin{matrix}\blacksquare & \square \\ \square & \blacksquare\end{matrix}\right) \neq \mathbb{P}\left(\begin{matrix}\blacksquare & \blacksquare \\ \square & \square\end{matrix}\right)$$

$$p_1 p_2 \neq p_3 p_4$$

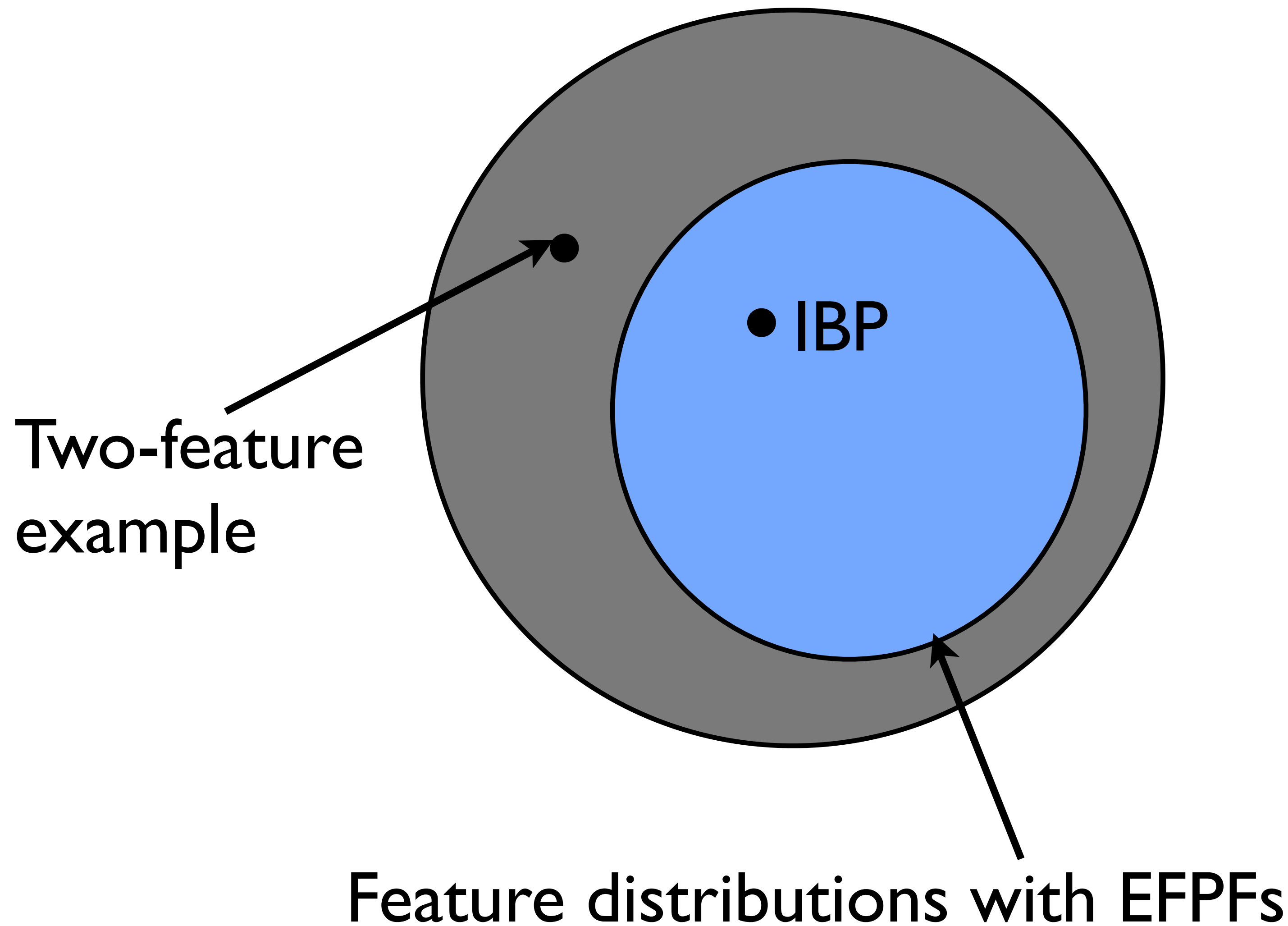
[Broderick, Jordan, Pitman 2013]

Exchangeable probability functions

Exchangeable cluster distributions
= Cluster distributions with EPPFs



Exchangeable feature distributions



[Broderick, Jordan, Pitman 2013]

Paintboxes

Exchangeable partition: Kingman paintbox

[Kingman 1978]

Paintboxes

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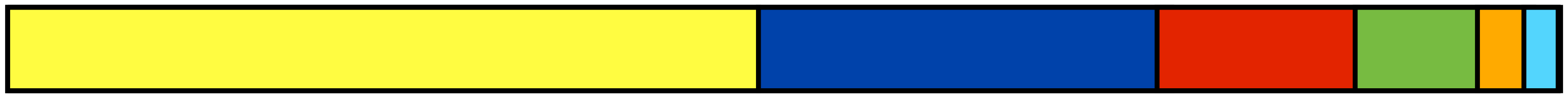

[Kingman 1978]

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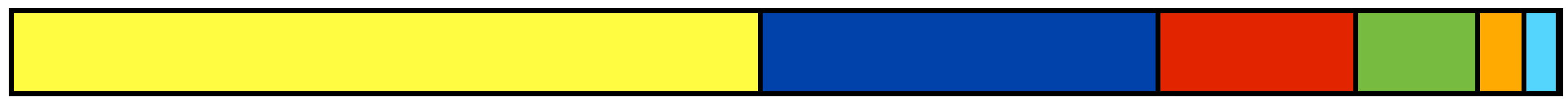

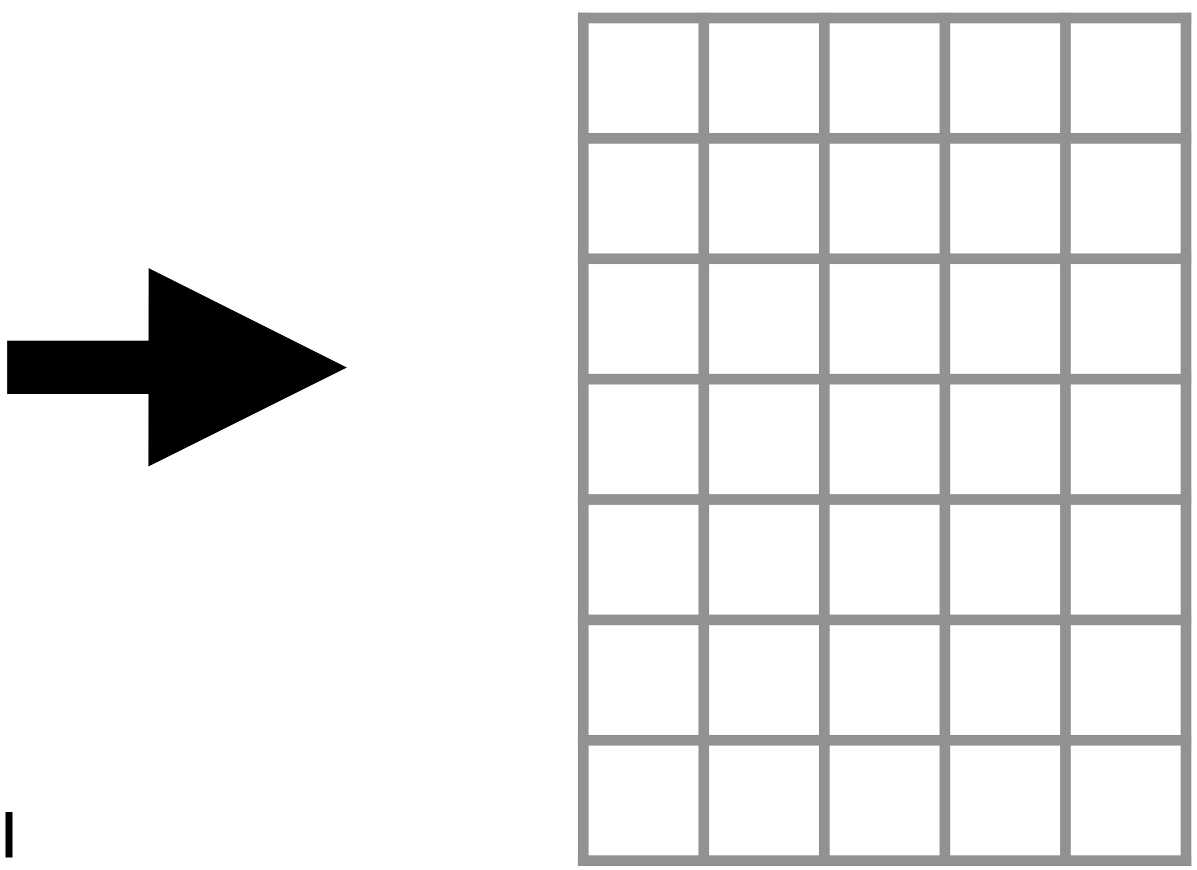

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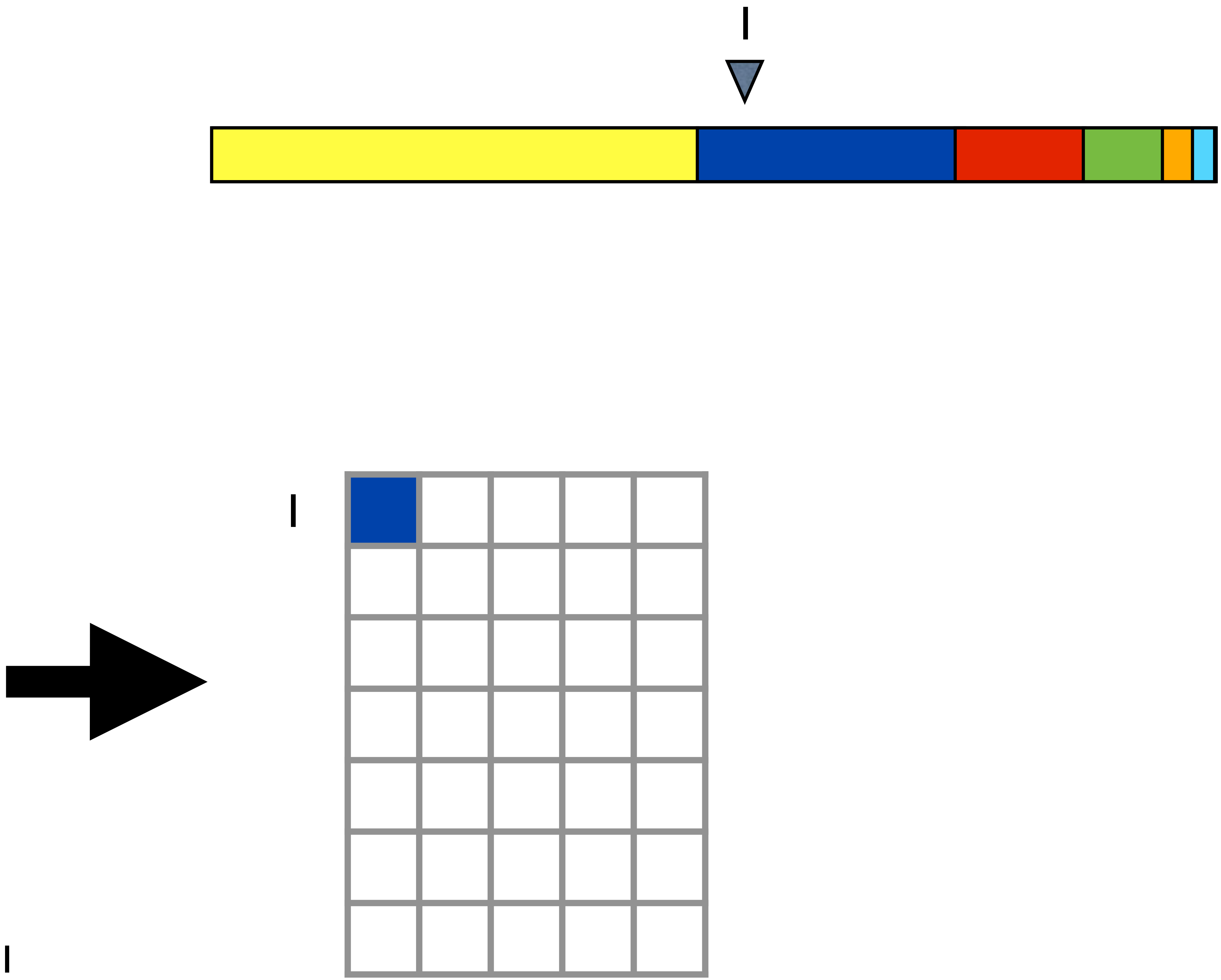

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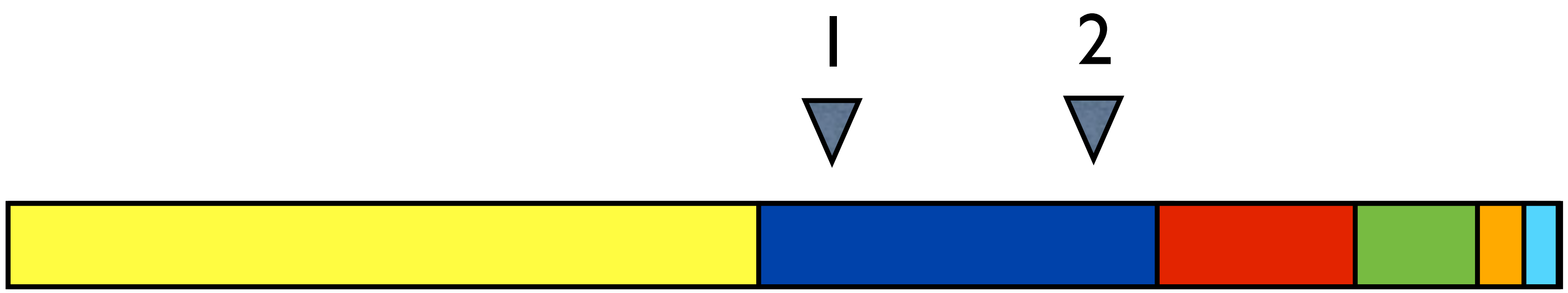

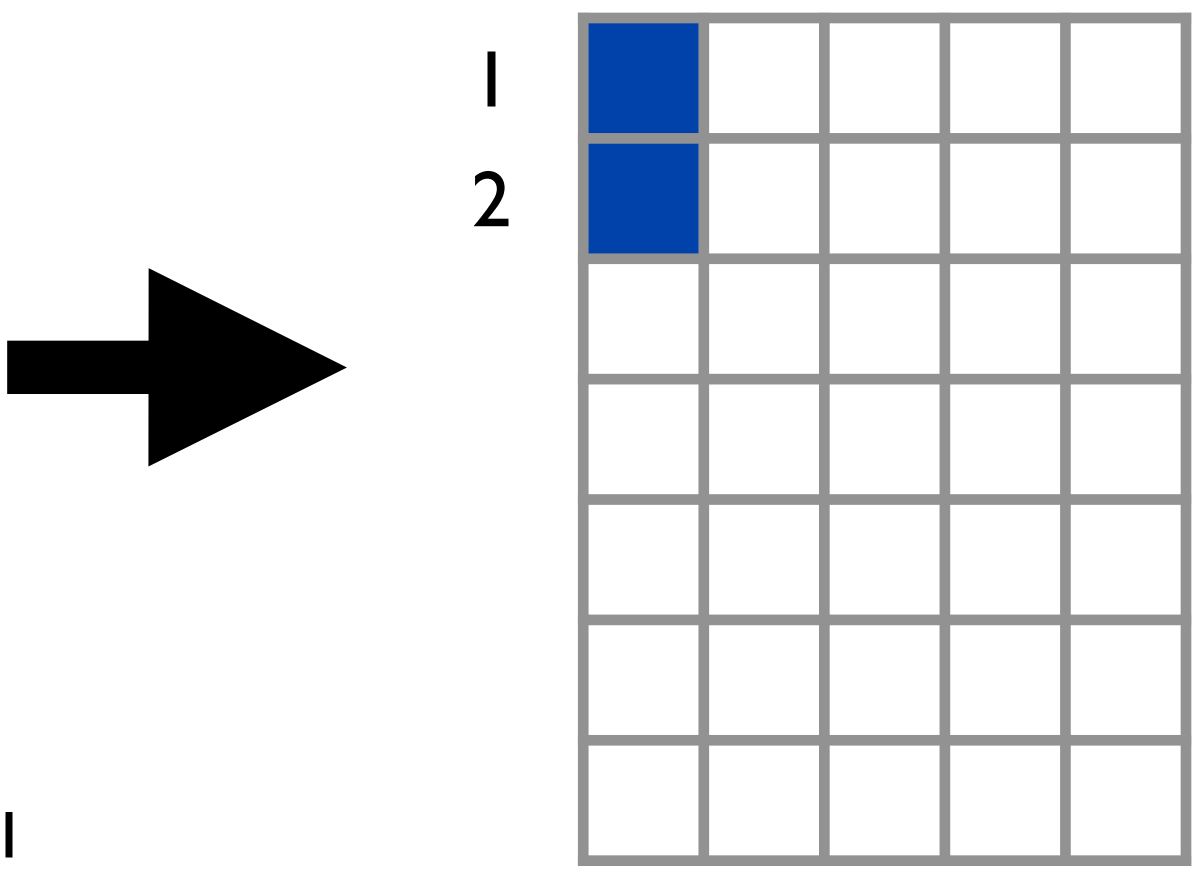

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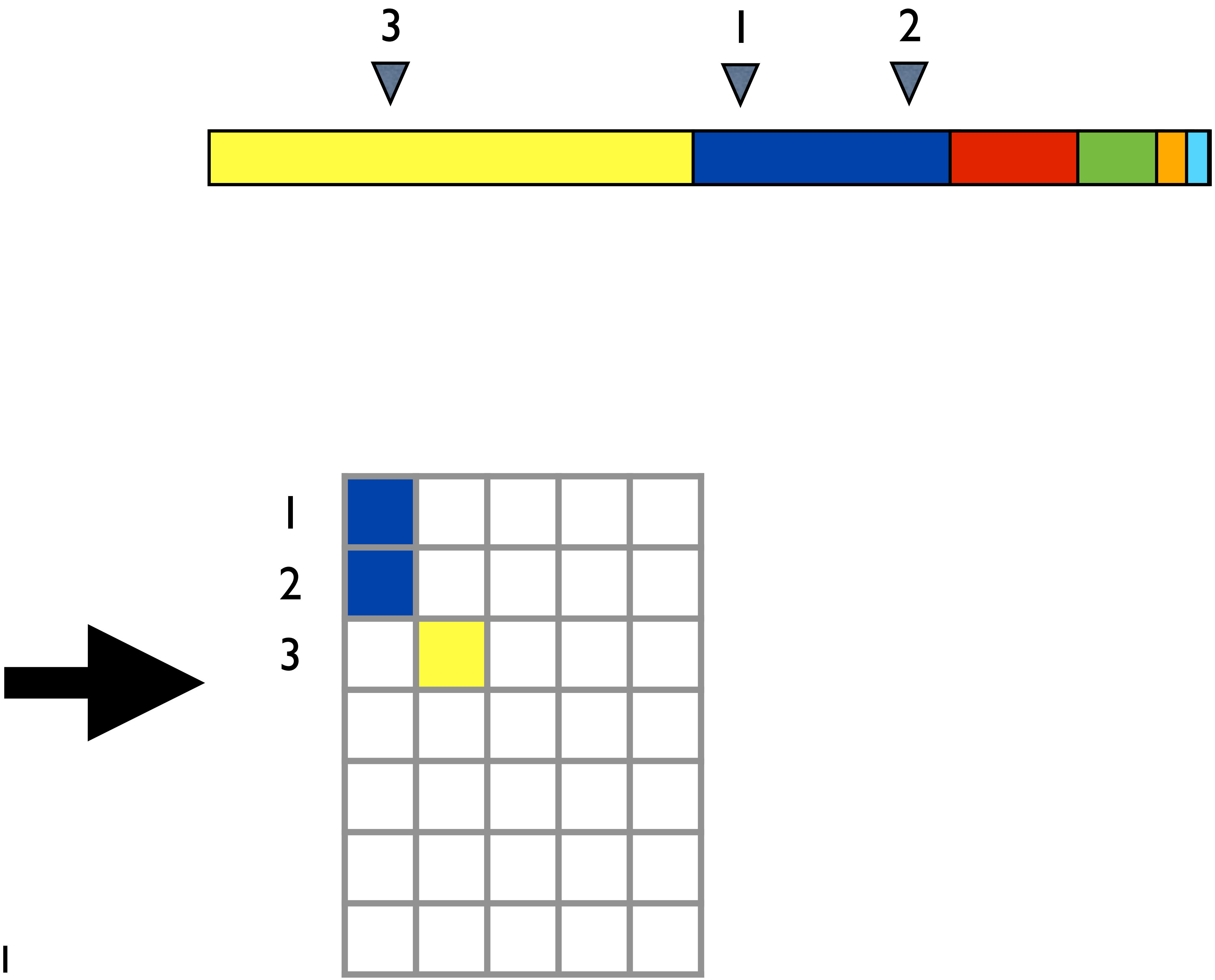

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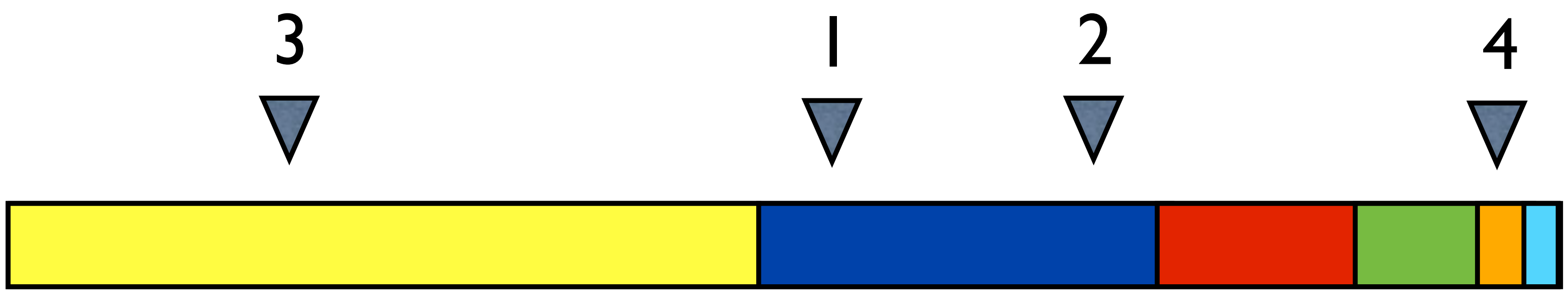

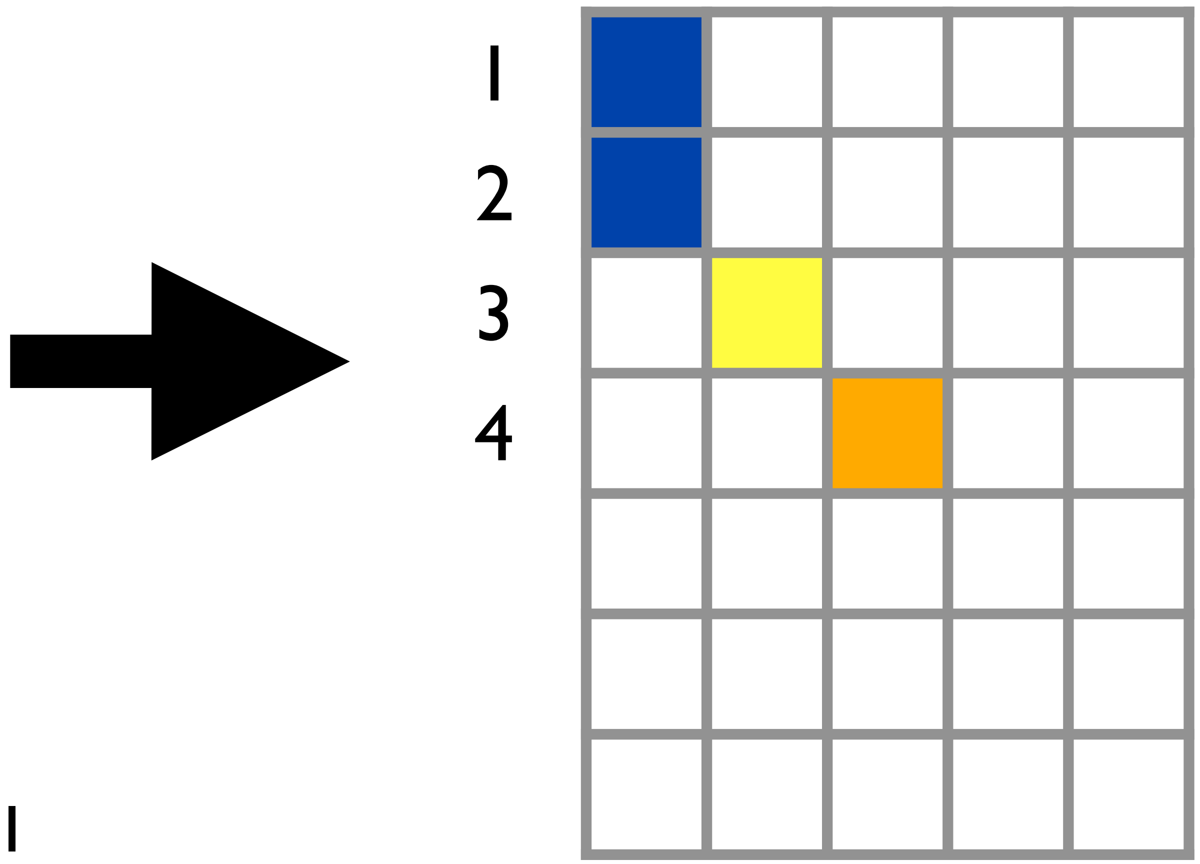

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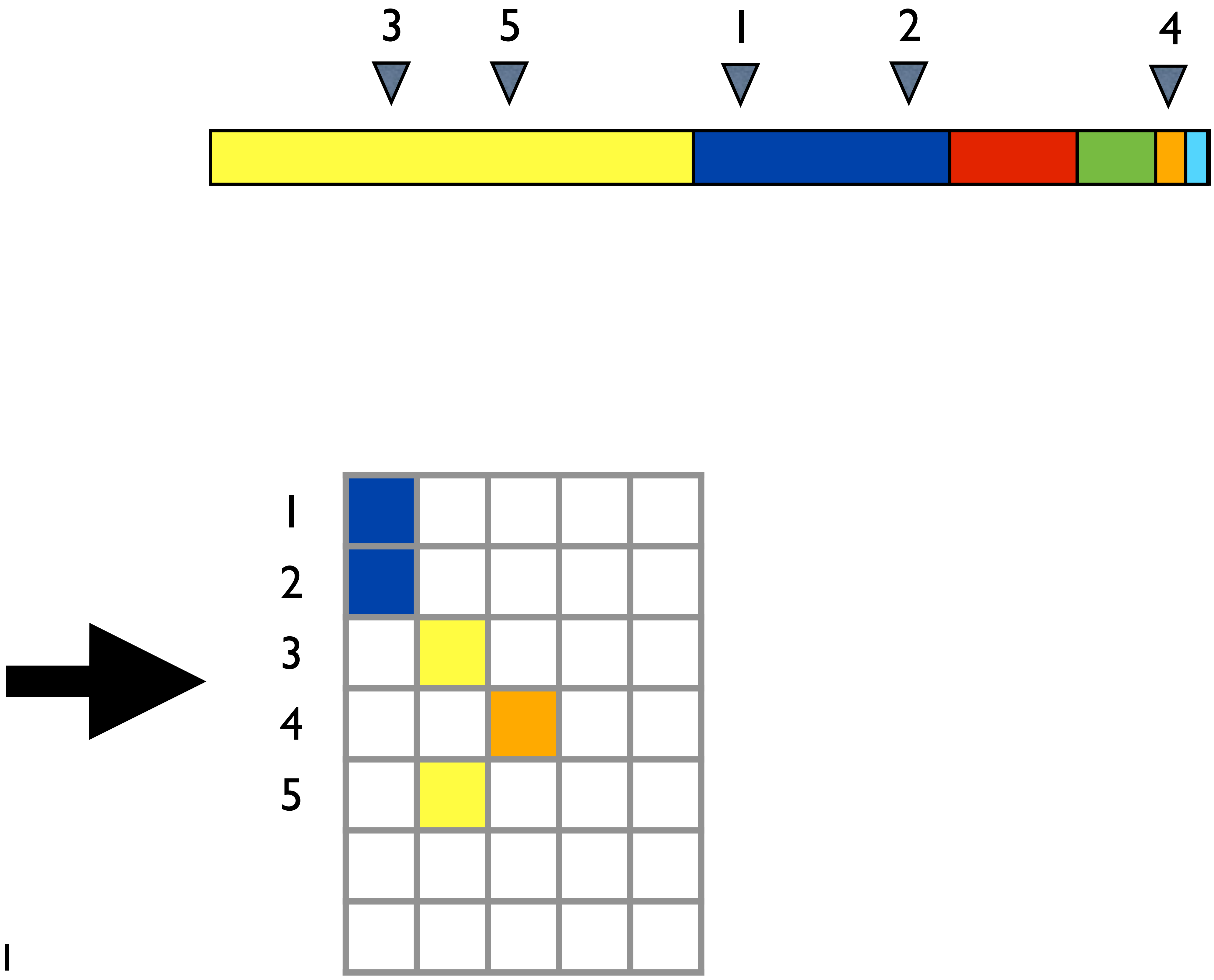

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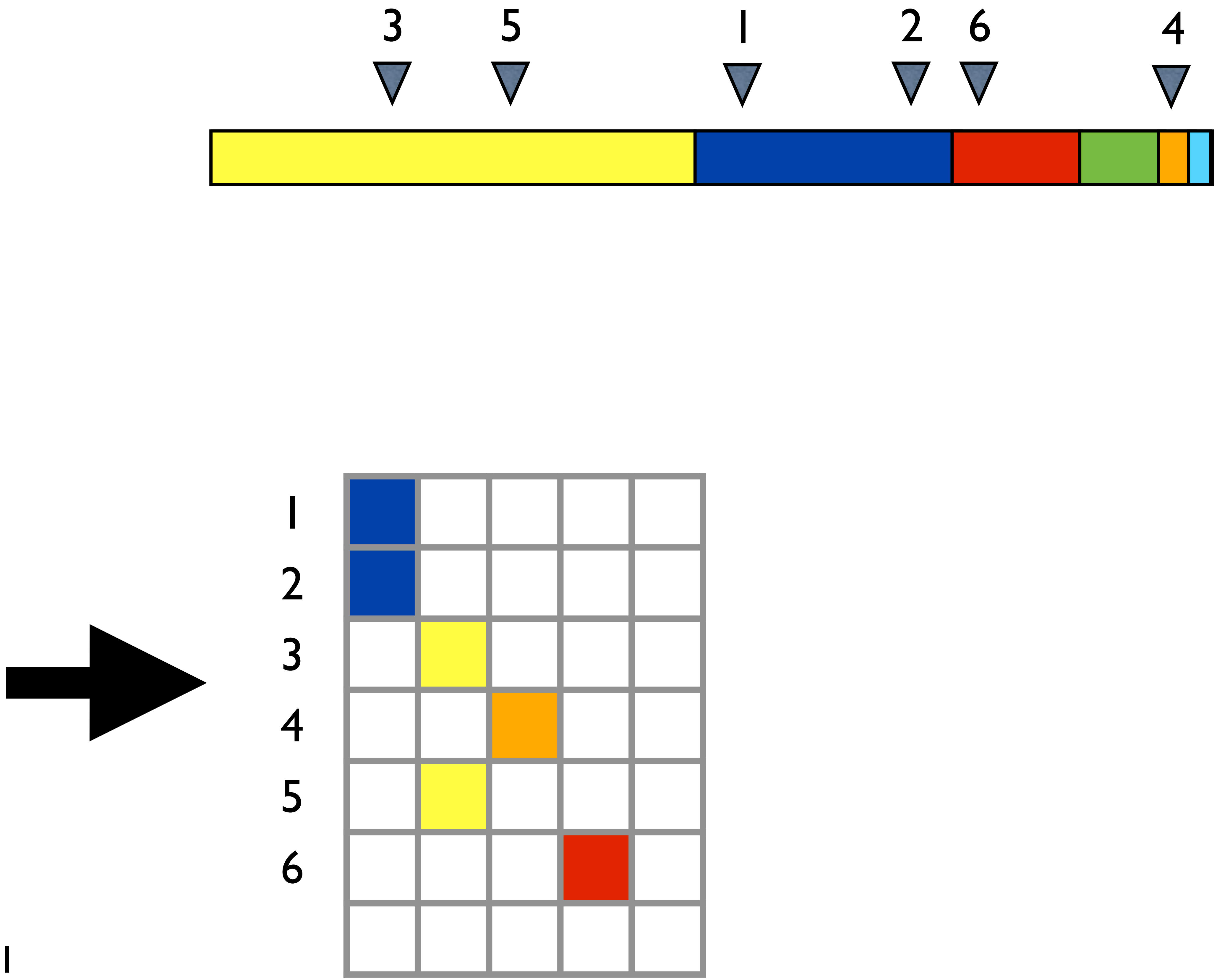

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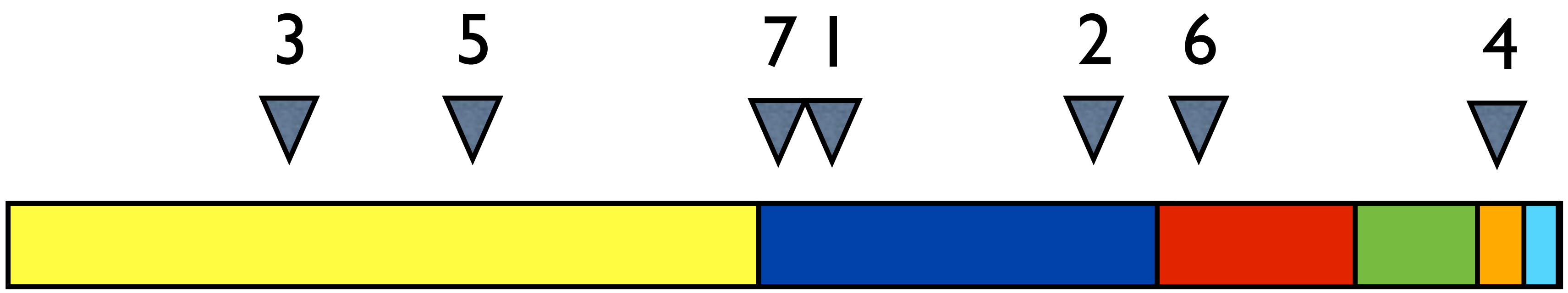

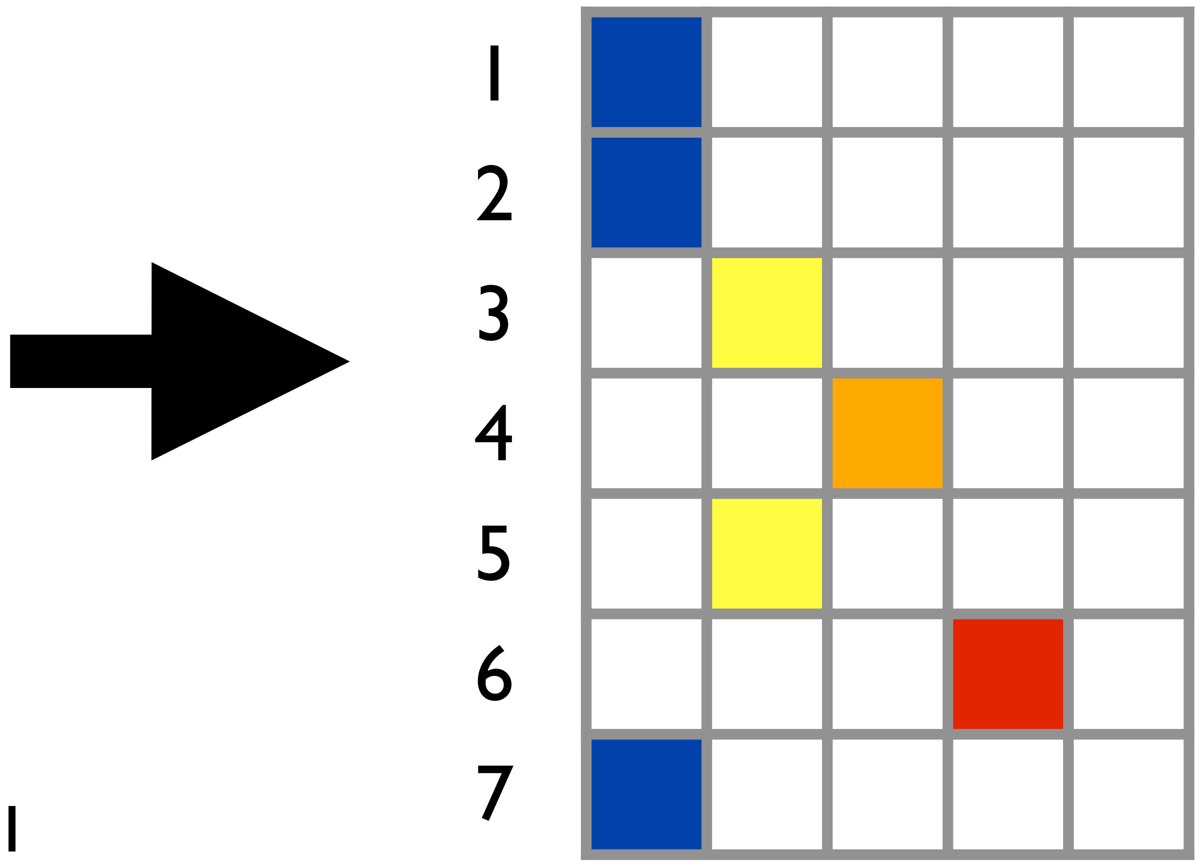

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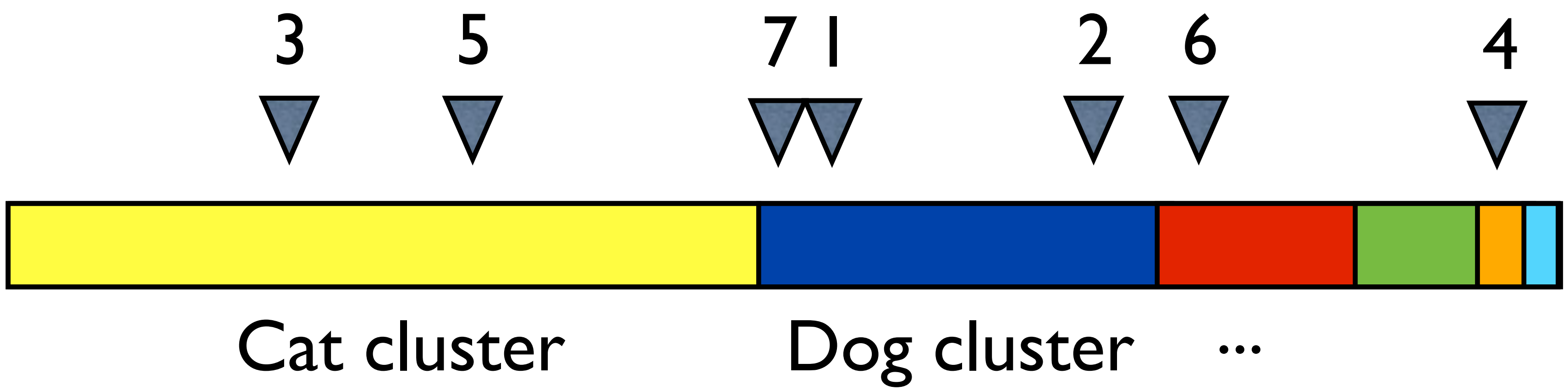

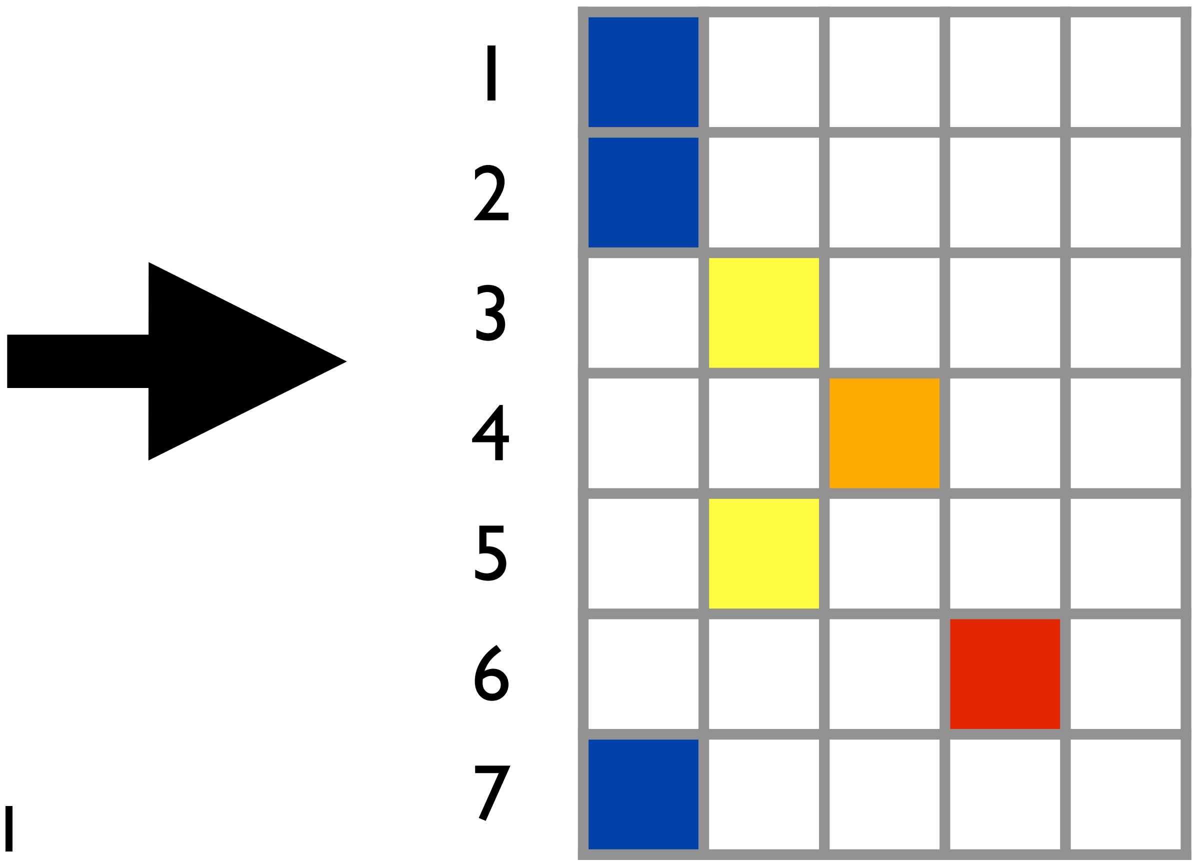

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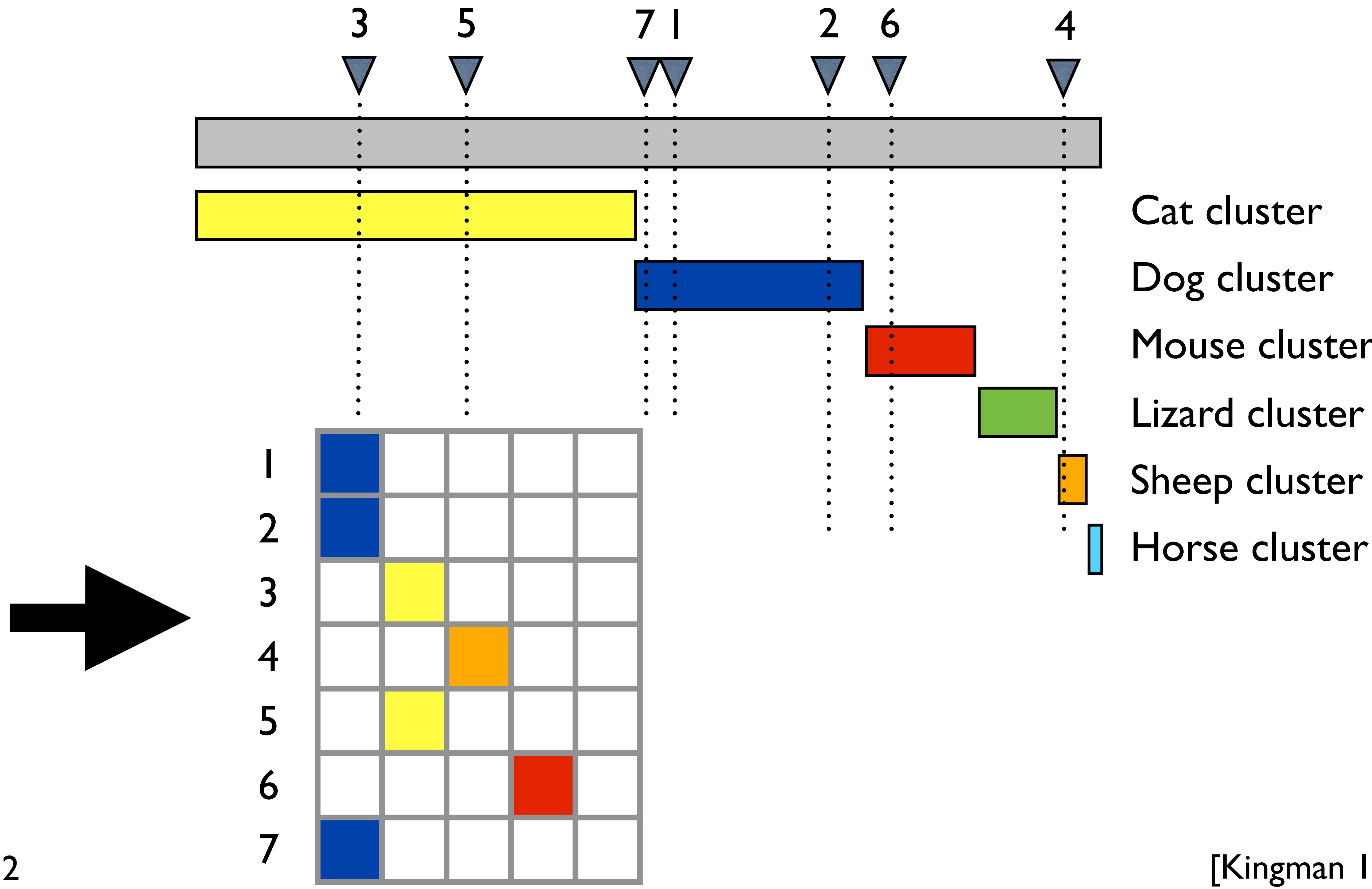

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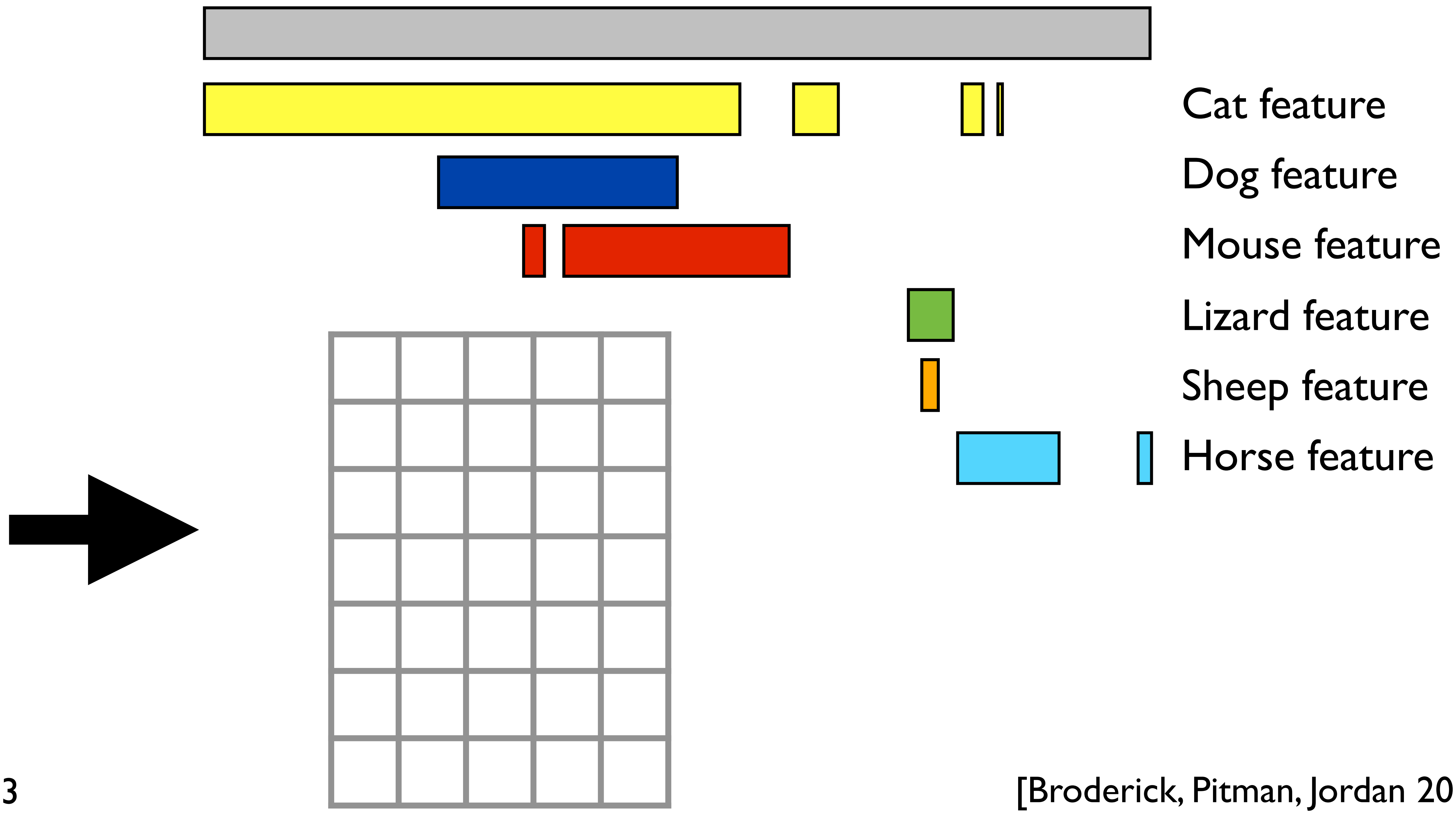
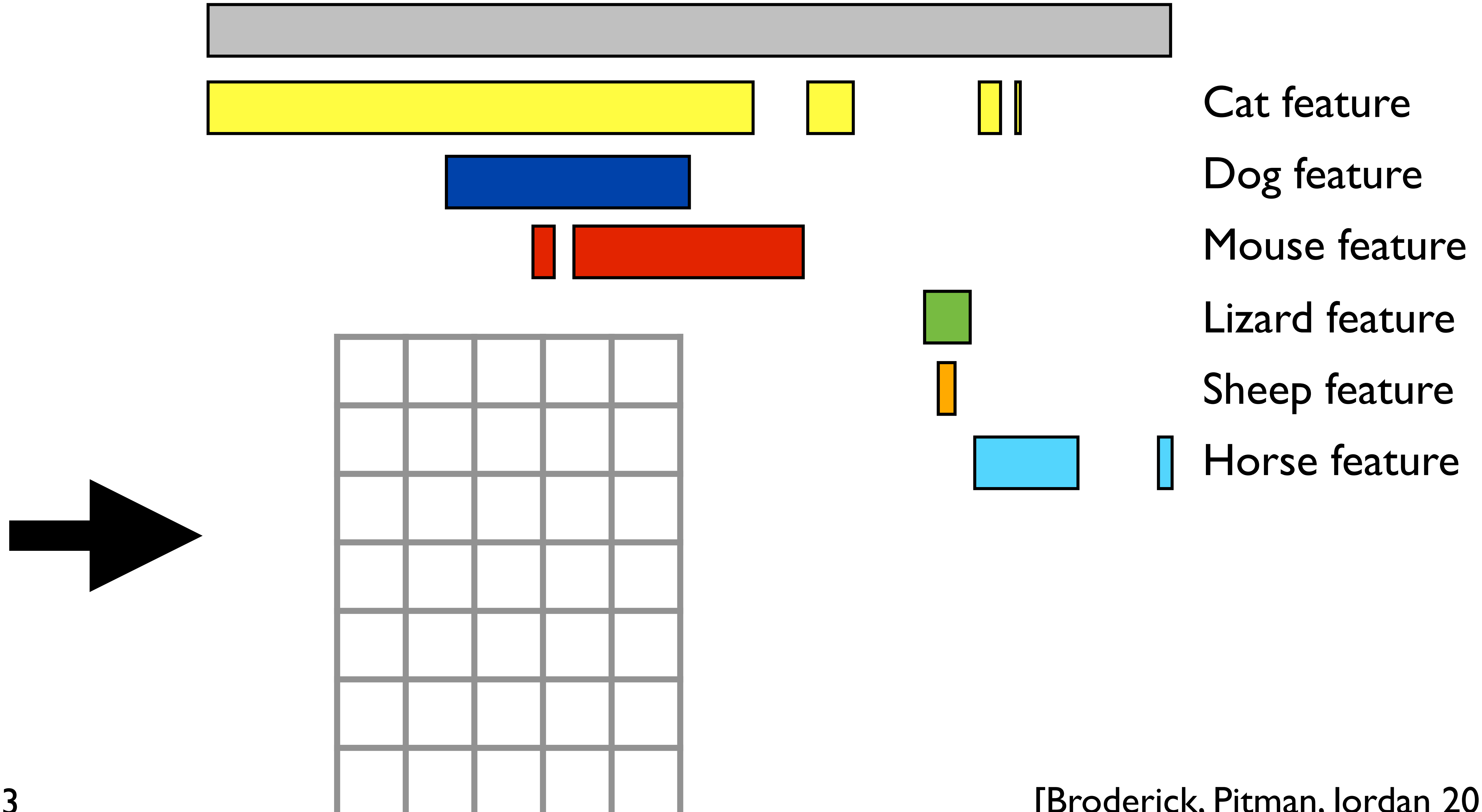
Paintboxes



[Broderick, Pitman, Jordan 2013]

Paintboxes

Exchangeable feature allocation: feature paintbox



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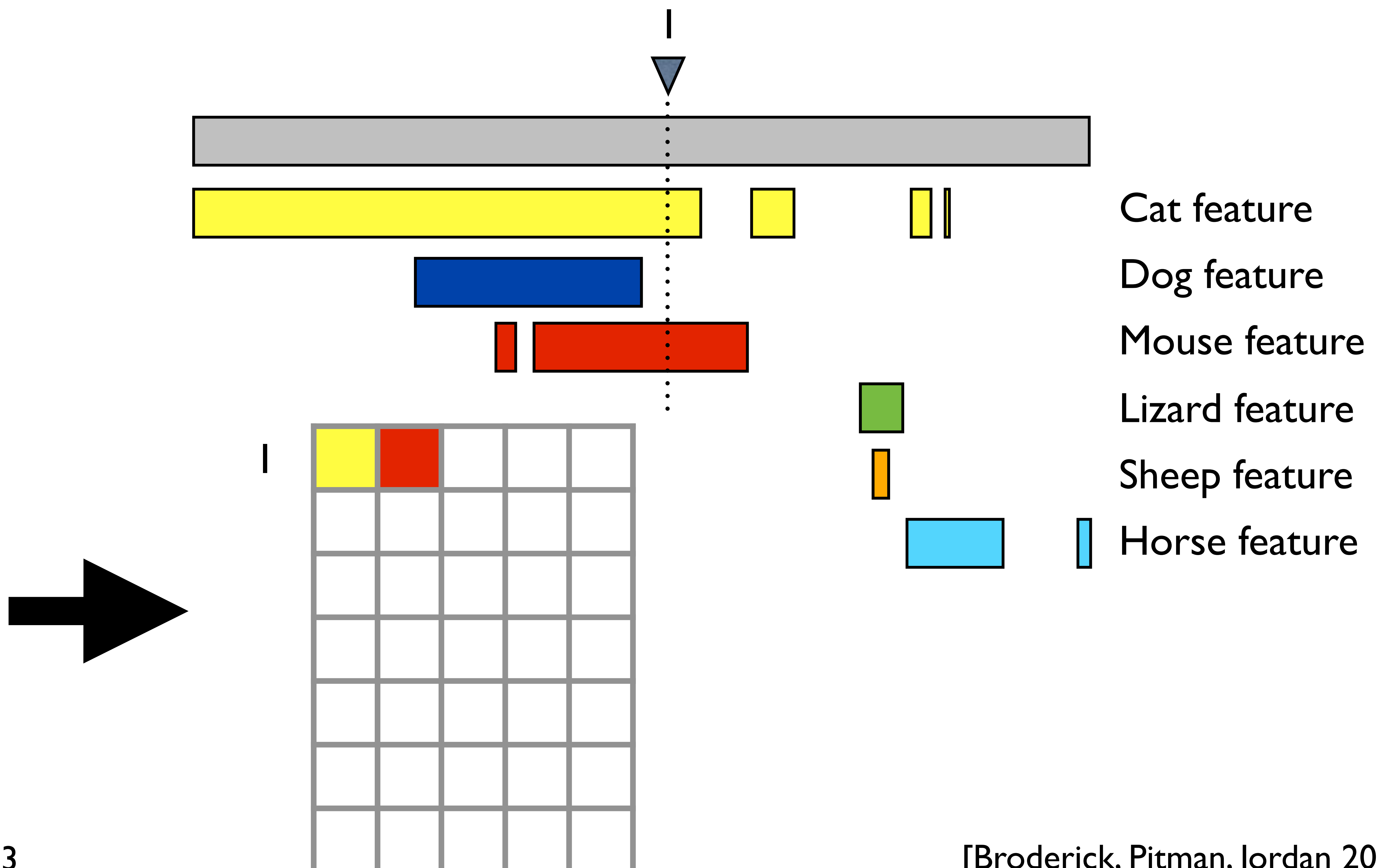

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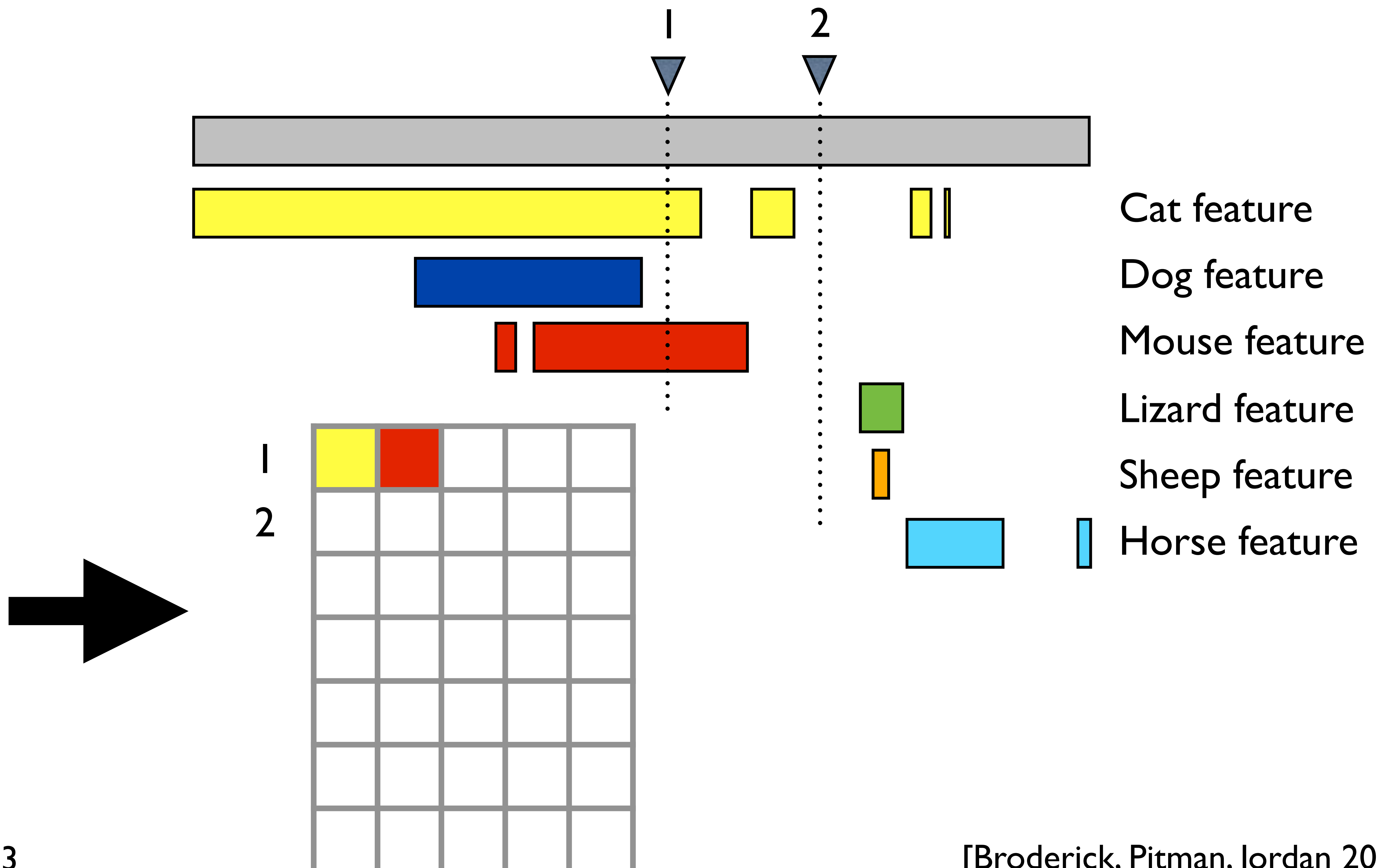

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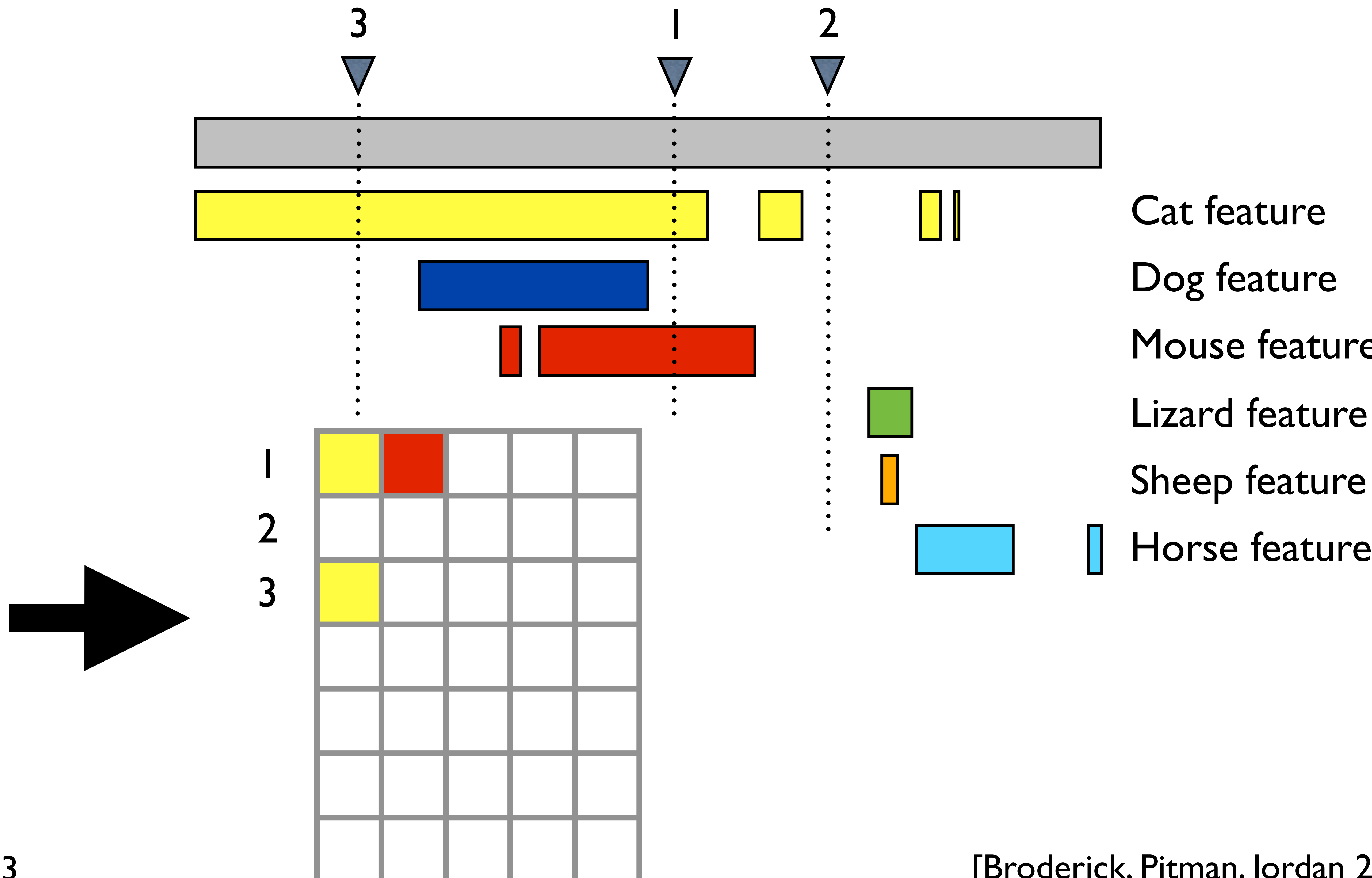

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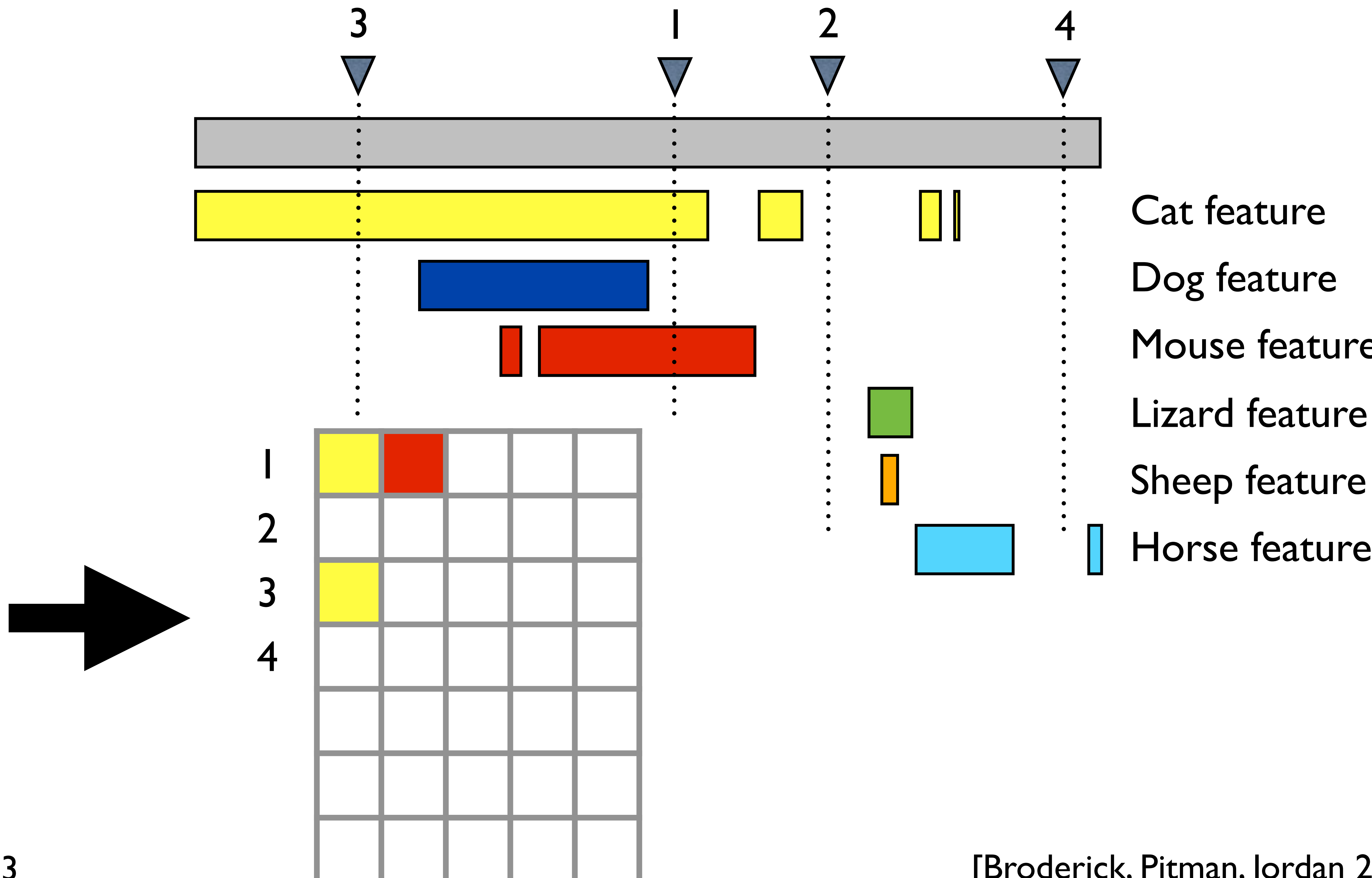

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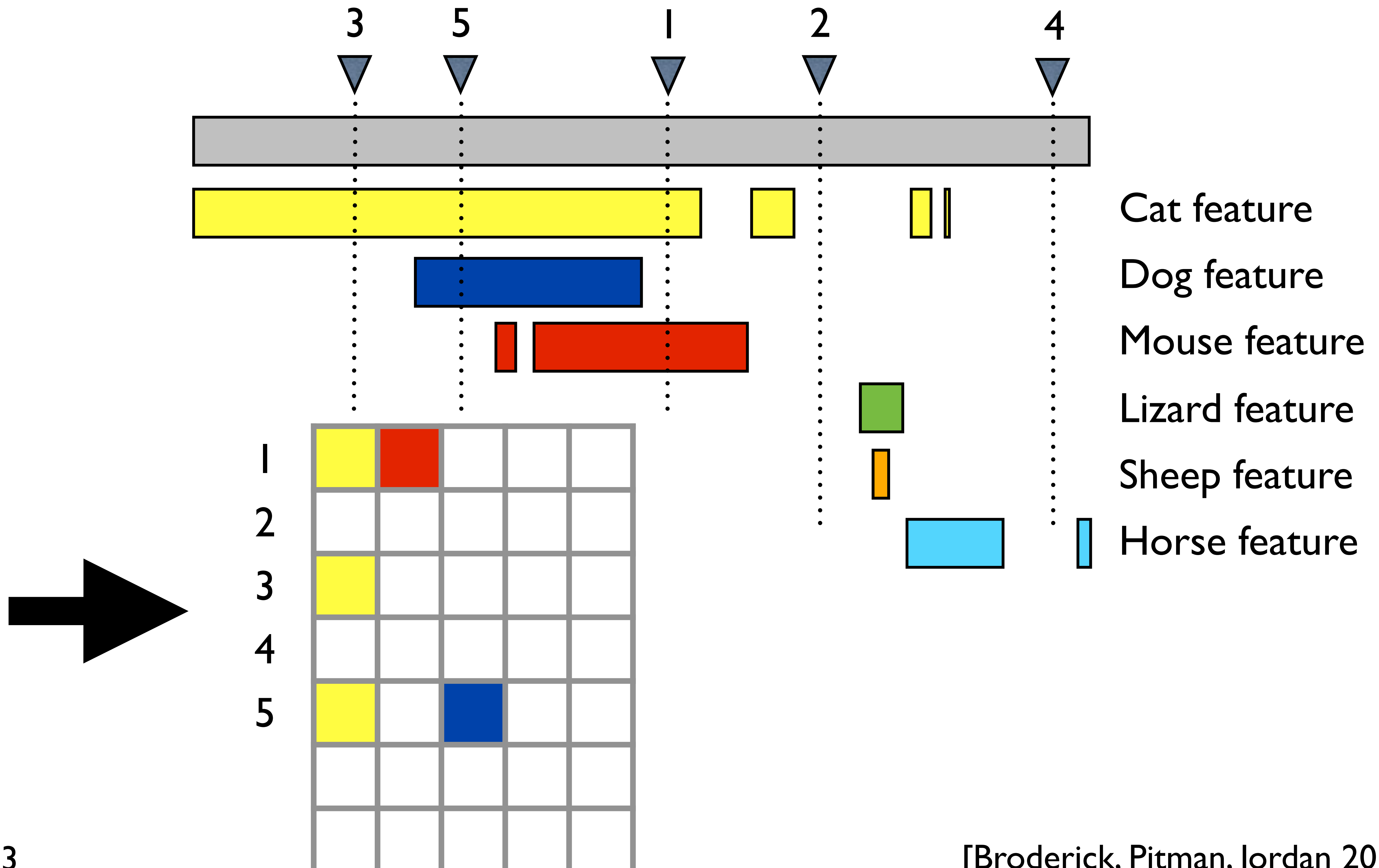

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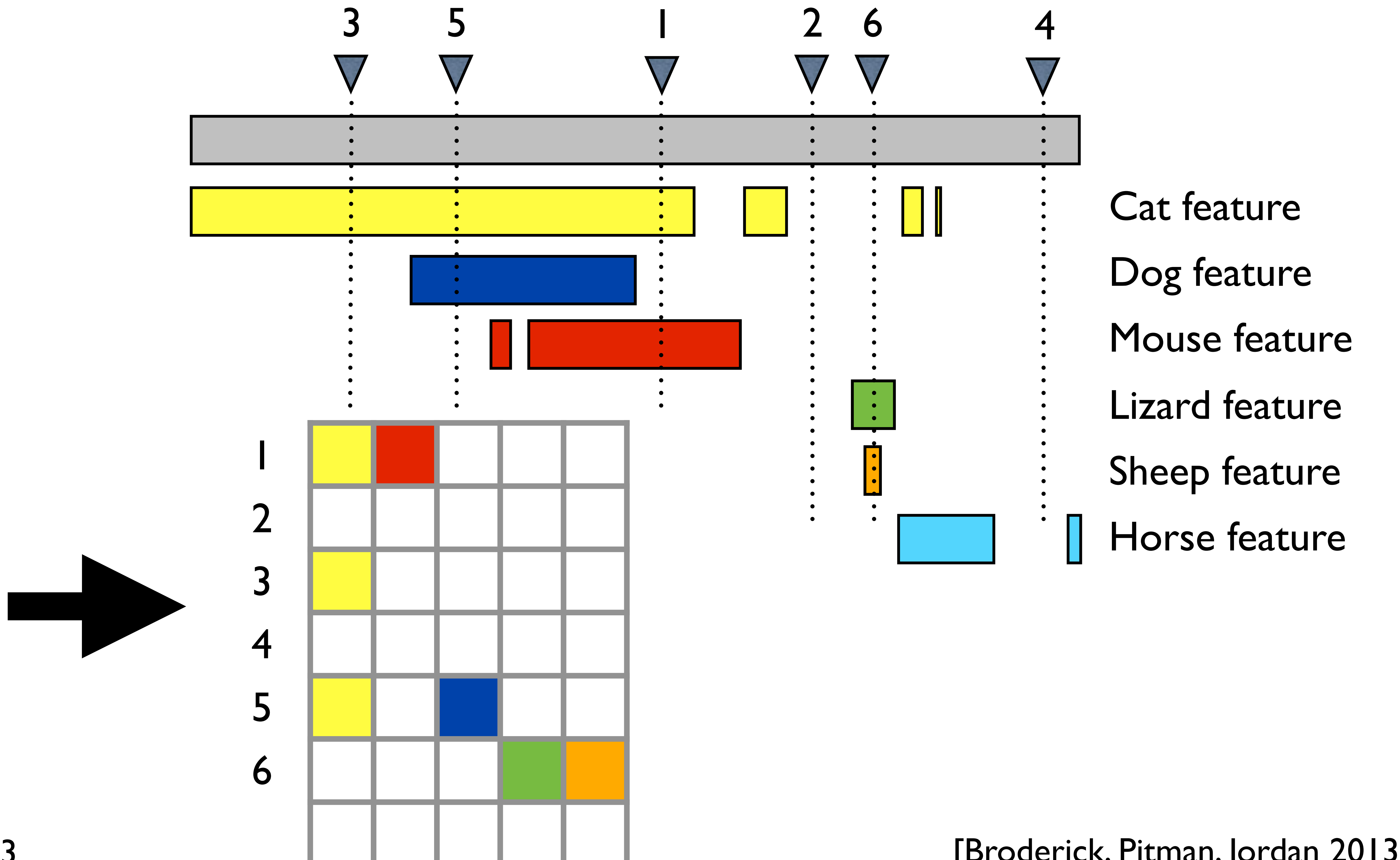

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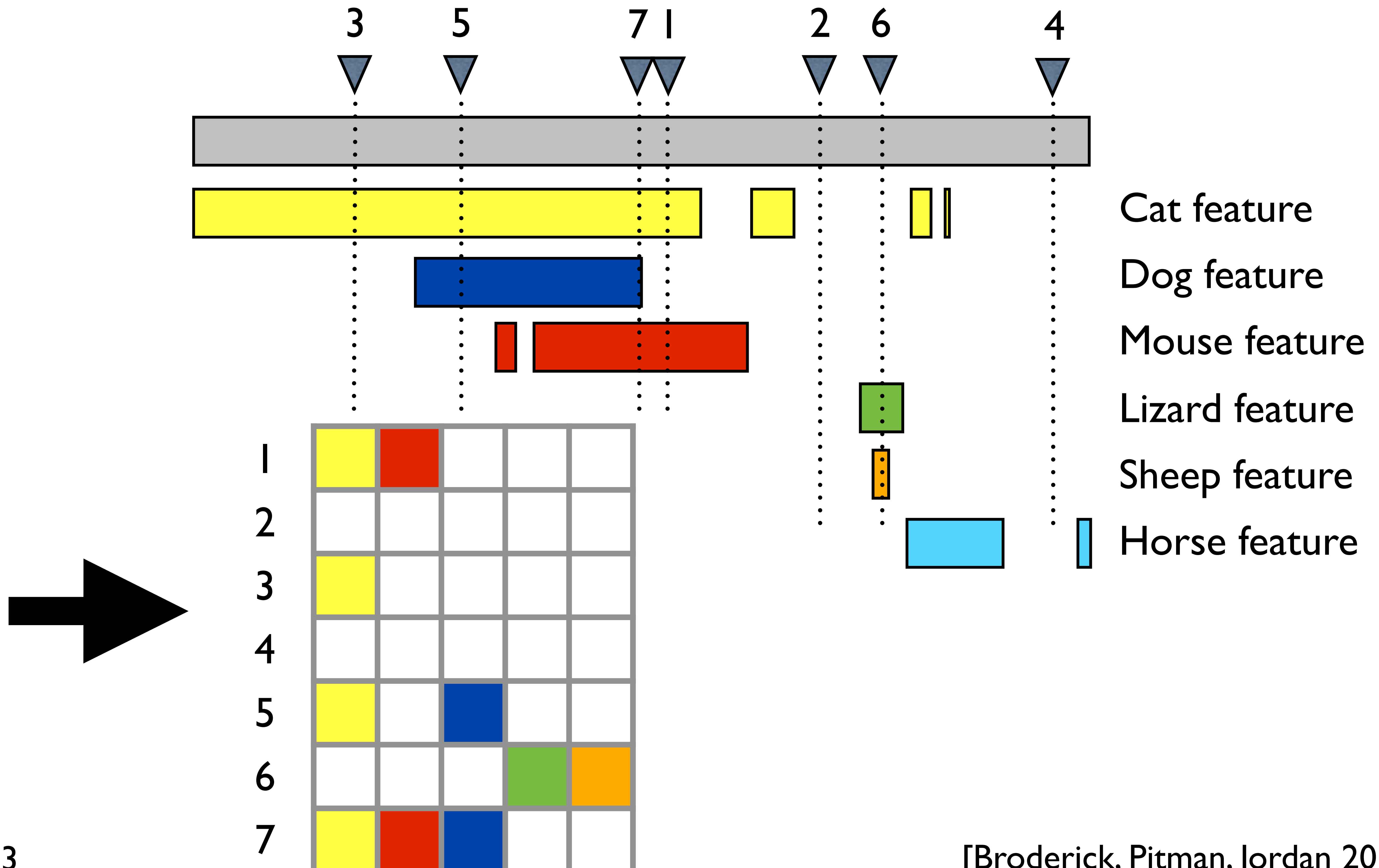

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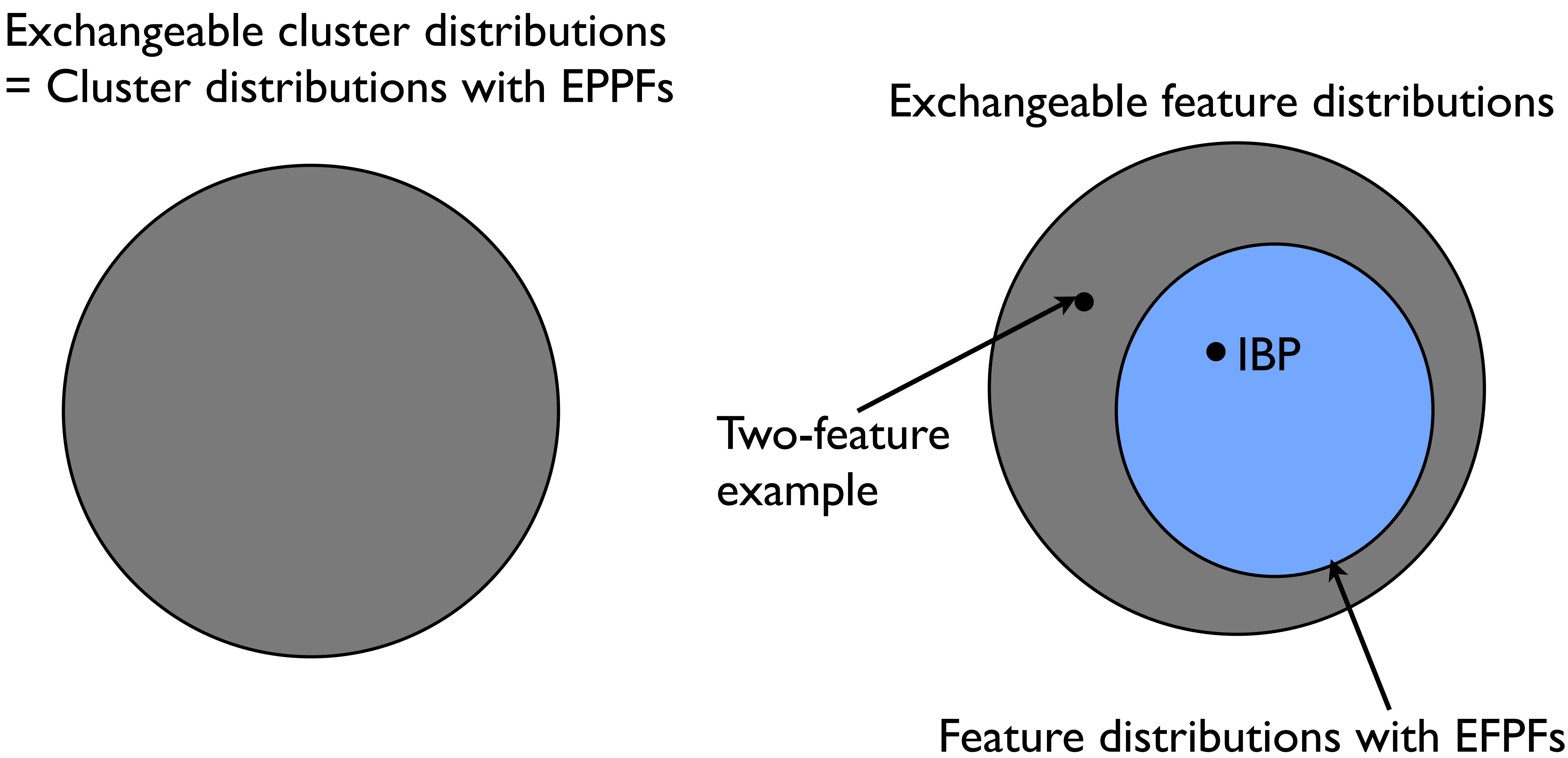
Paintboxes



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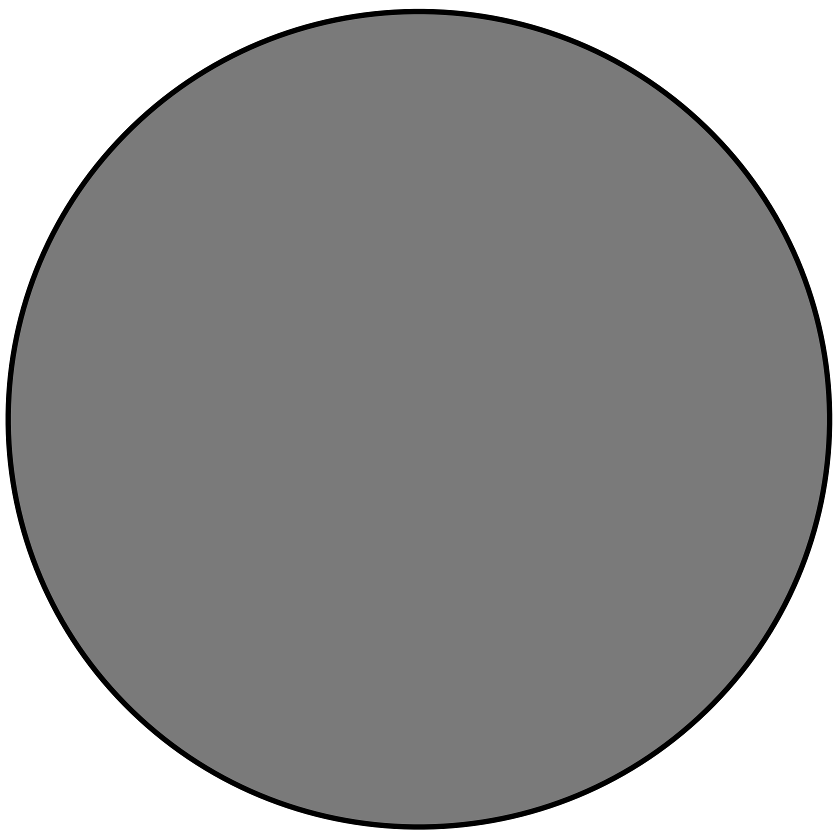
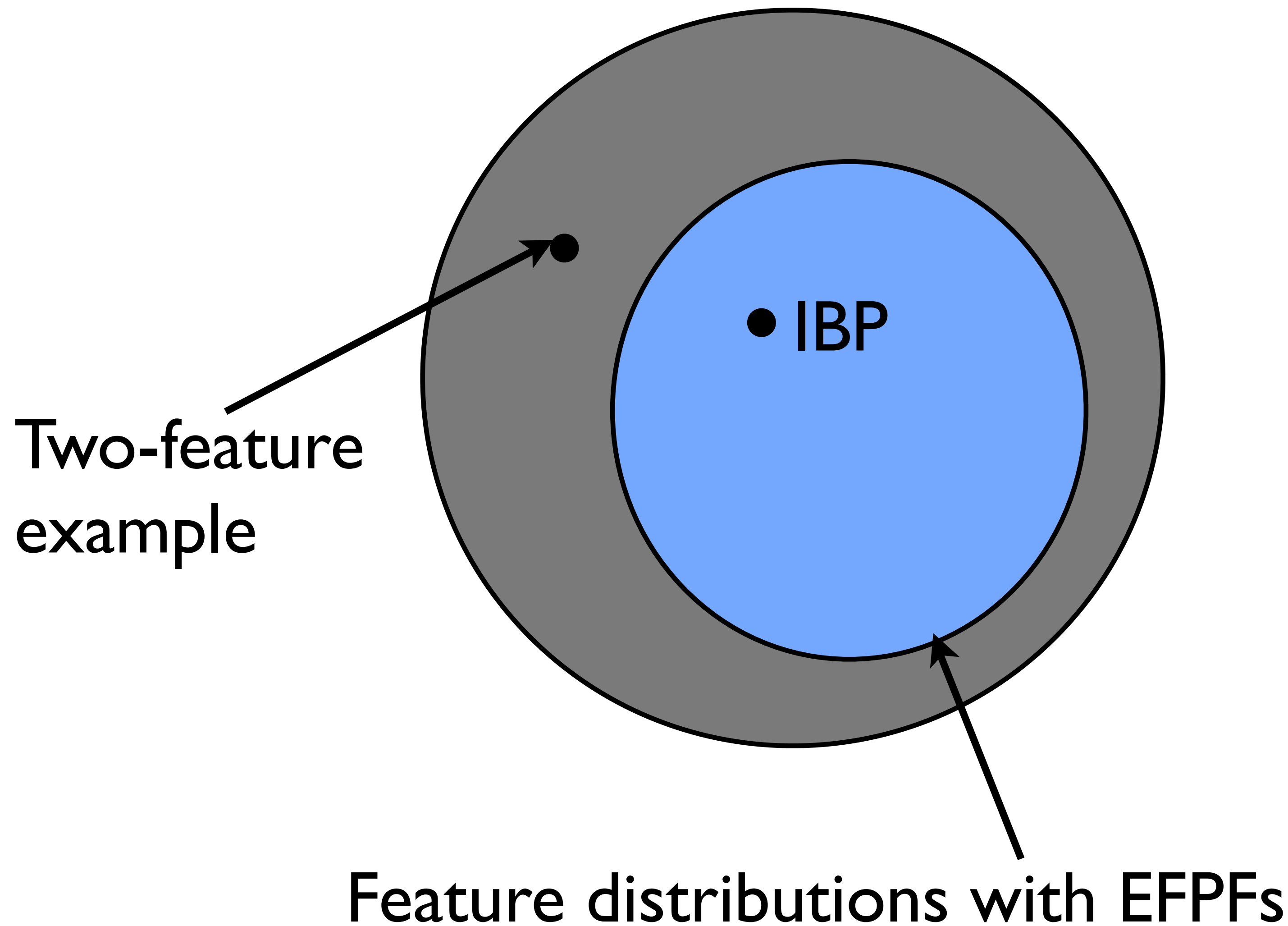
Paintboxes

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions

Exchangeable feature distributions
= Feature paintbox allocations

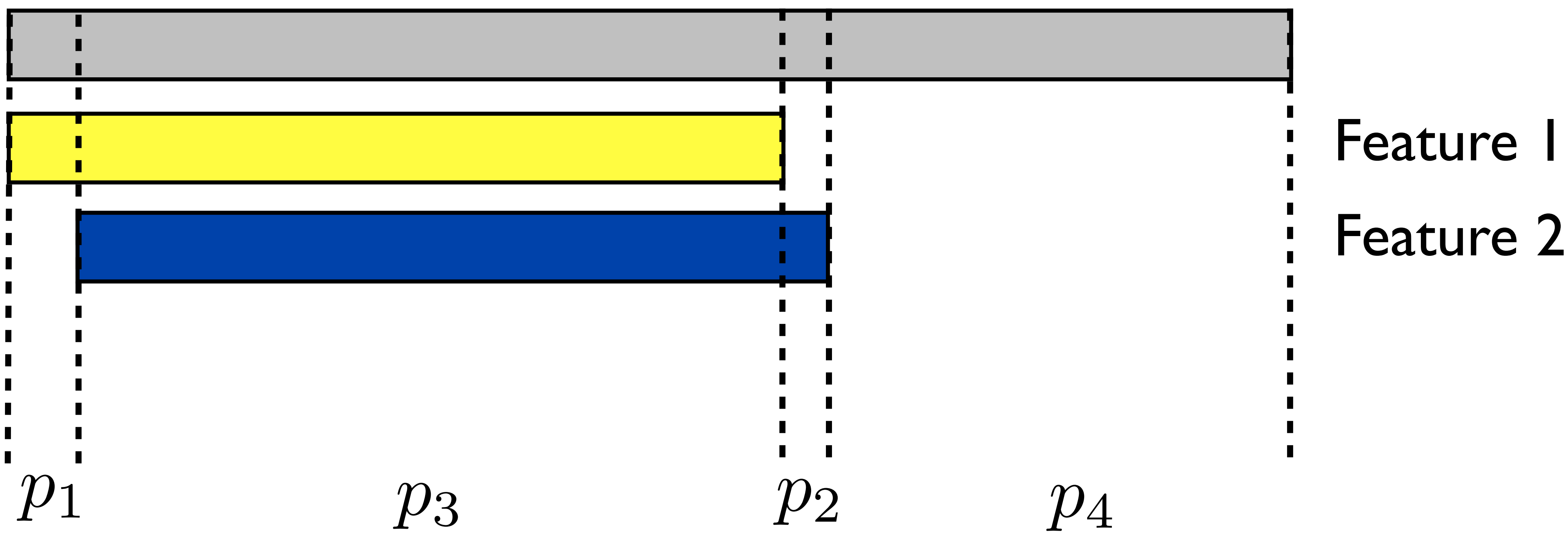




[Broderick, Pitman, Jordan 2013]

Paintboxes

Two feature example



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Paintboxes

Indian buffet process: beta feature frequencies

[Thibaux, Jordan 2007]

Paintboxes

Indian buffet process: beta feature frequencies

For *m* = 1, 2, ...

1. Draw $K_m^+ = \mathrm{Poisson}\left(\gamma \dfrac{\theta}{\theta + m - 1}\right)$

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2. For k = $K_{m-1} + 1, \ldots, K_m$

   Draw a frequency of size

$$q_k \sim \mathrm{Beta}(1, \theta + m - 1)$$

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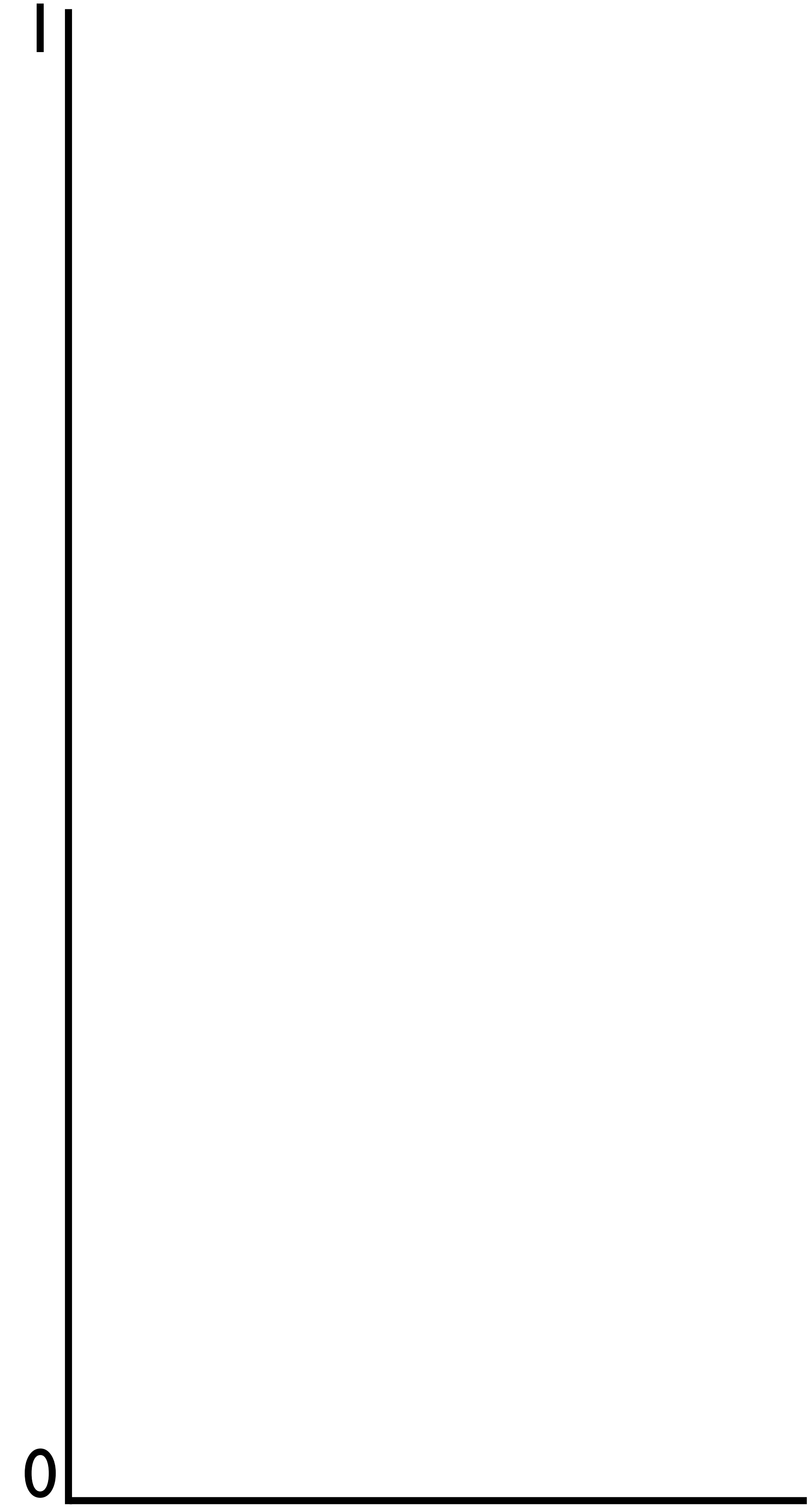

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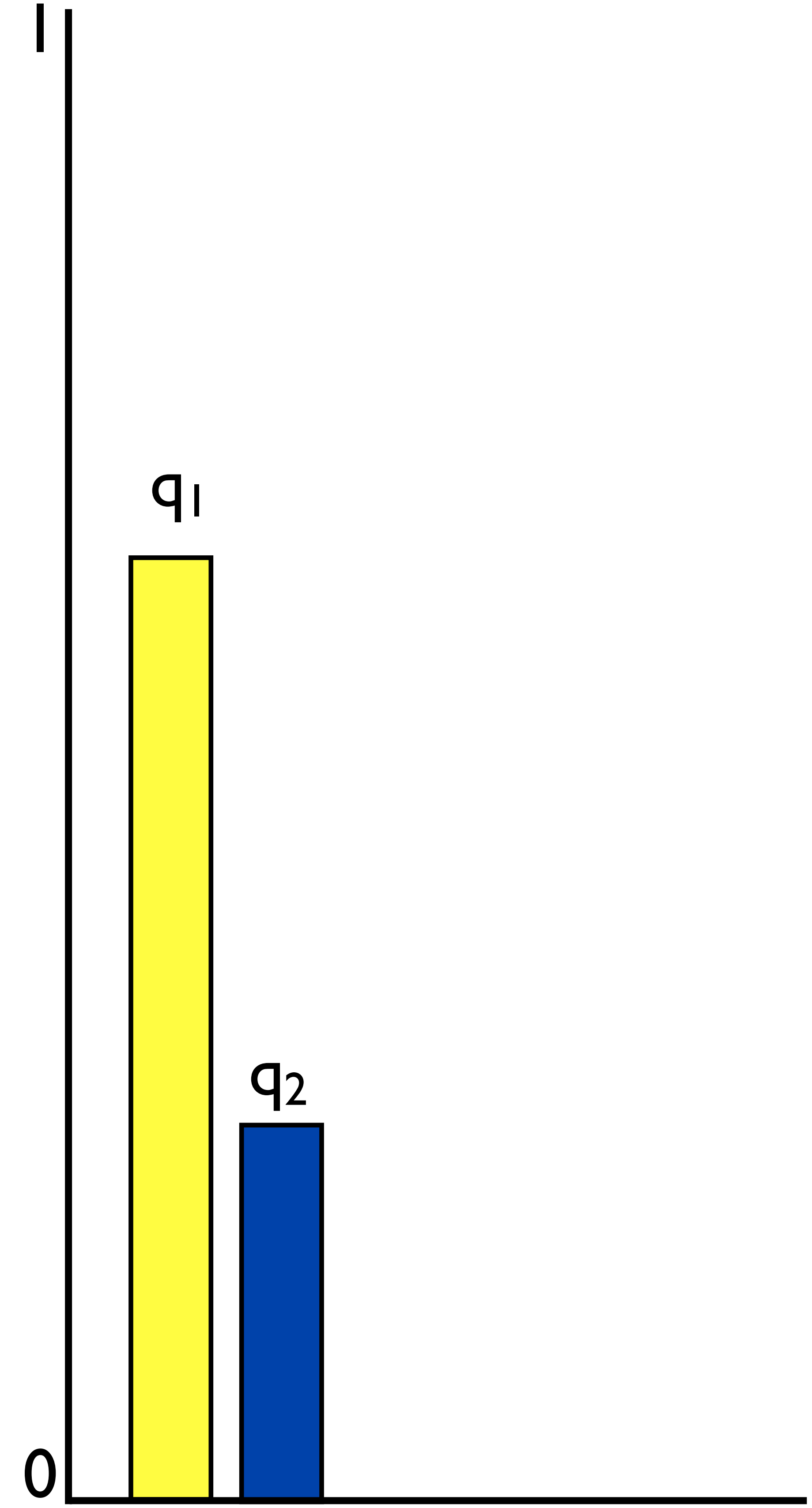

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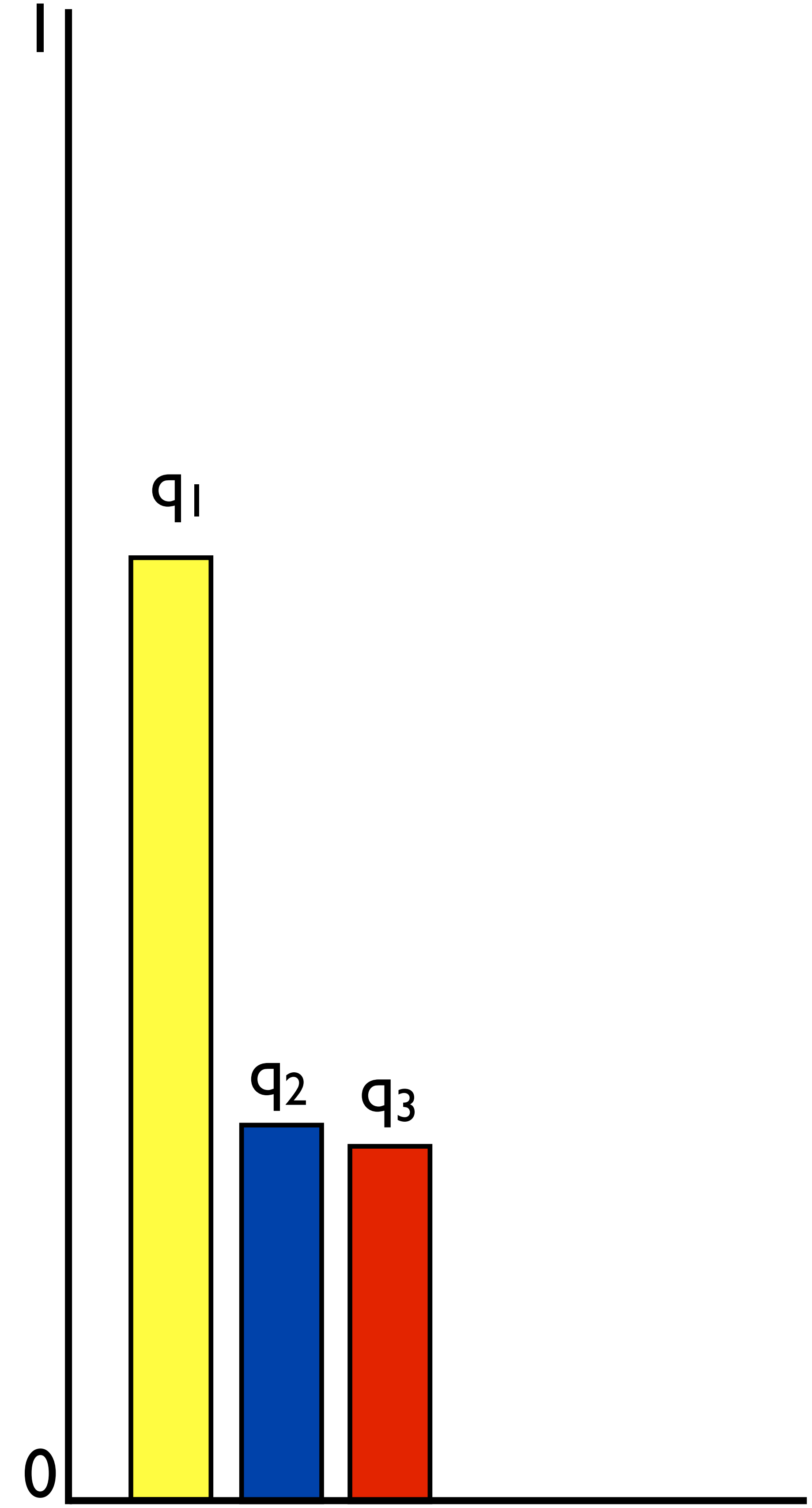

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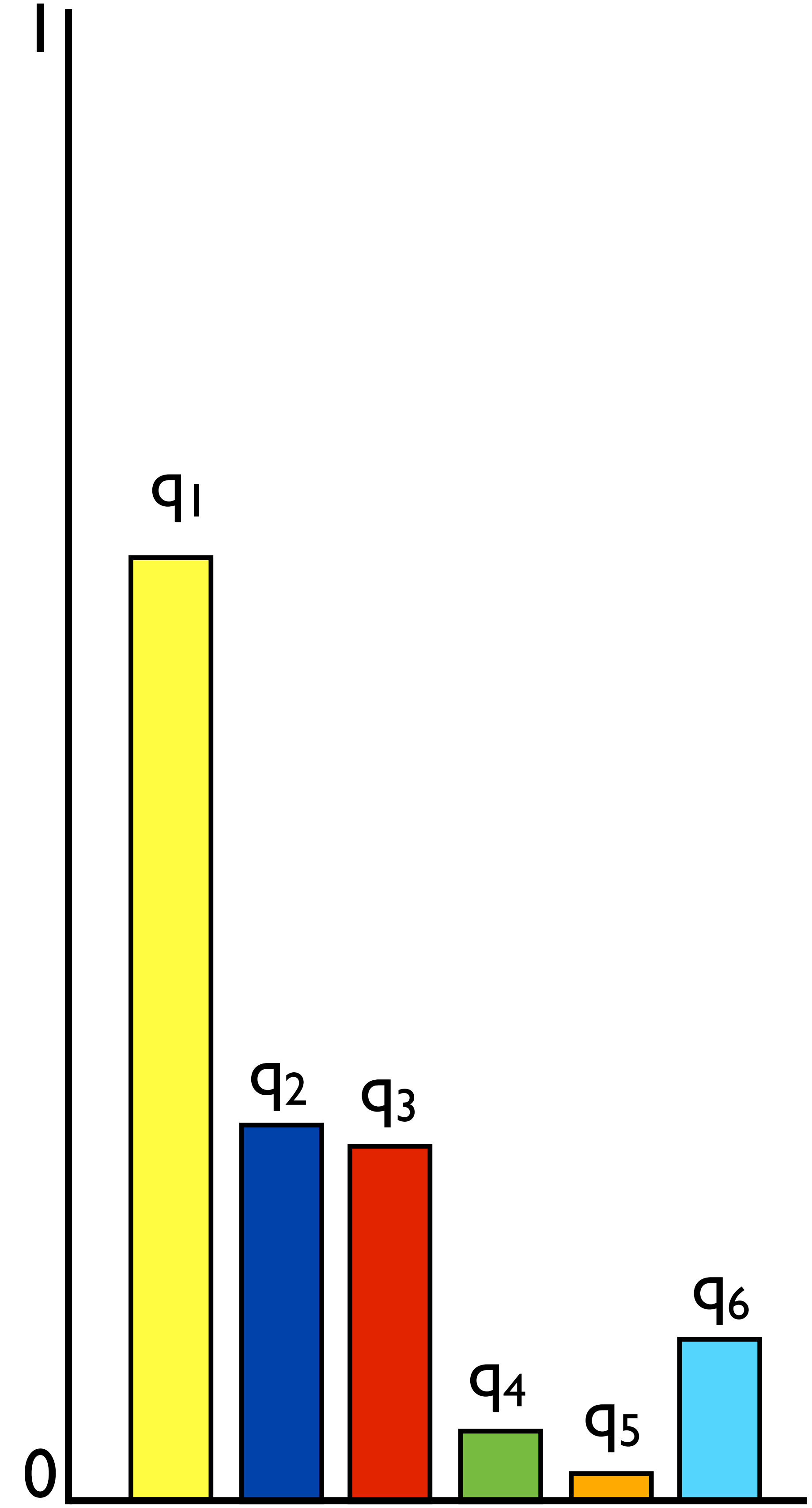

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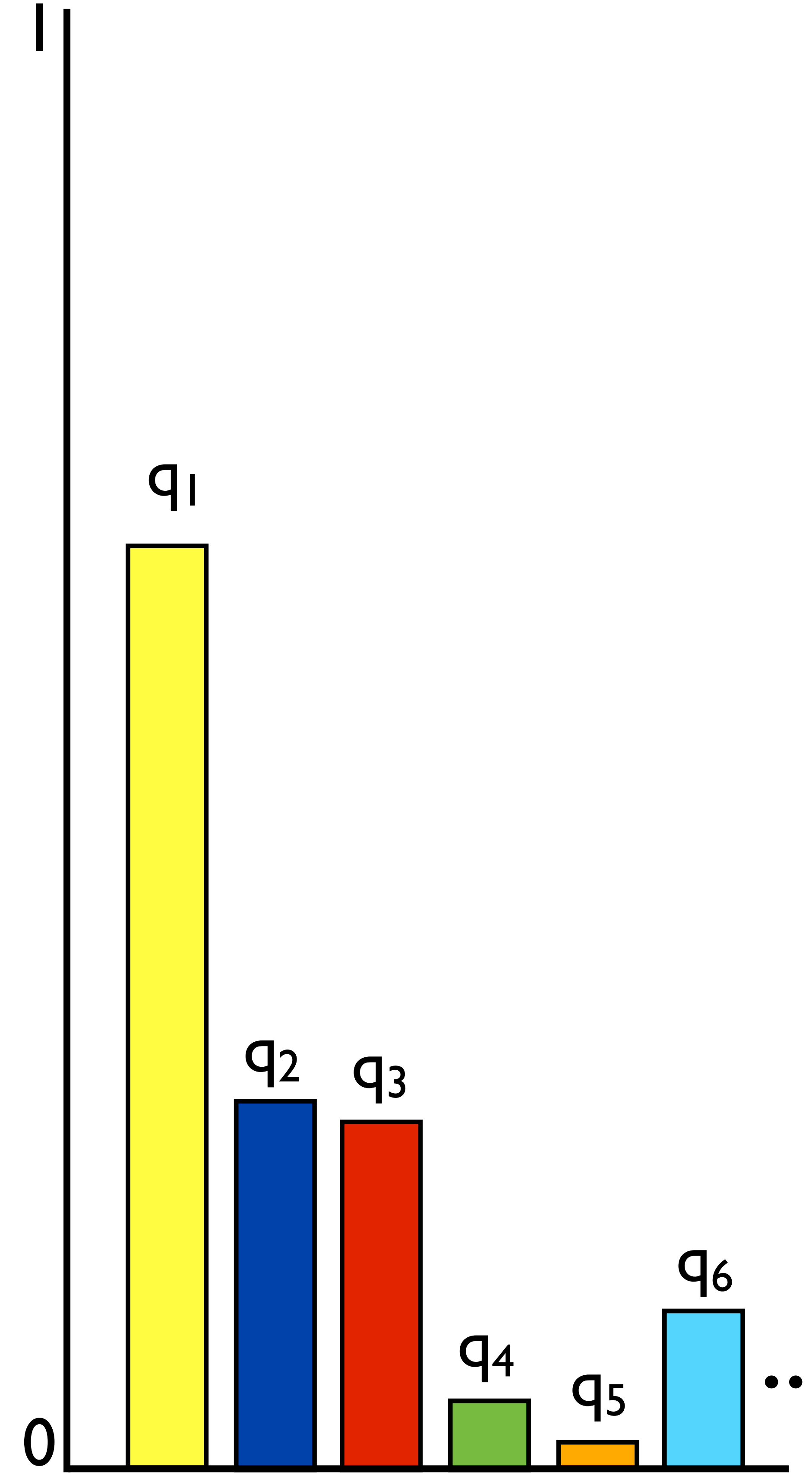

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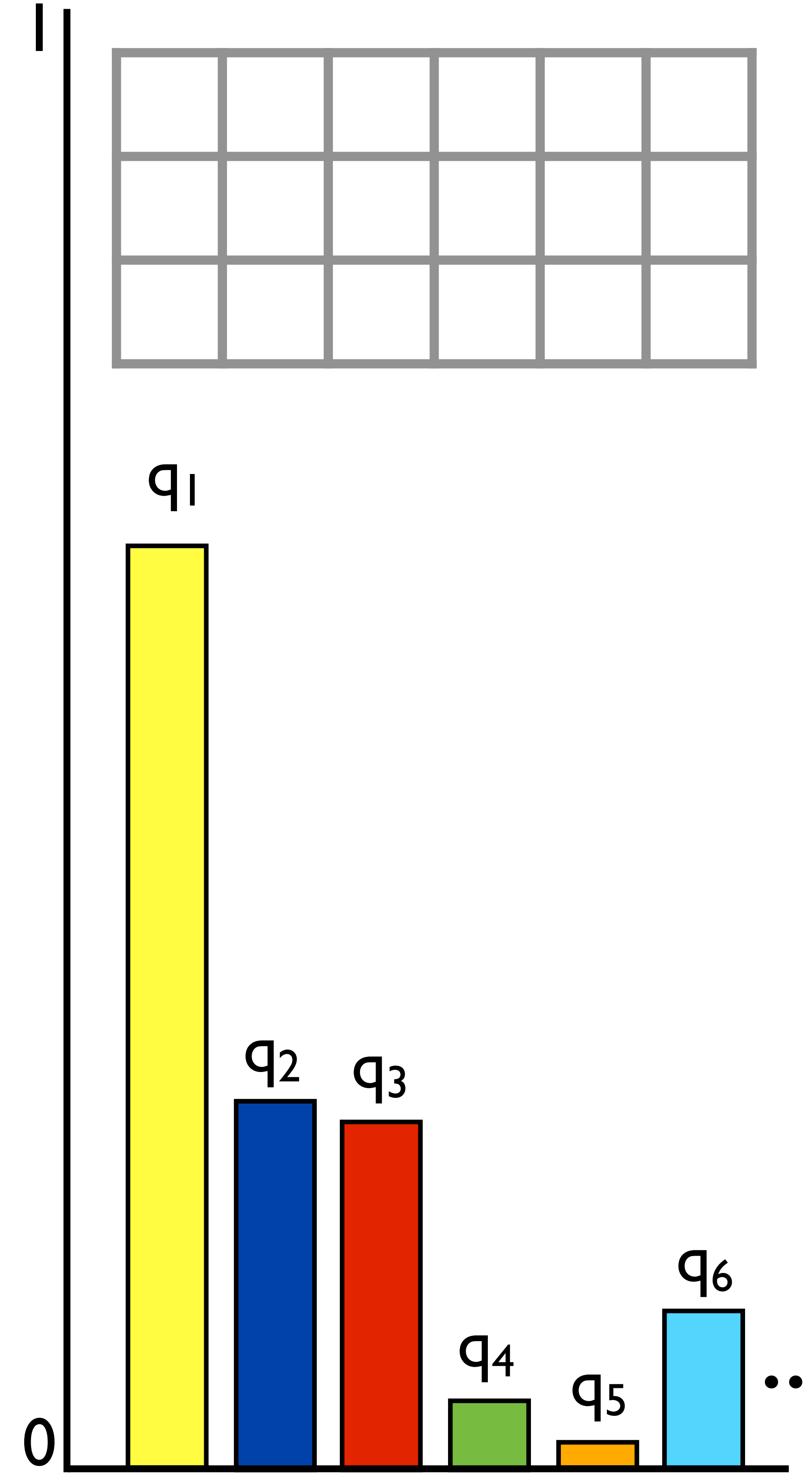

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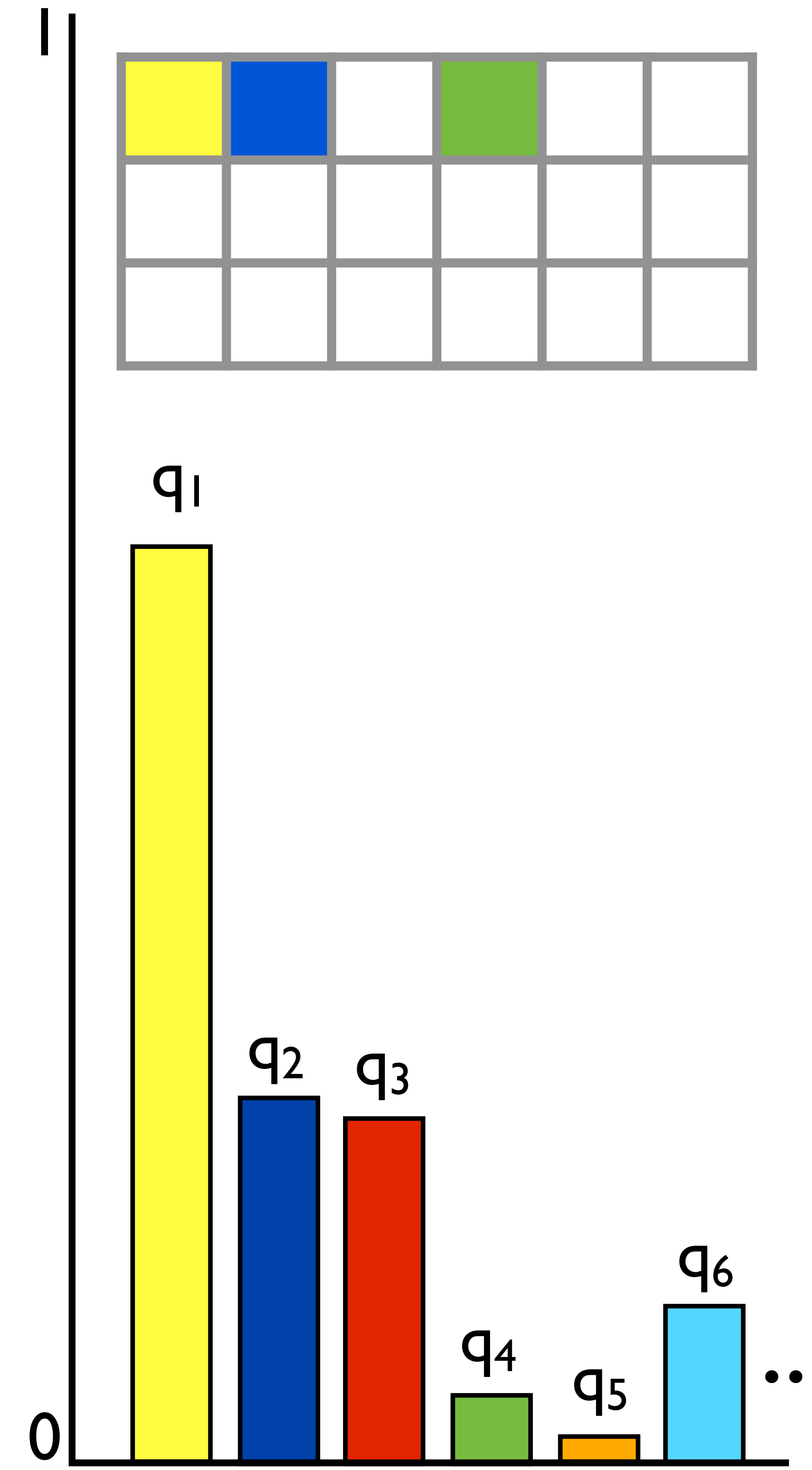

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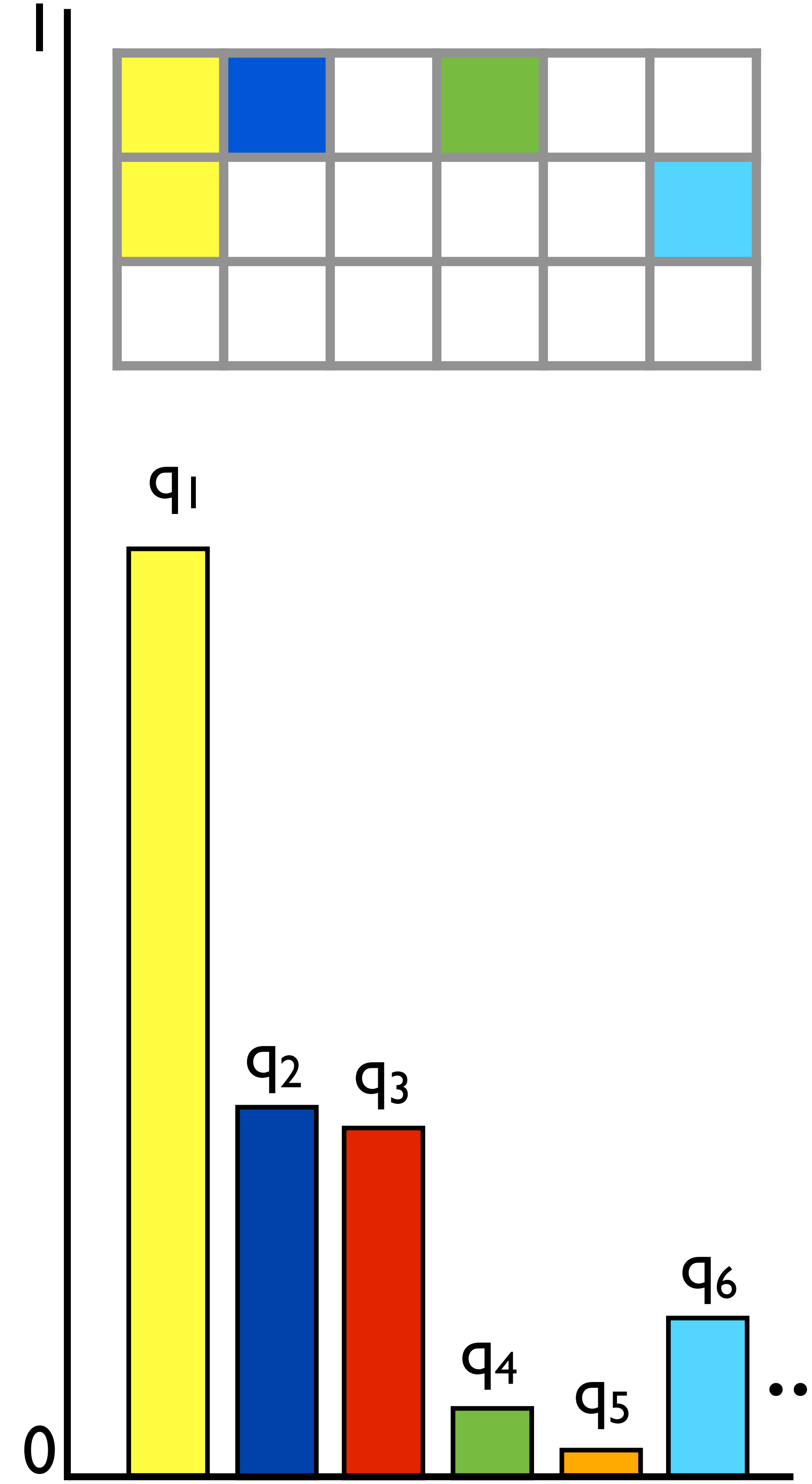

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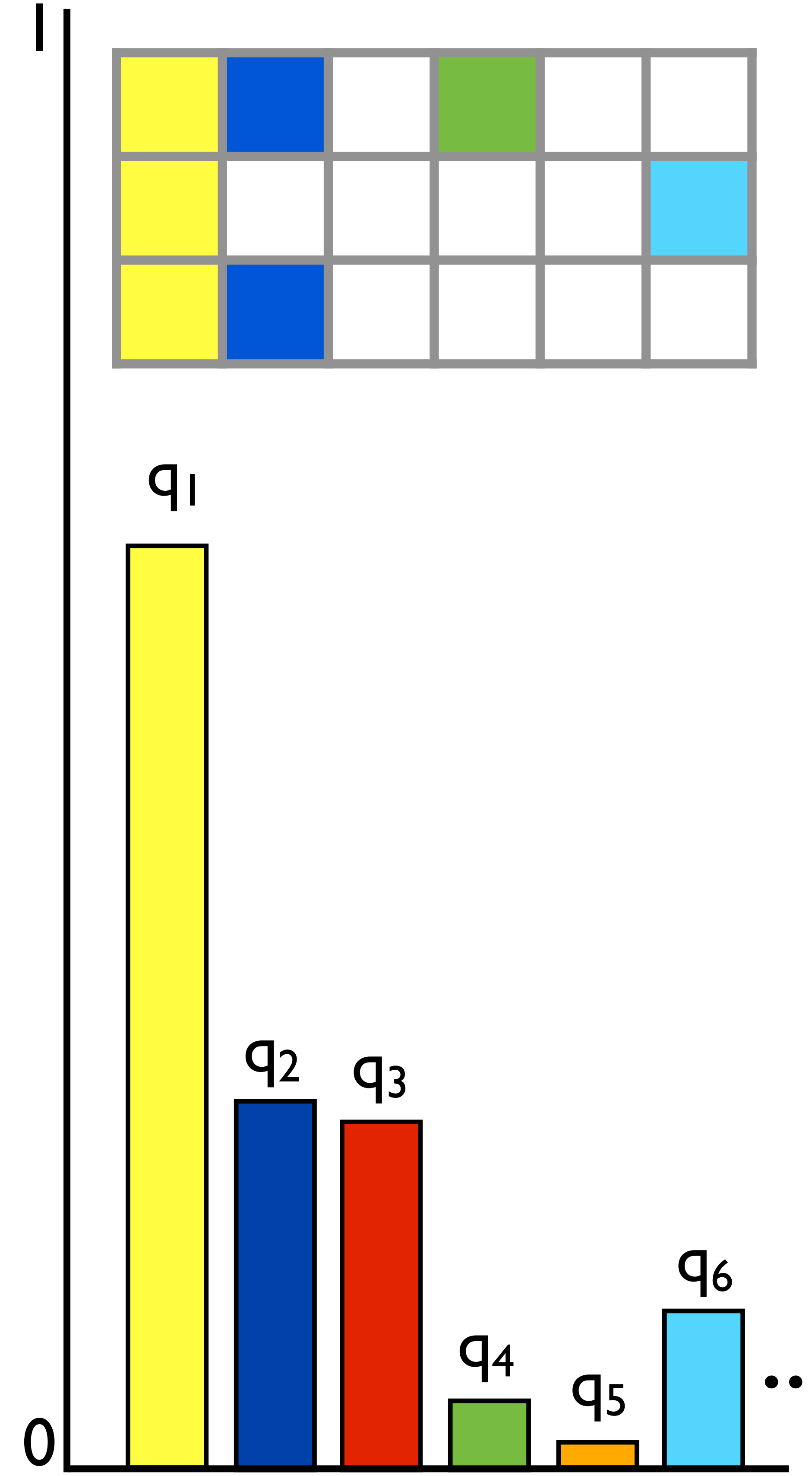

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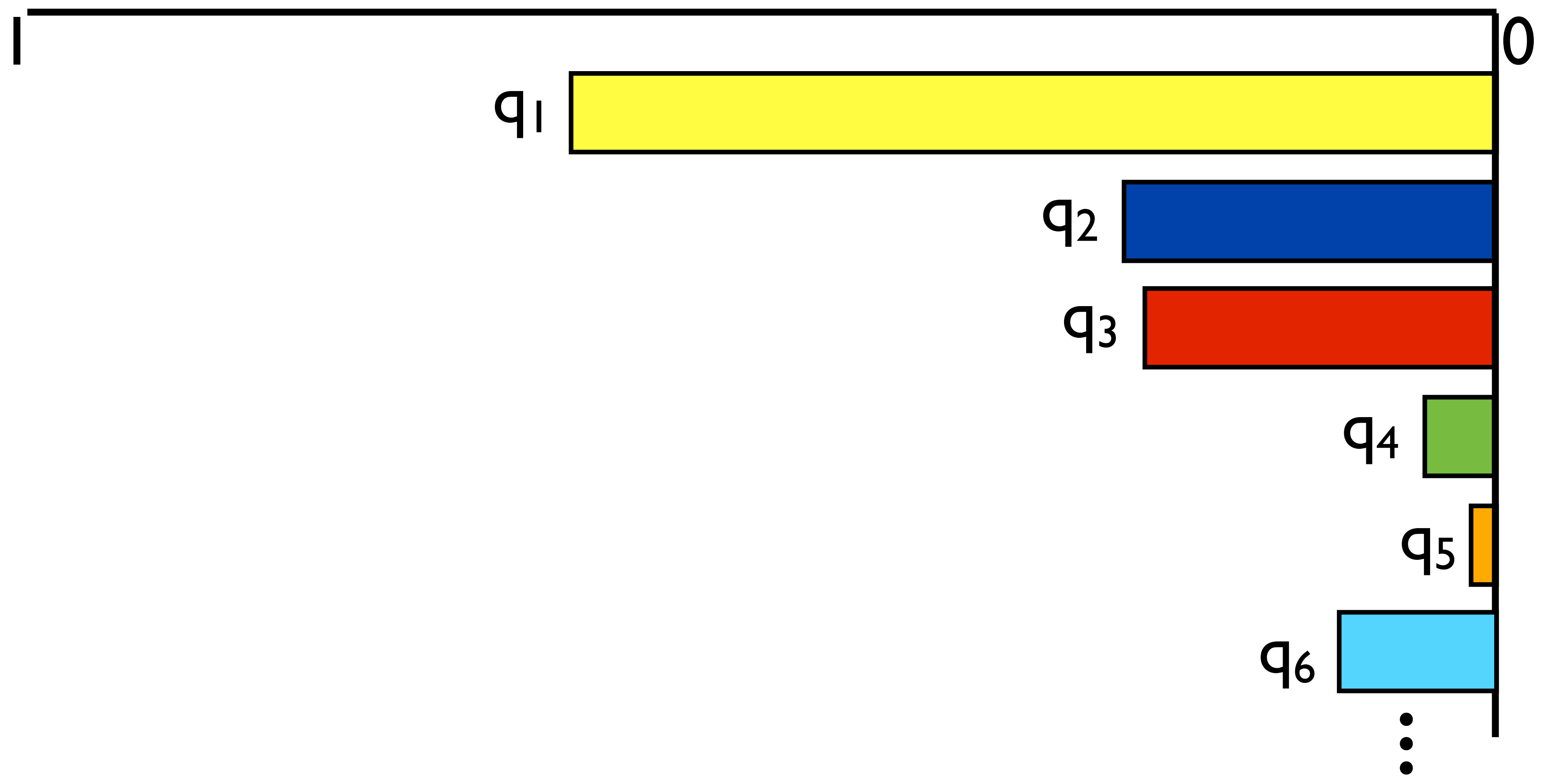

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Indian buffet process: beta feature frequencies

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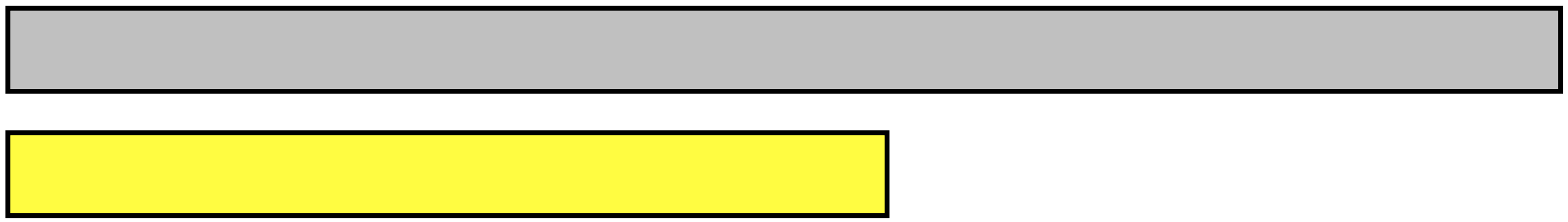

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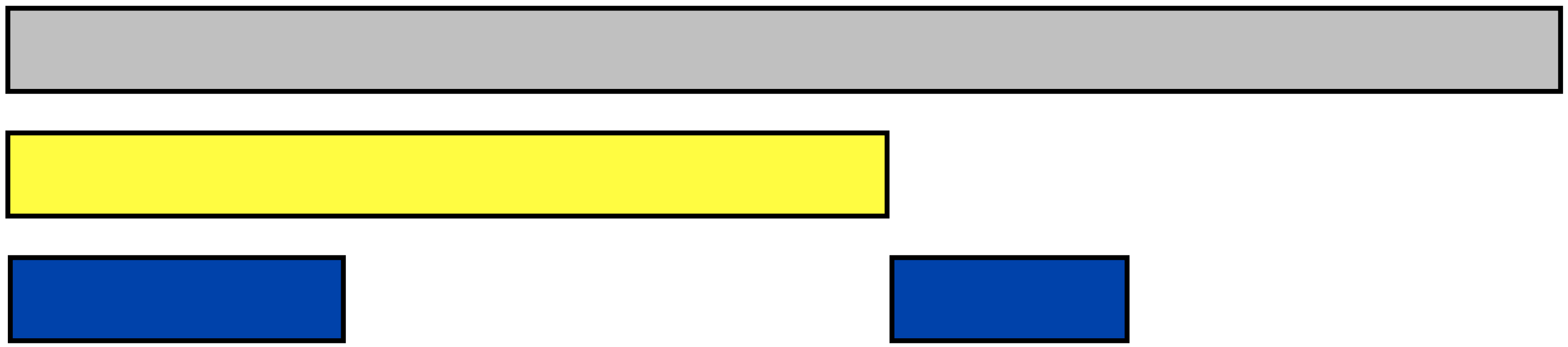

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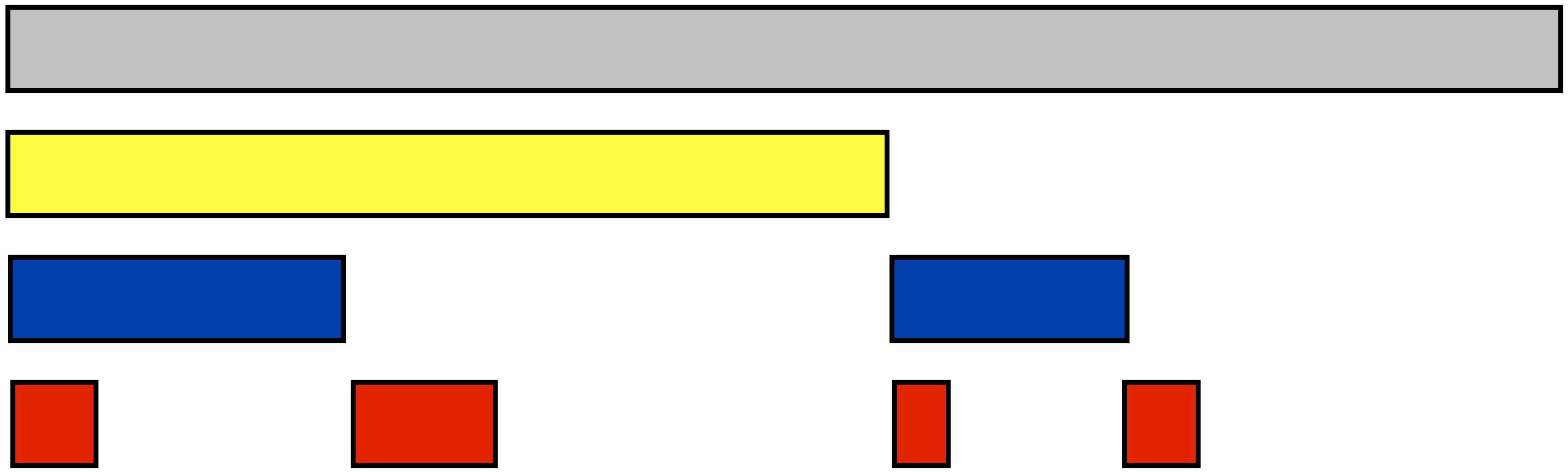

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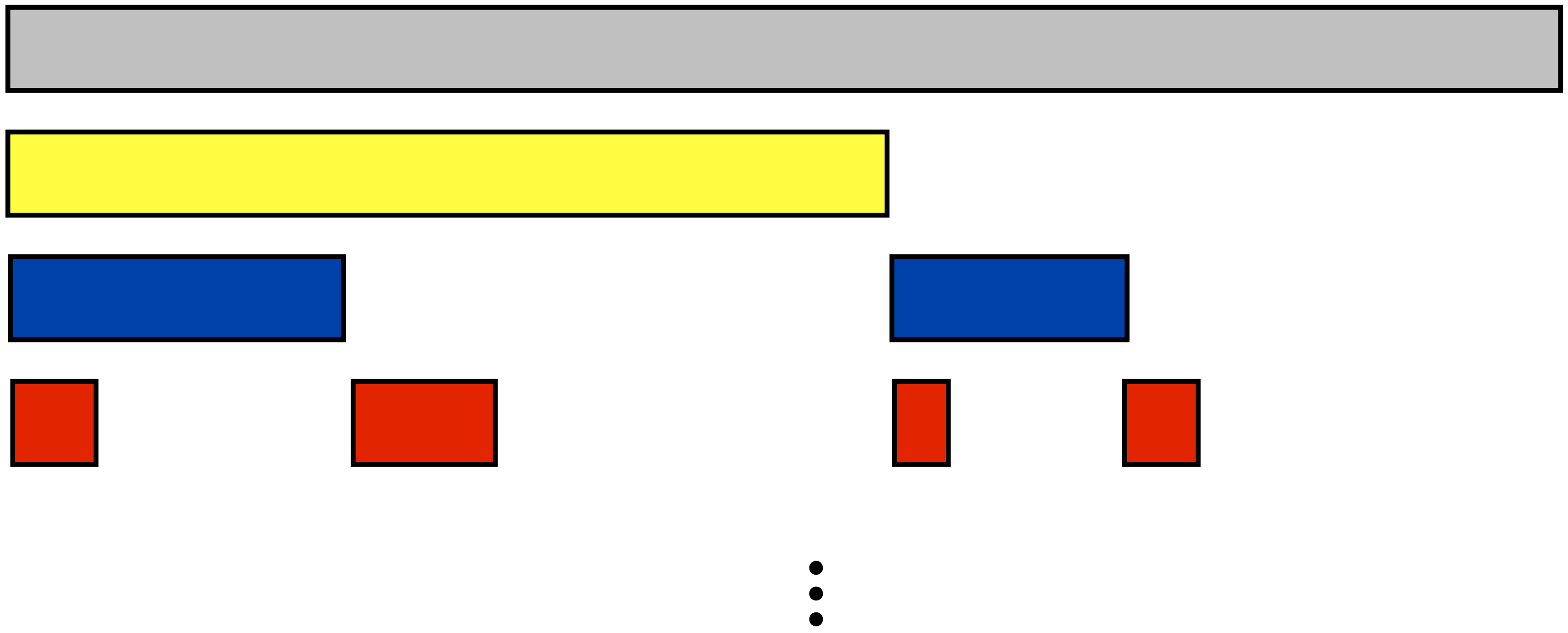
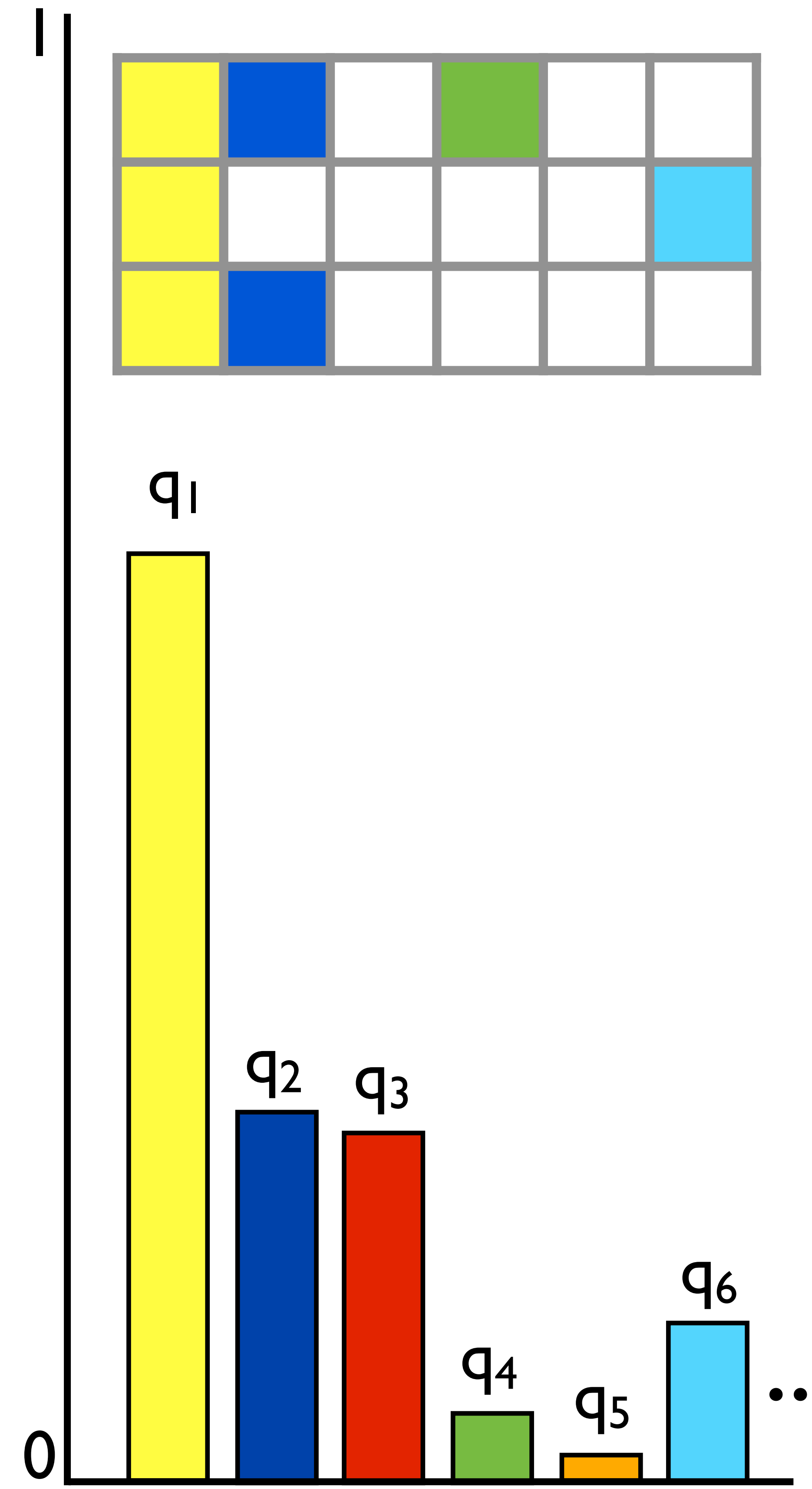
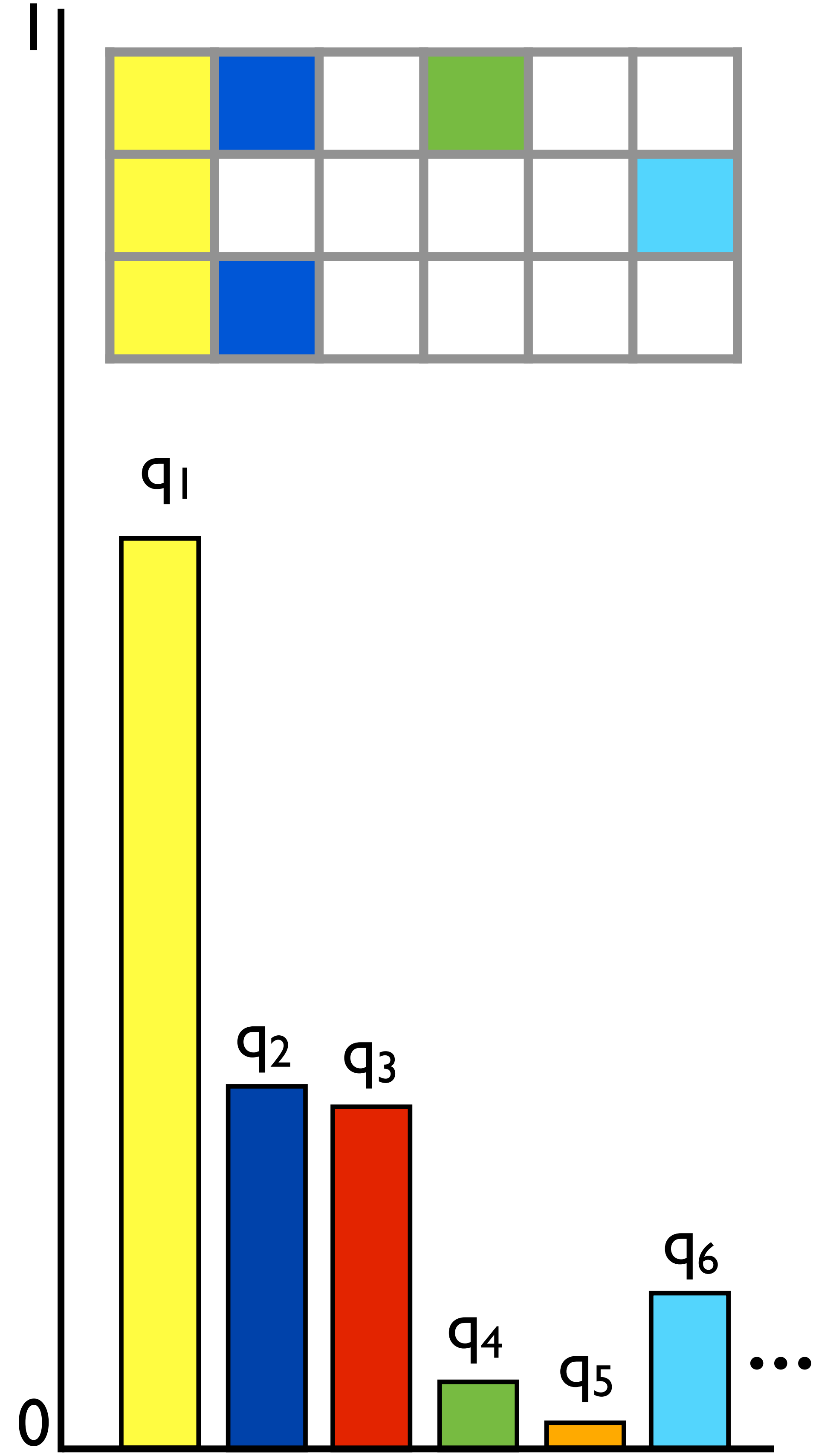


Paintboxes



Paintboxes

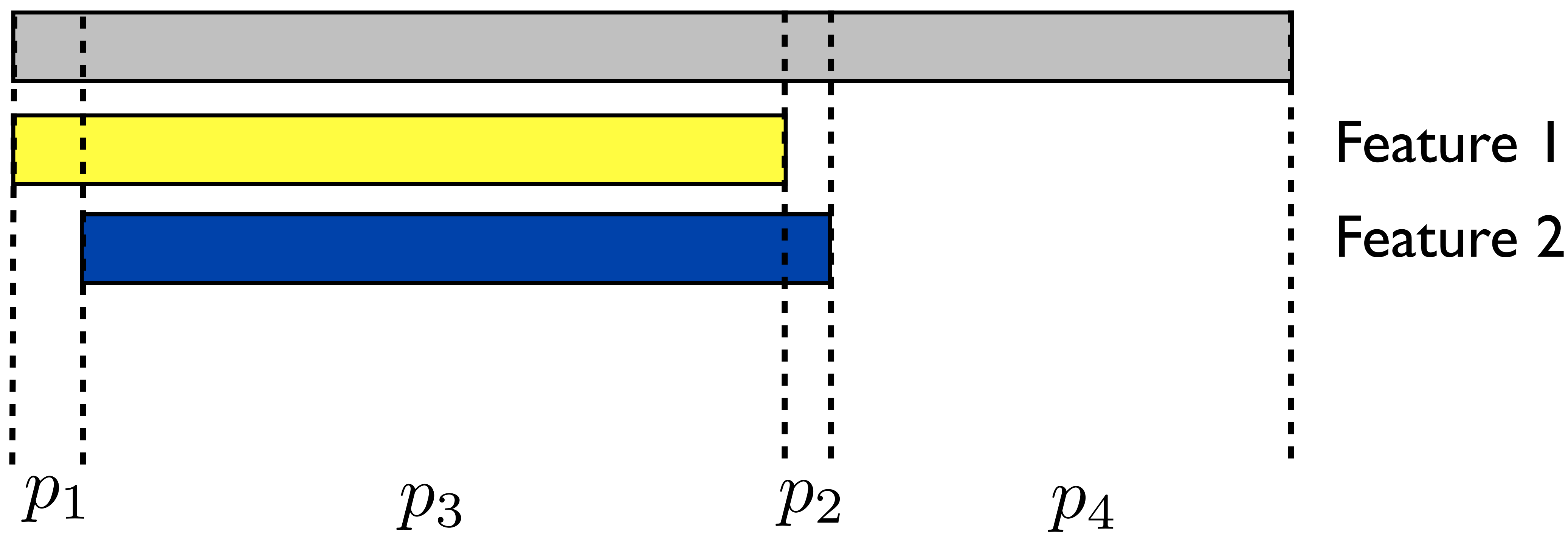
"Feature frequency models"



[Broderick, Pitman, Jordan 2013]

Paintboxes

Two feature example



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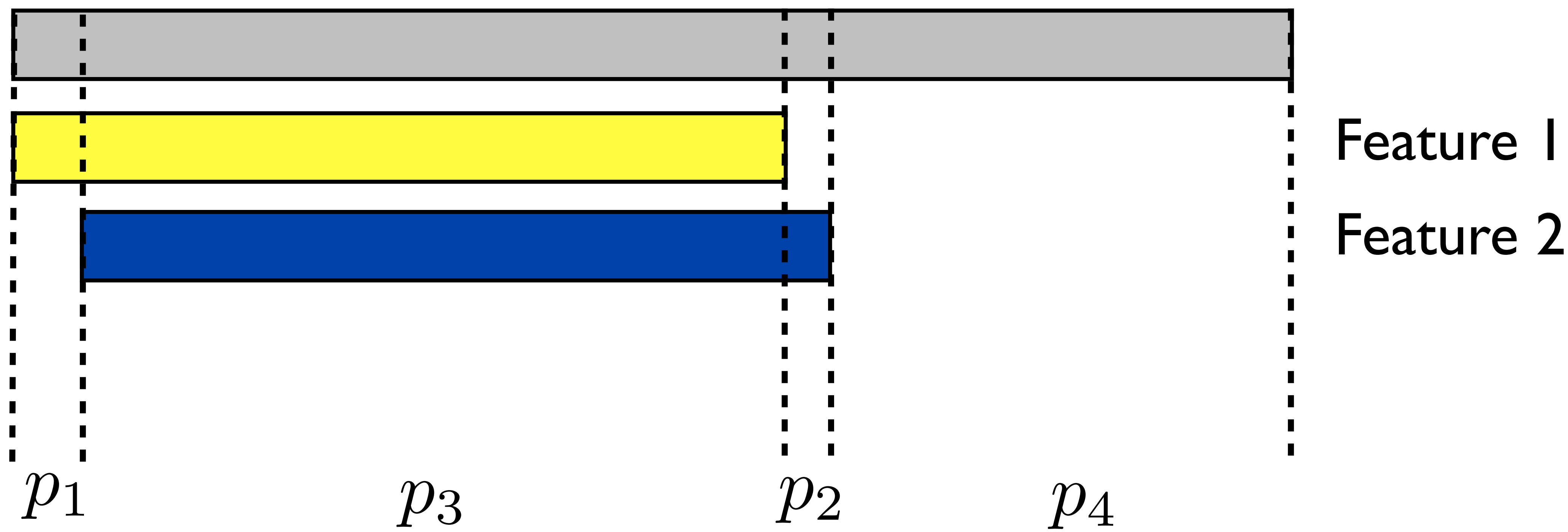
Paintboxes

Two feature example

Not a feature frequency model



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Paintboxes

Exchangeable cluster distributions
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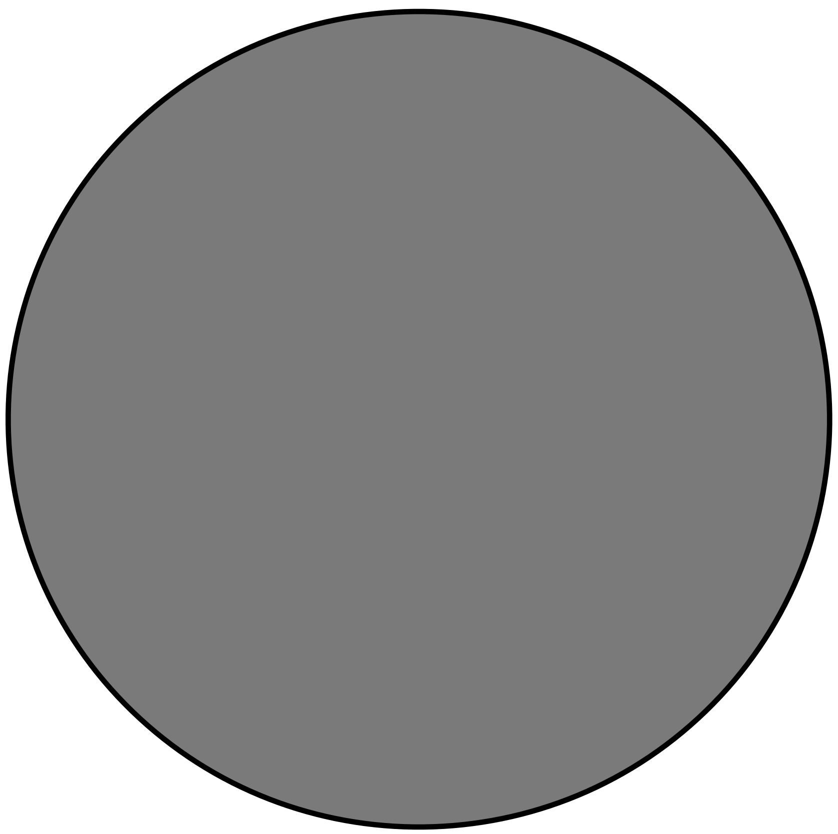

Exchangeable feature distributions
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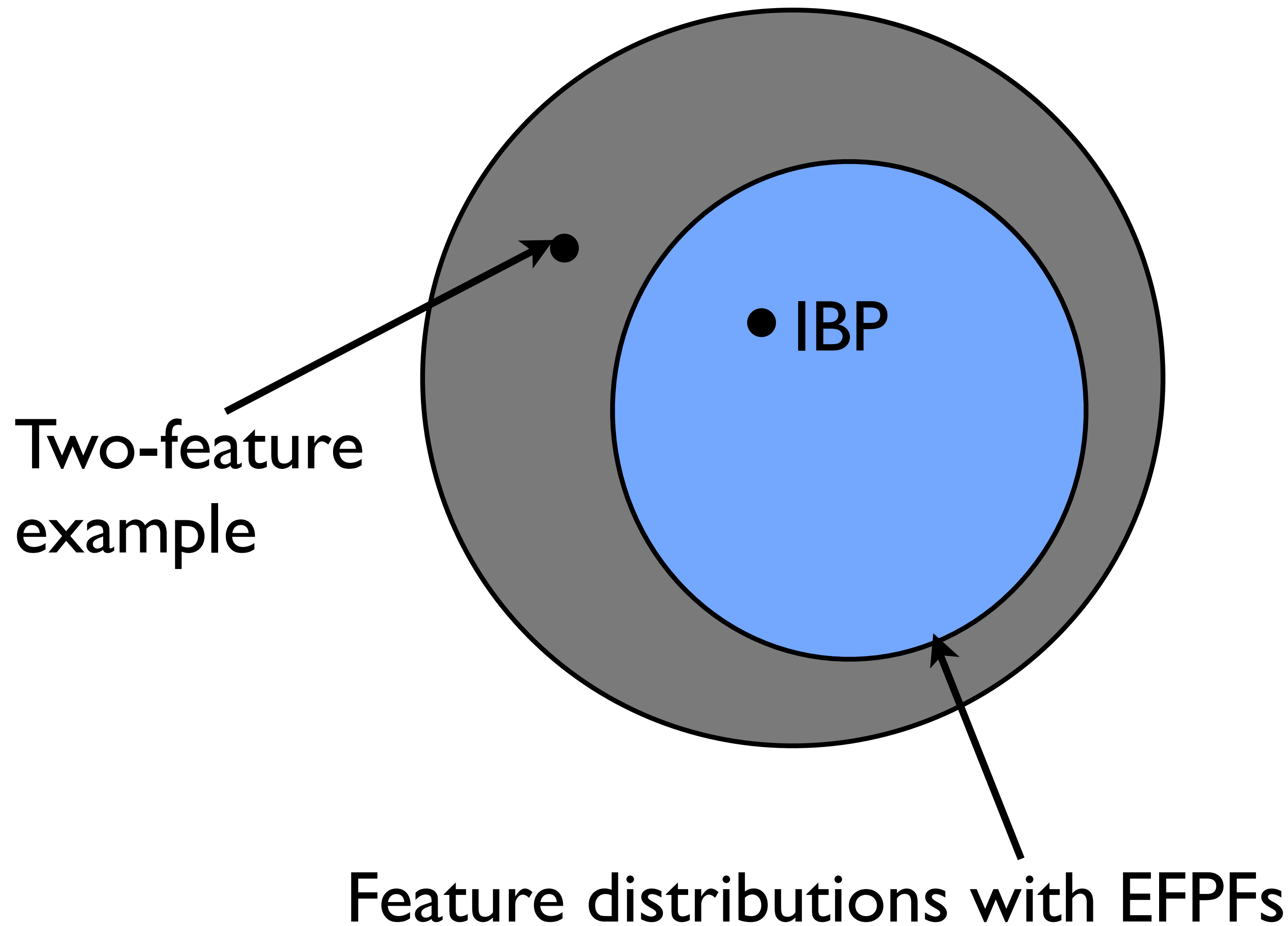

[Broderick, Pitman, Jordan 2013]

Paintboxes

Exchangeable cluster distributions
= Cluster distributions with EPPFs
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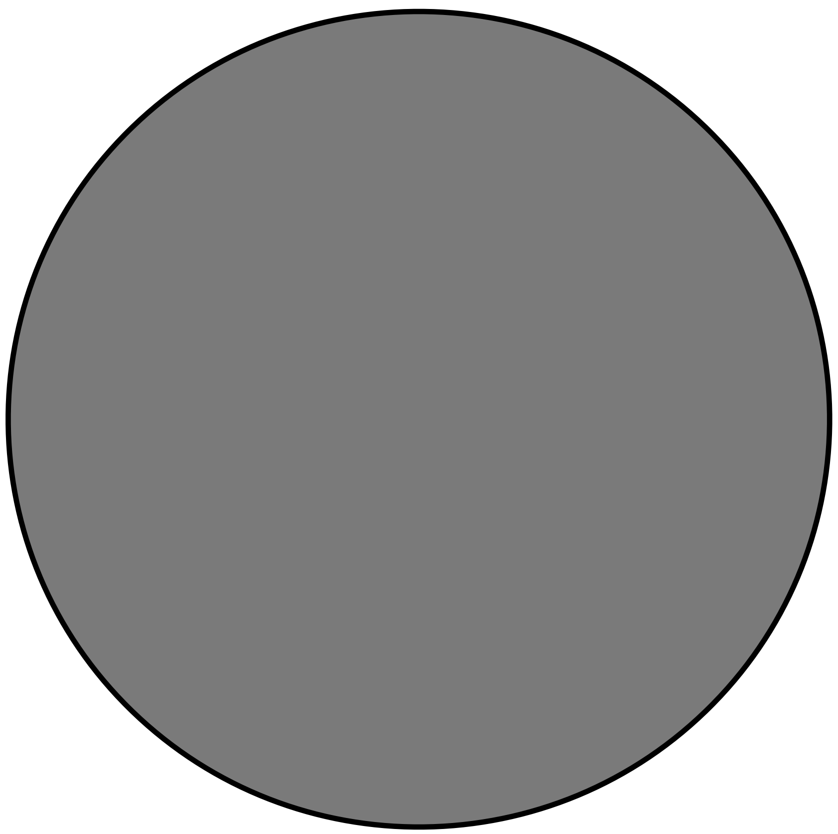
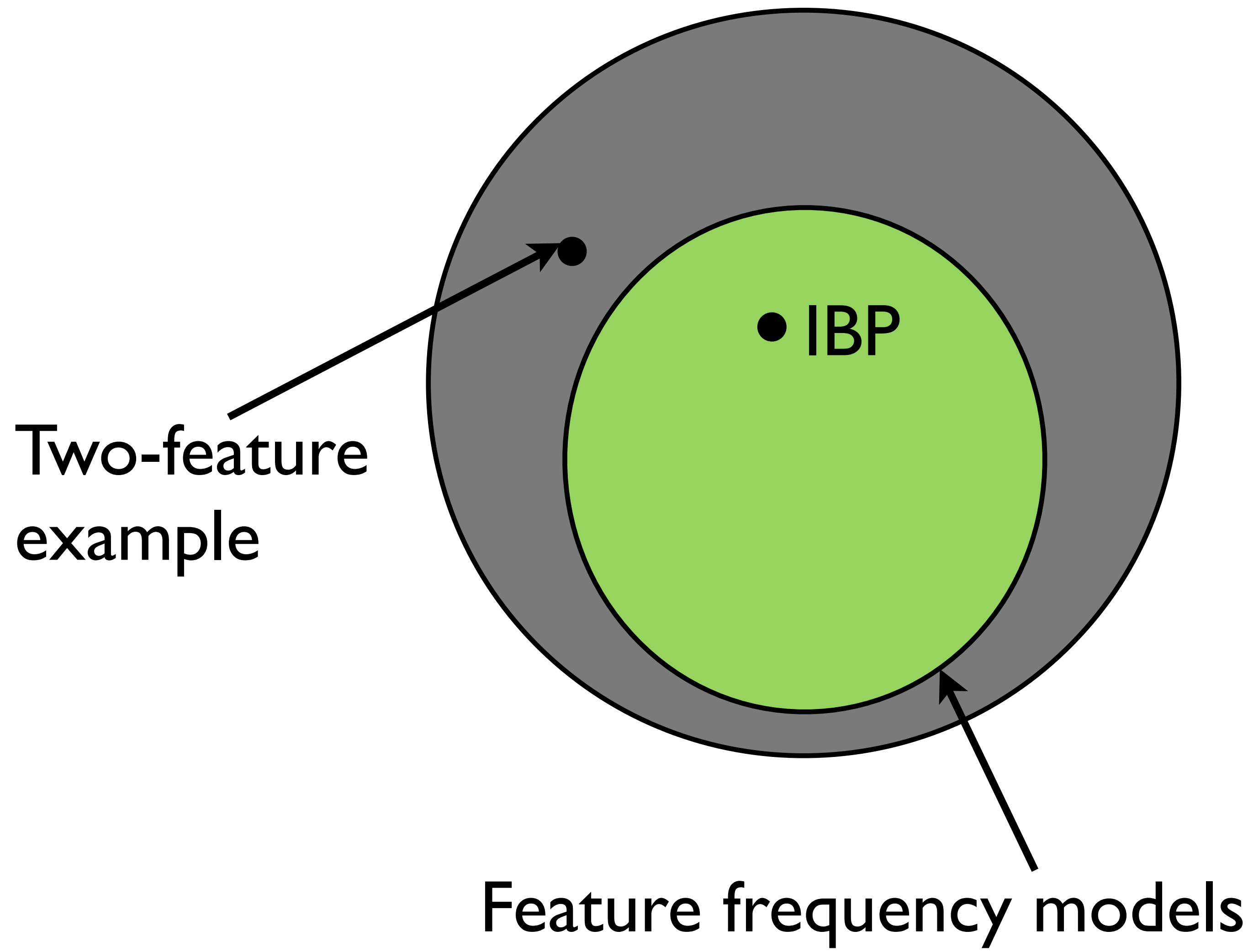
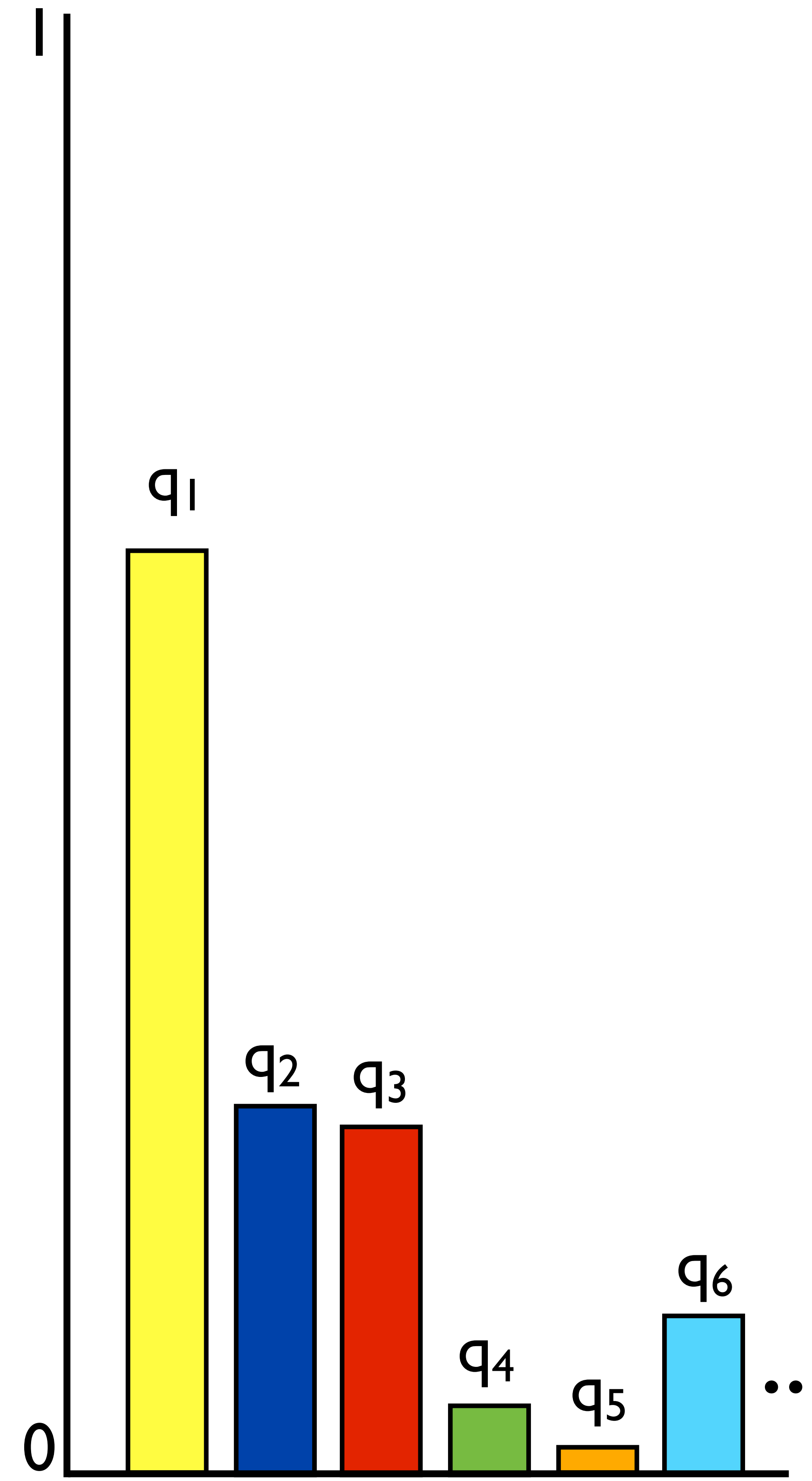


Exchangeable feature distributions
= Feature paintbox allocations

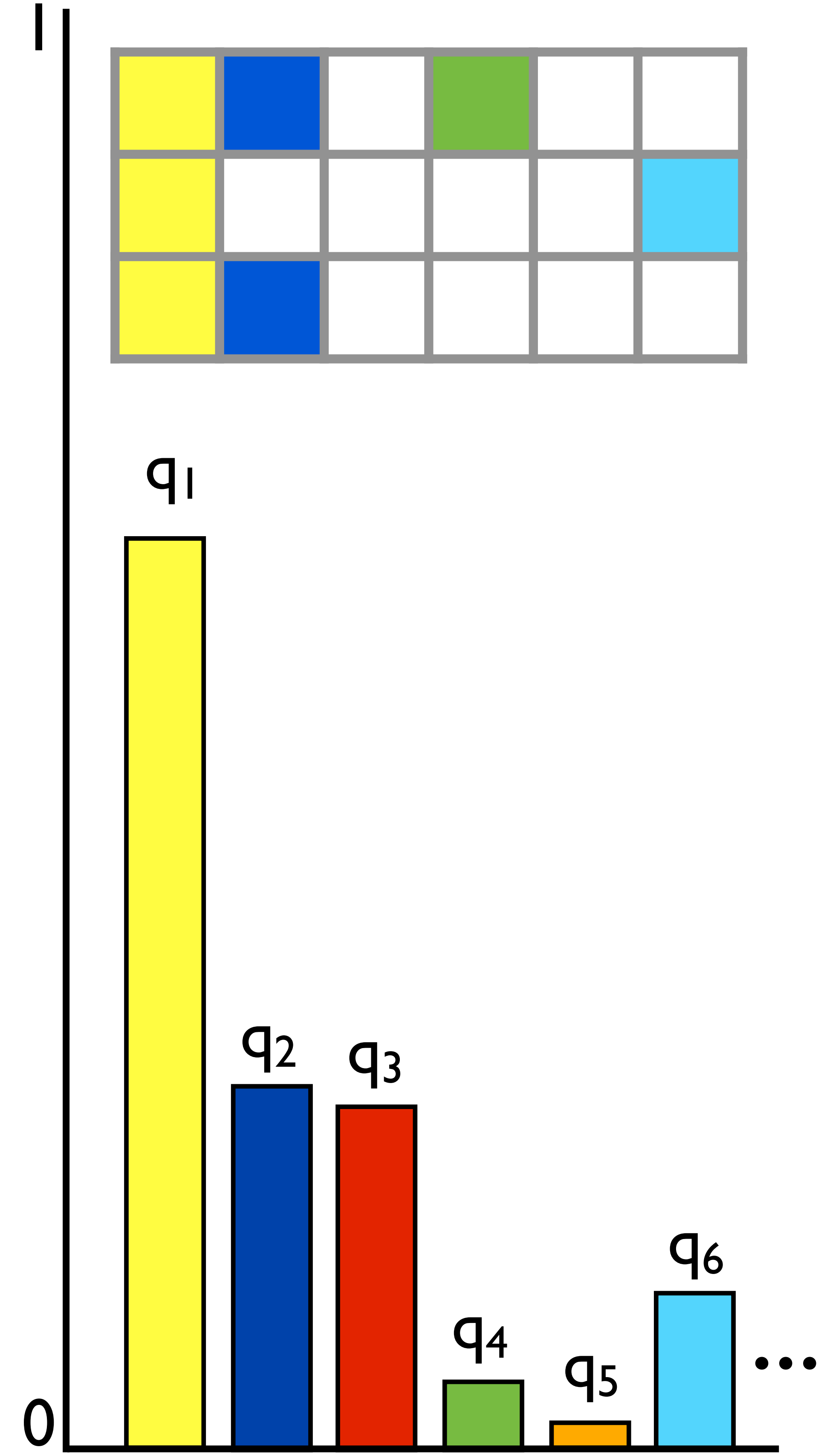


[Broderick, Pitman, Jordan 2013]

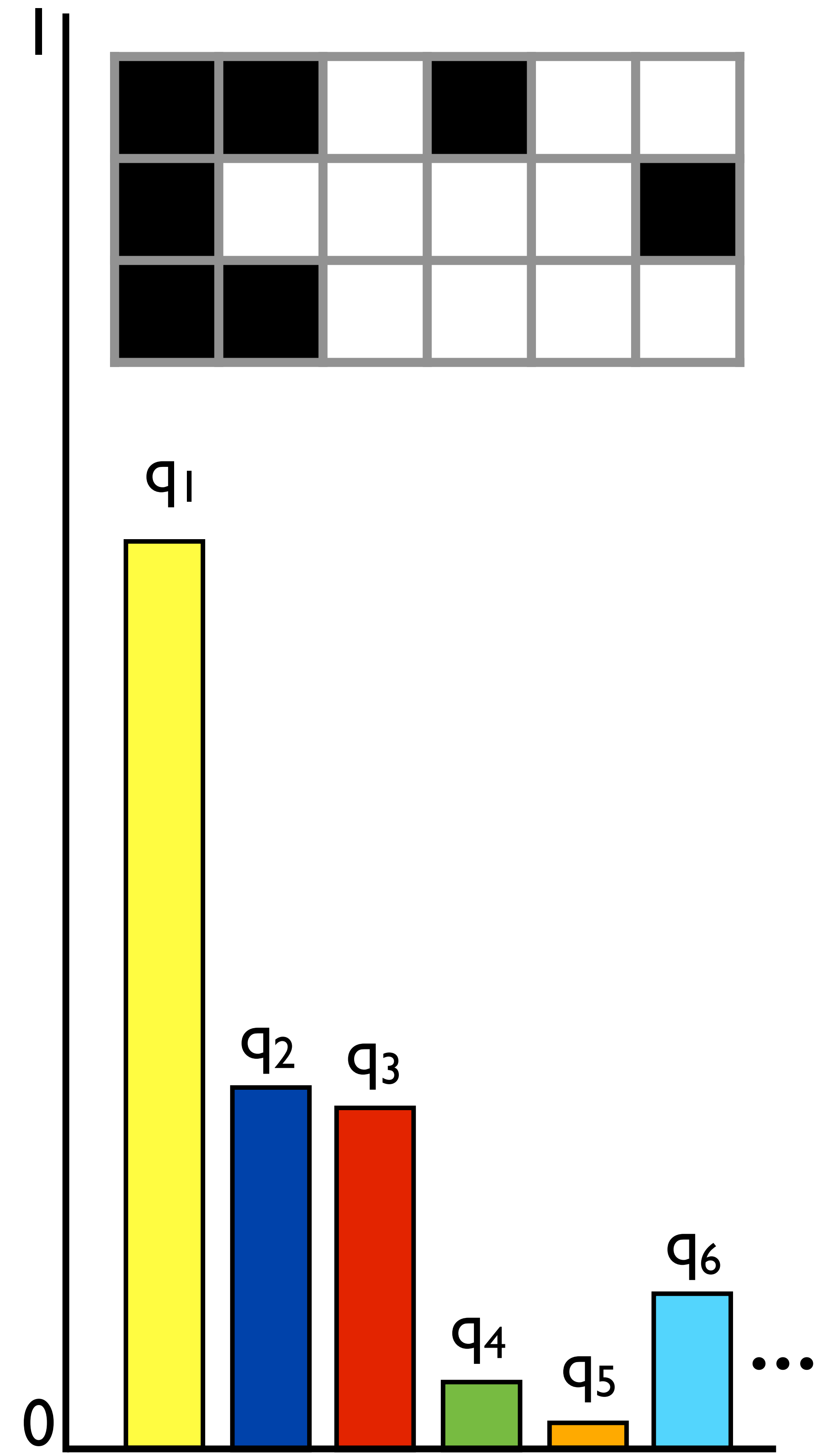
Feature frequency models: EFPFs?



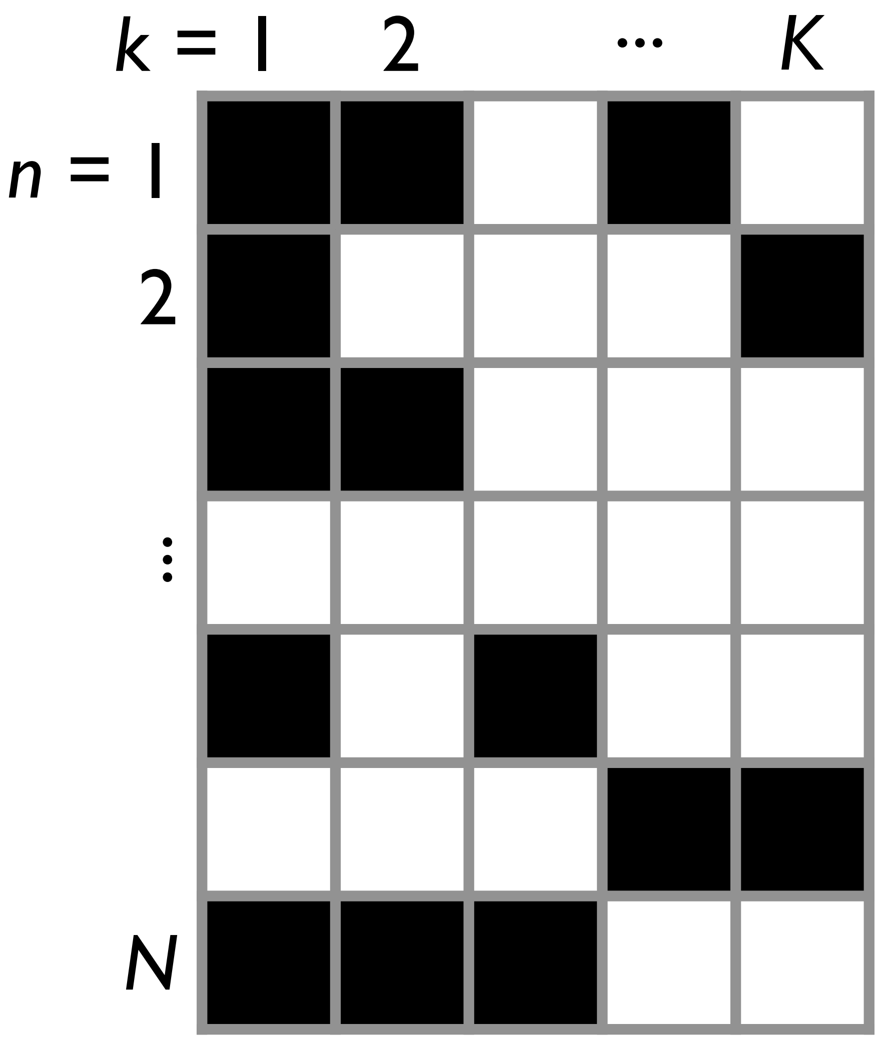
Feature frequency models: EFPFs?

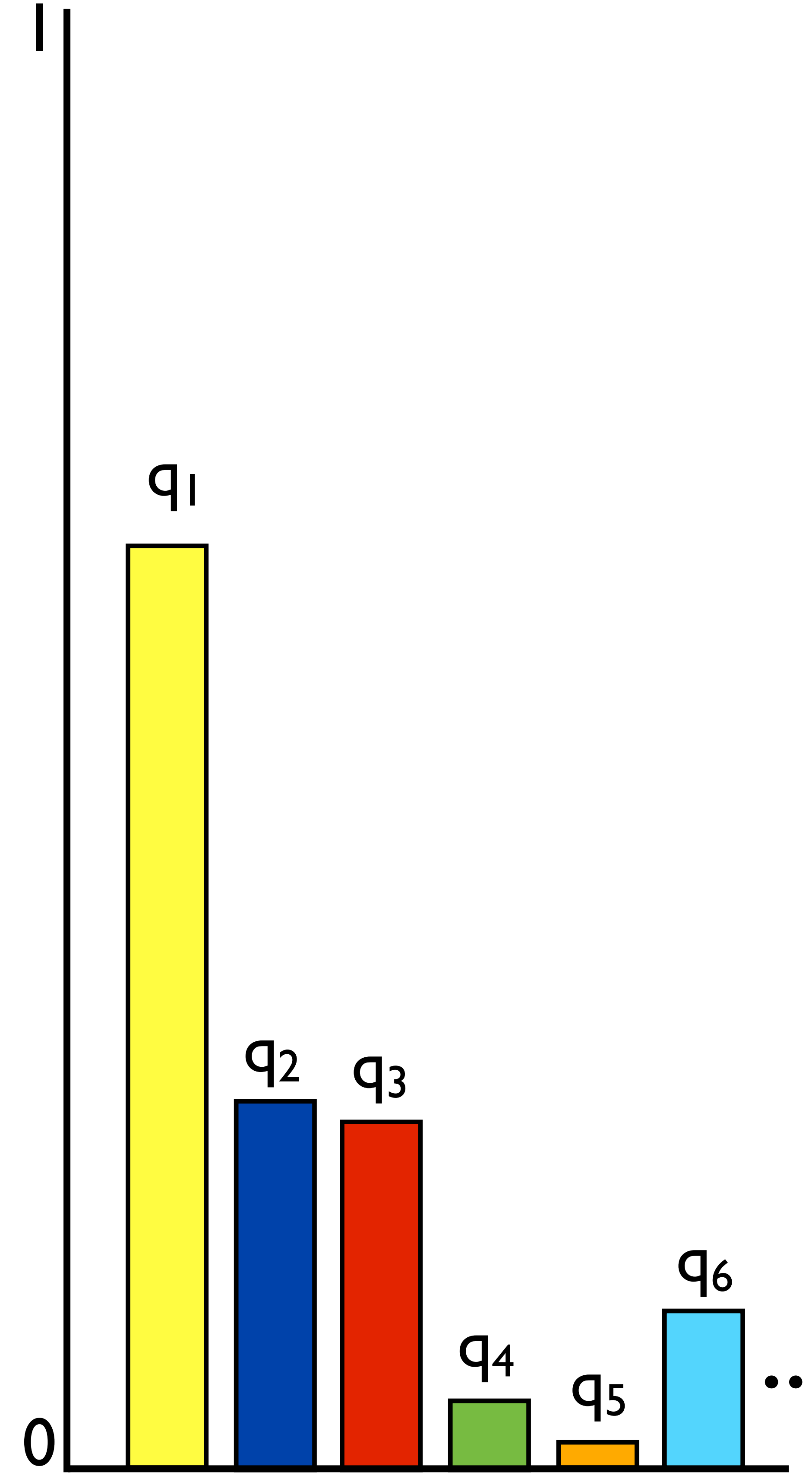


Feature frequency models: EFPFs?

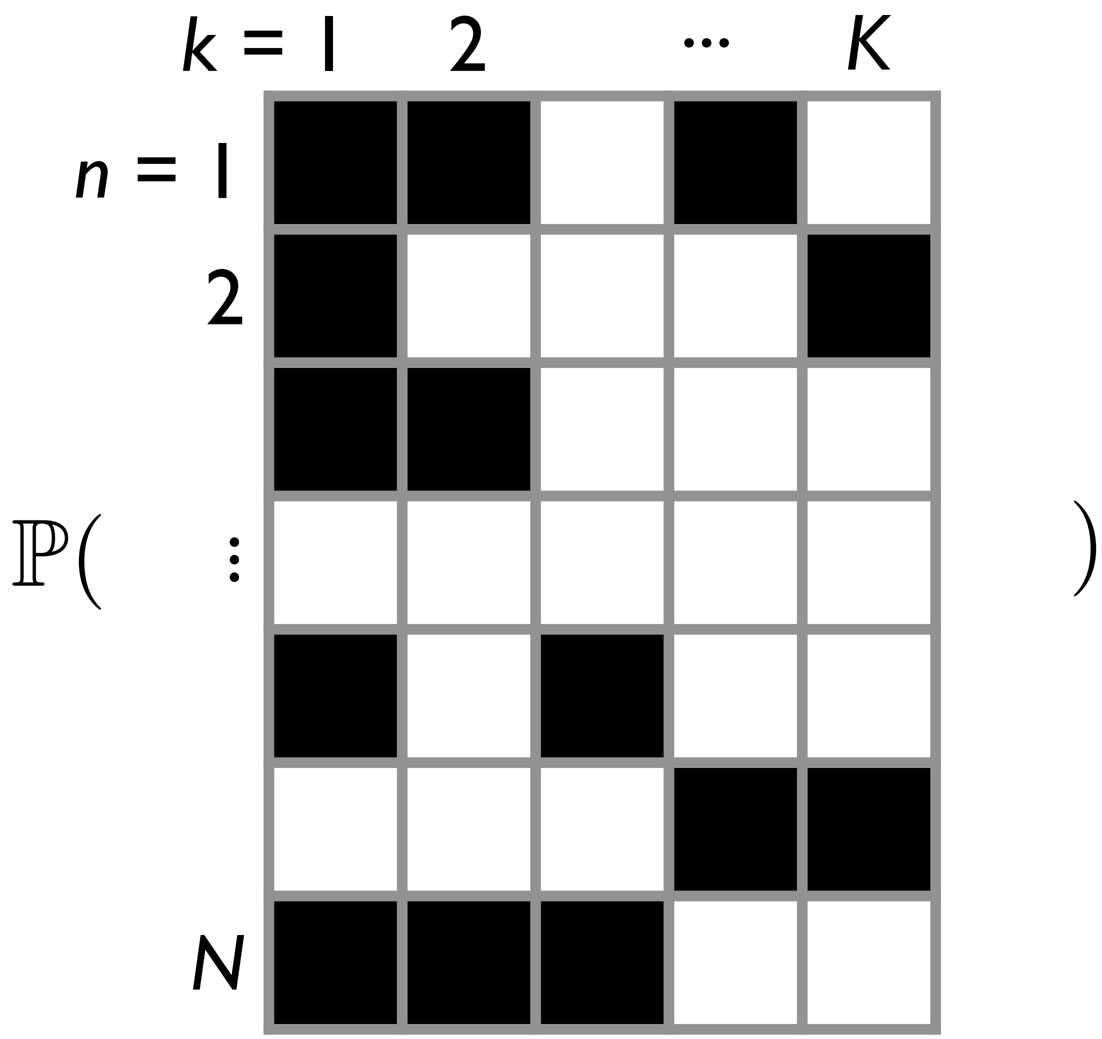


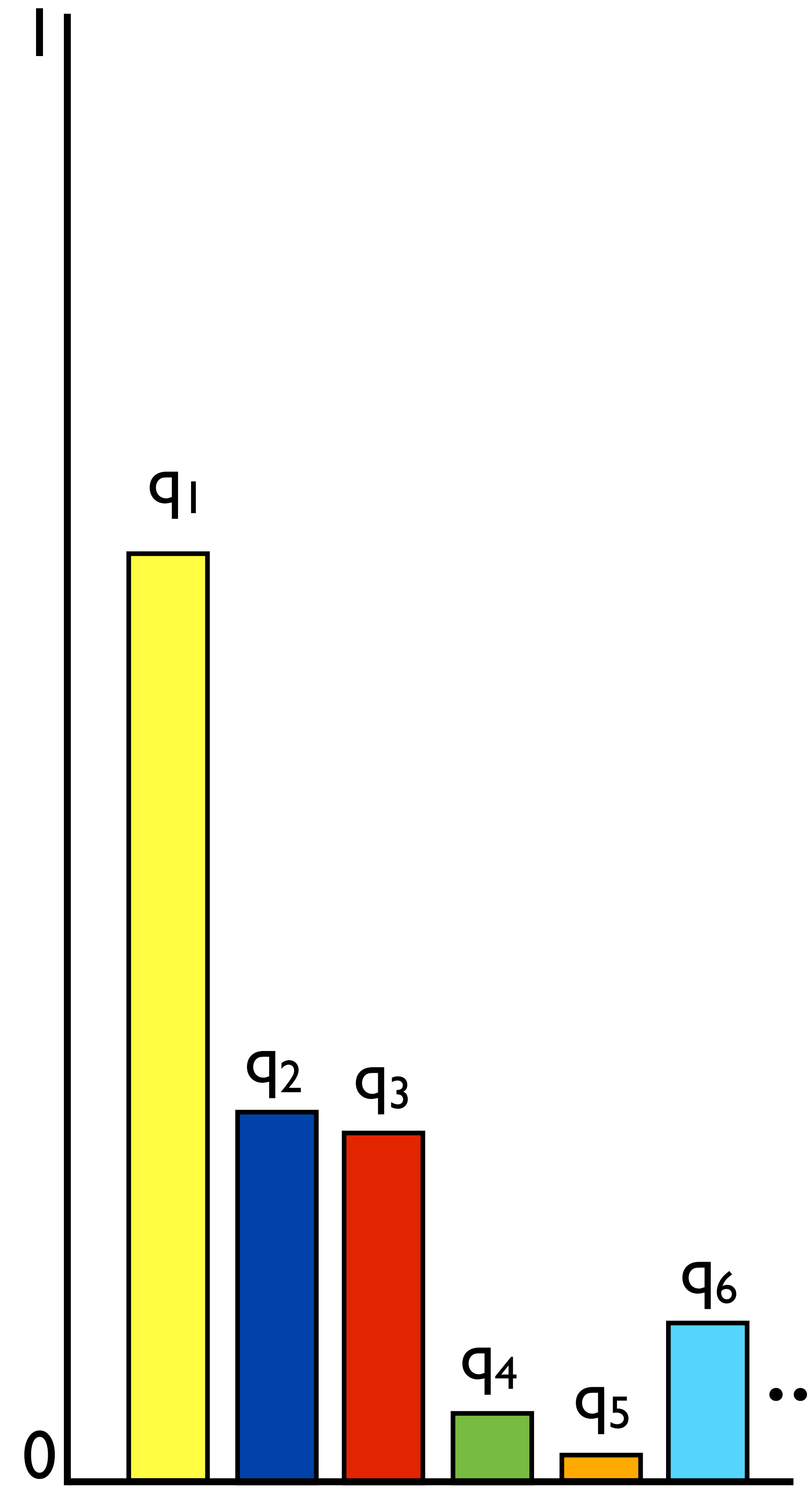
Feature frequency models: EFPFs?



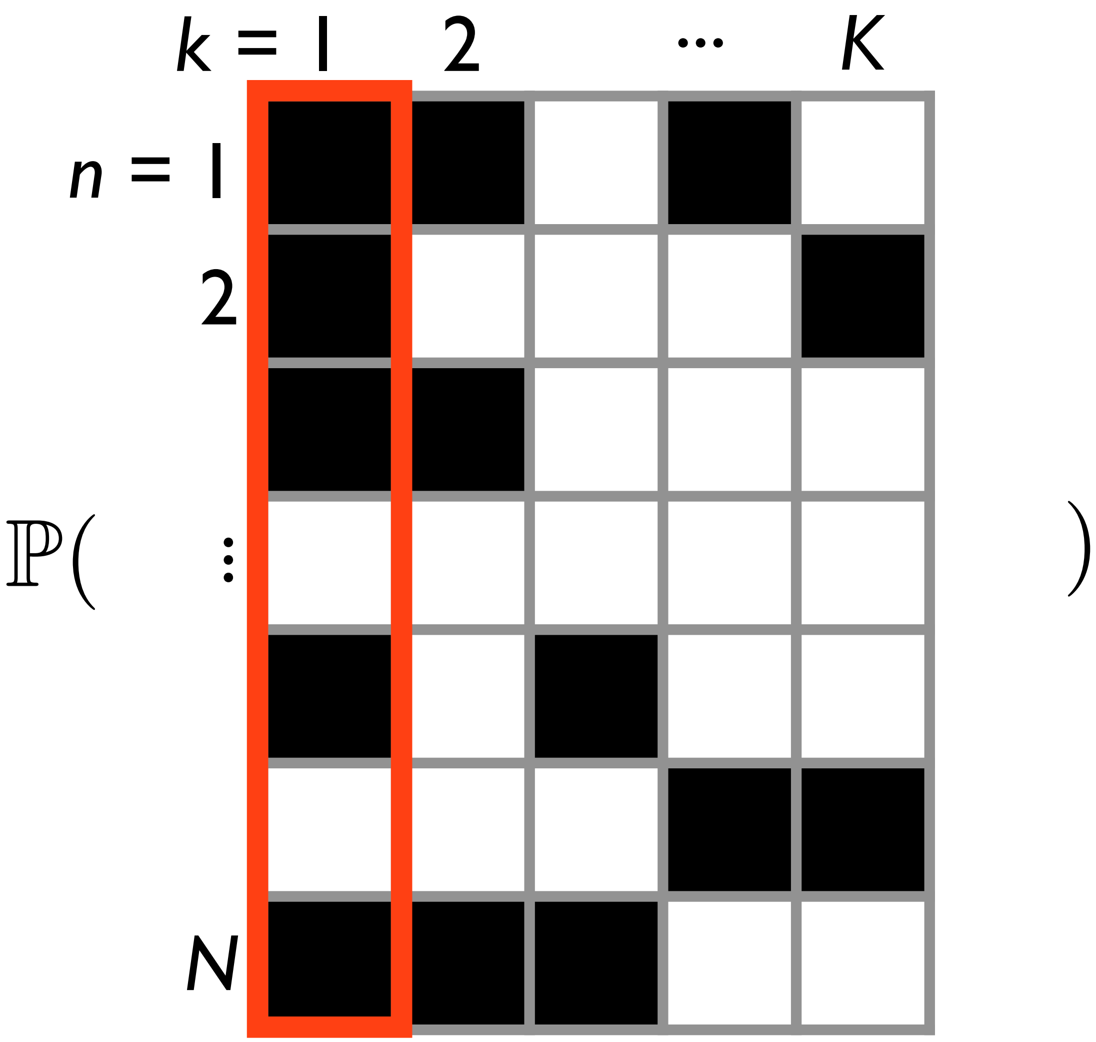


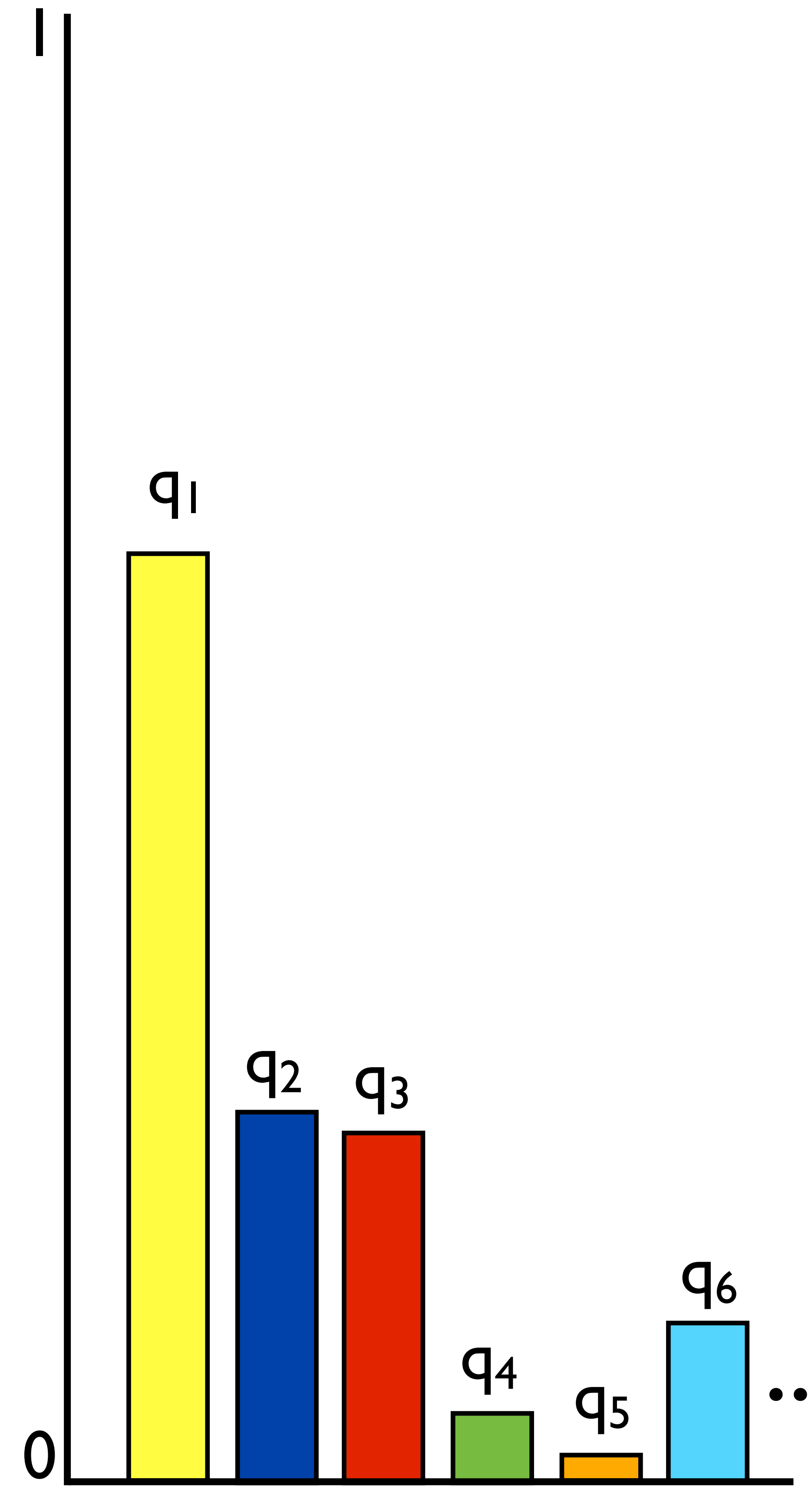
Feature frequency models: EFPFs?



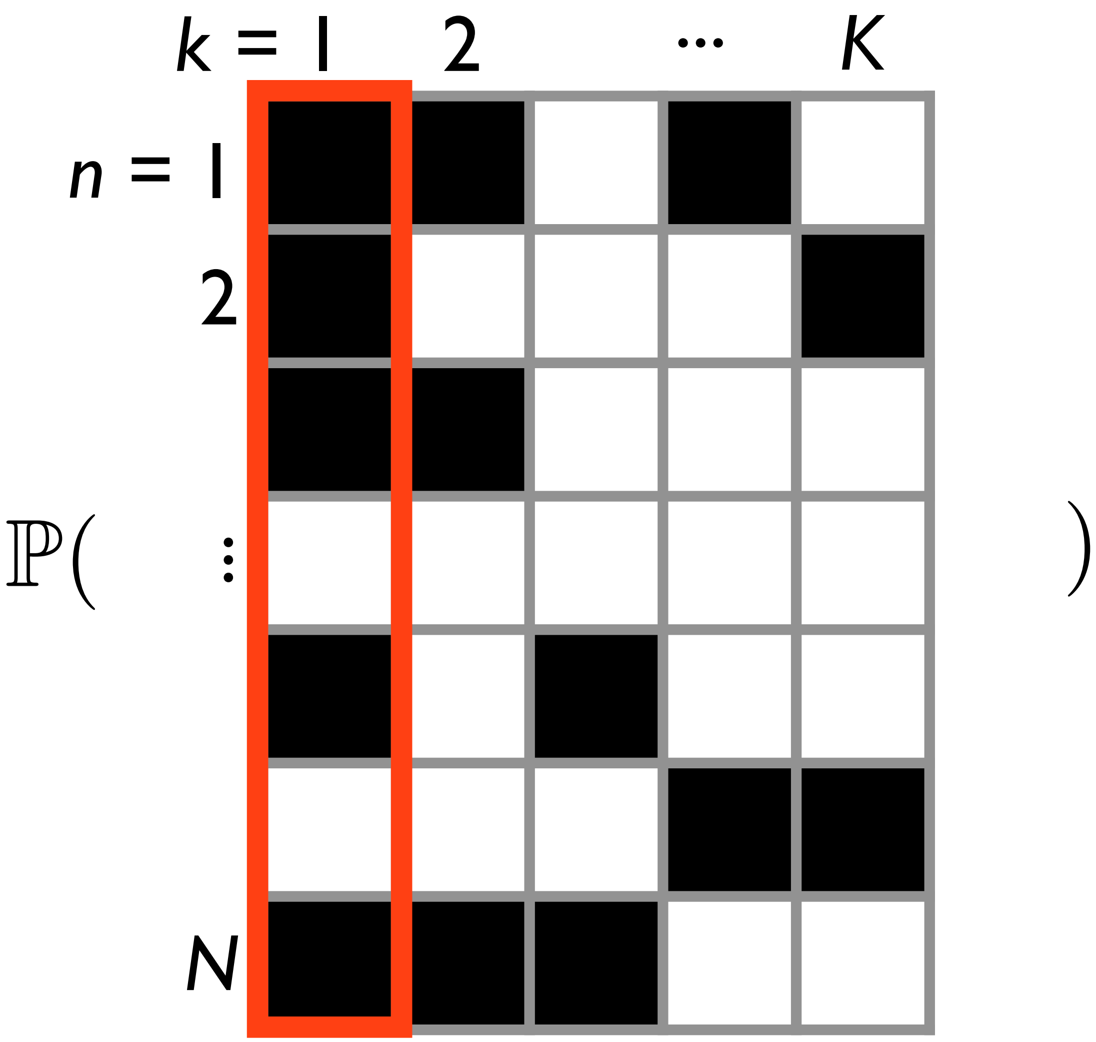


Feature frequency models: EFPFs?

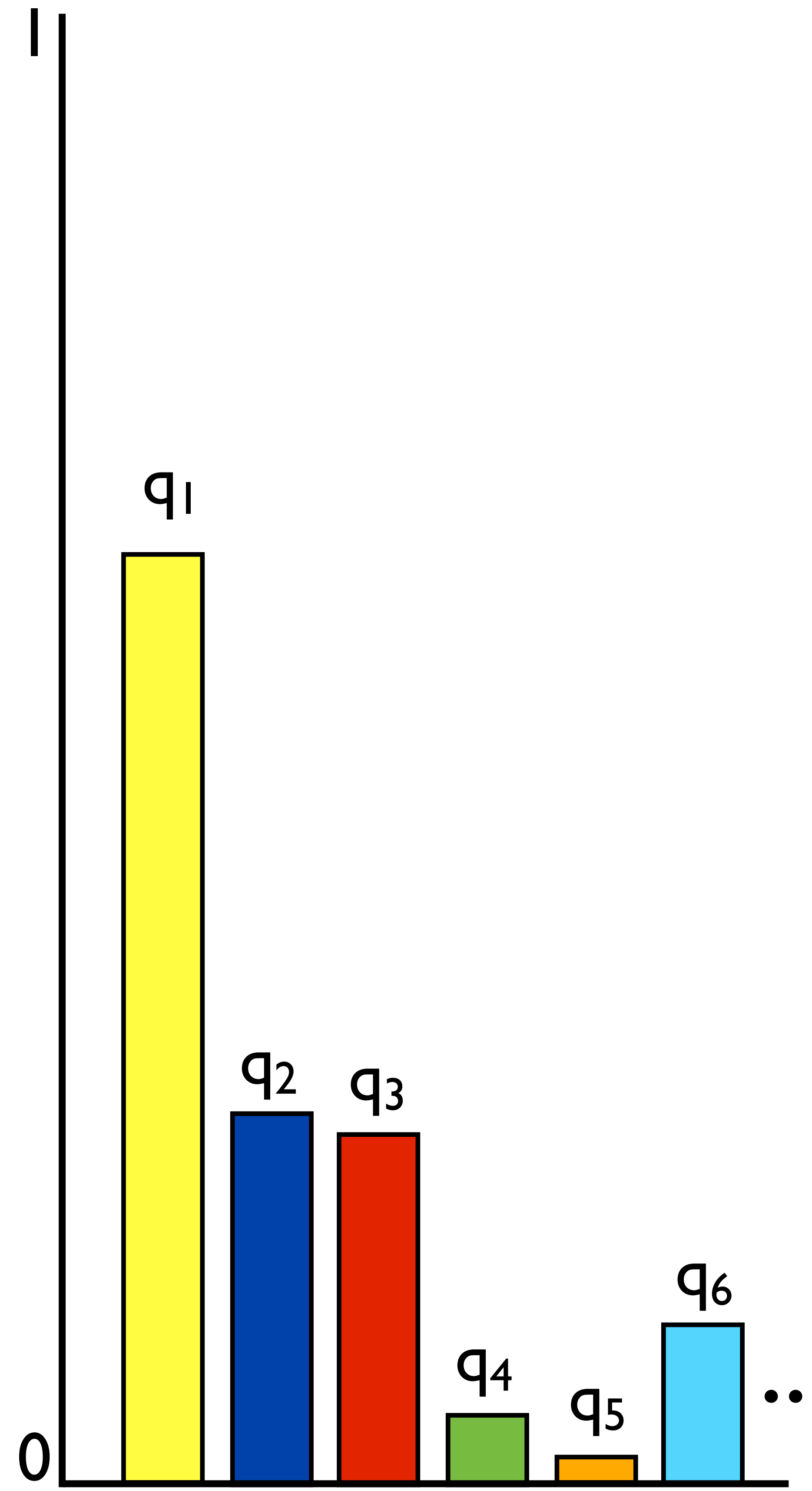




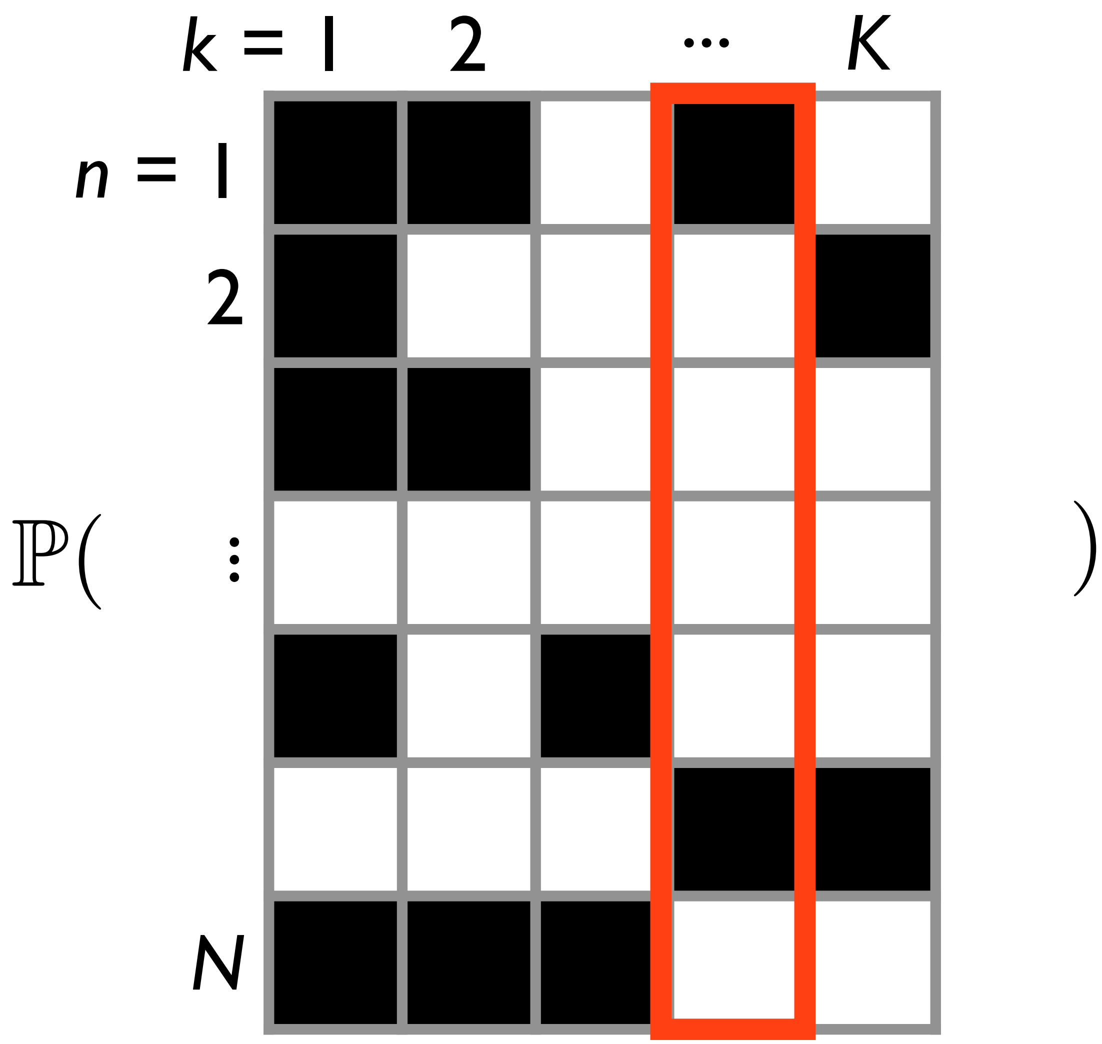
Feature frequency models: EFPFs?



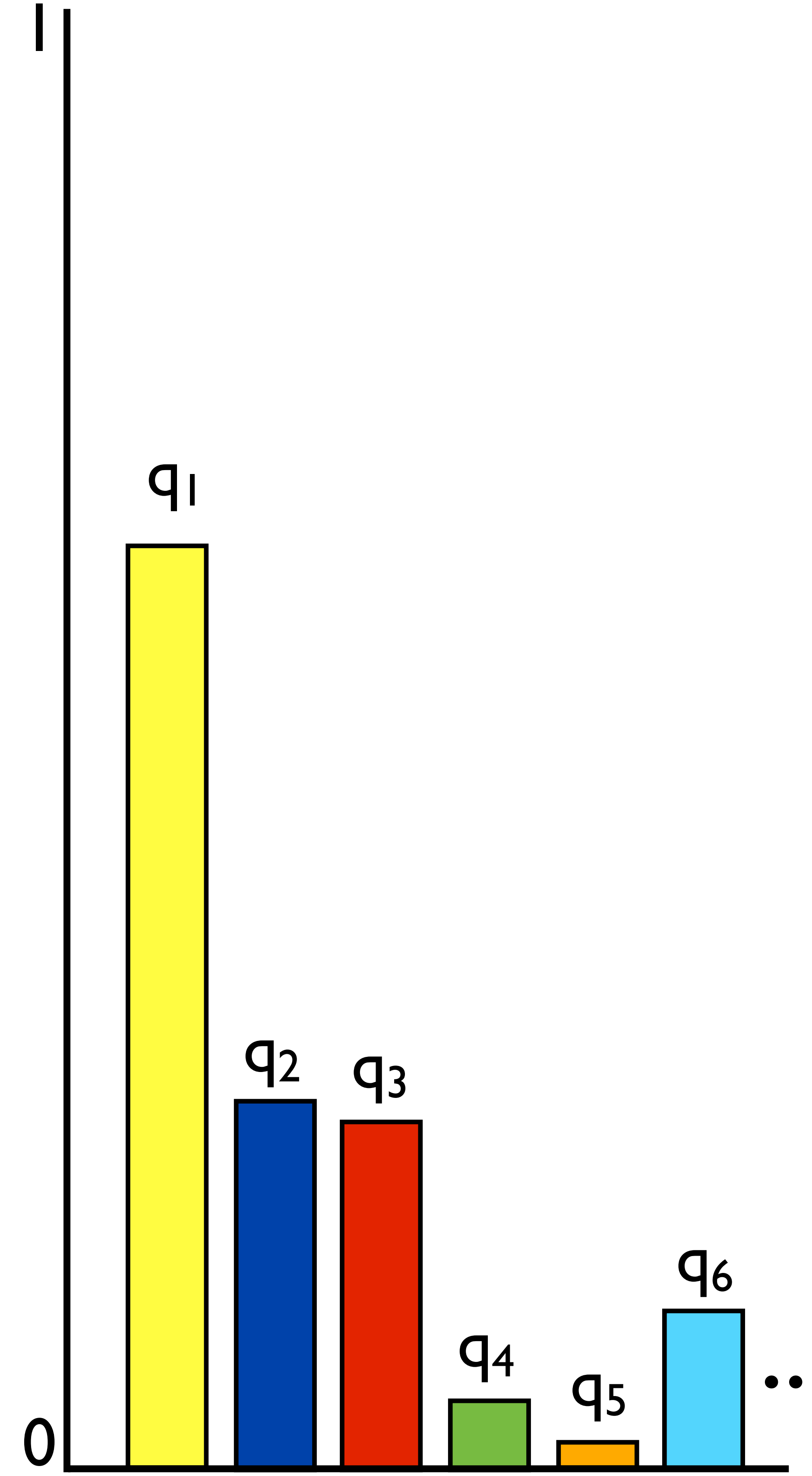
$$q_1^{S_{N,1}}(1 - q_1)^{N - S_{N,1}}$$



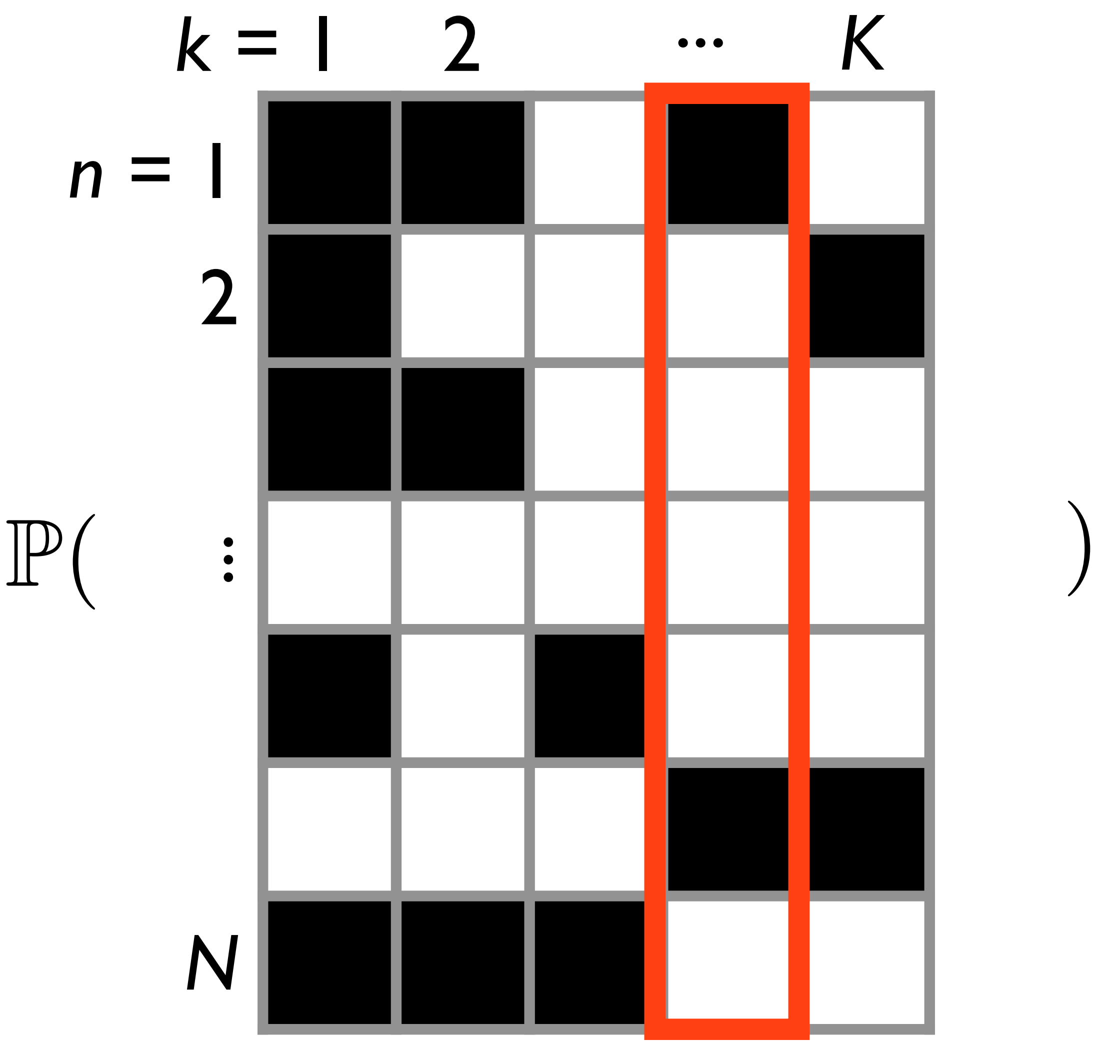
Feature frequency models: EFPFs?



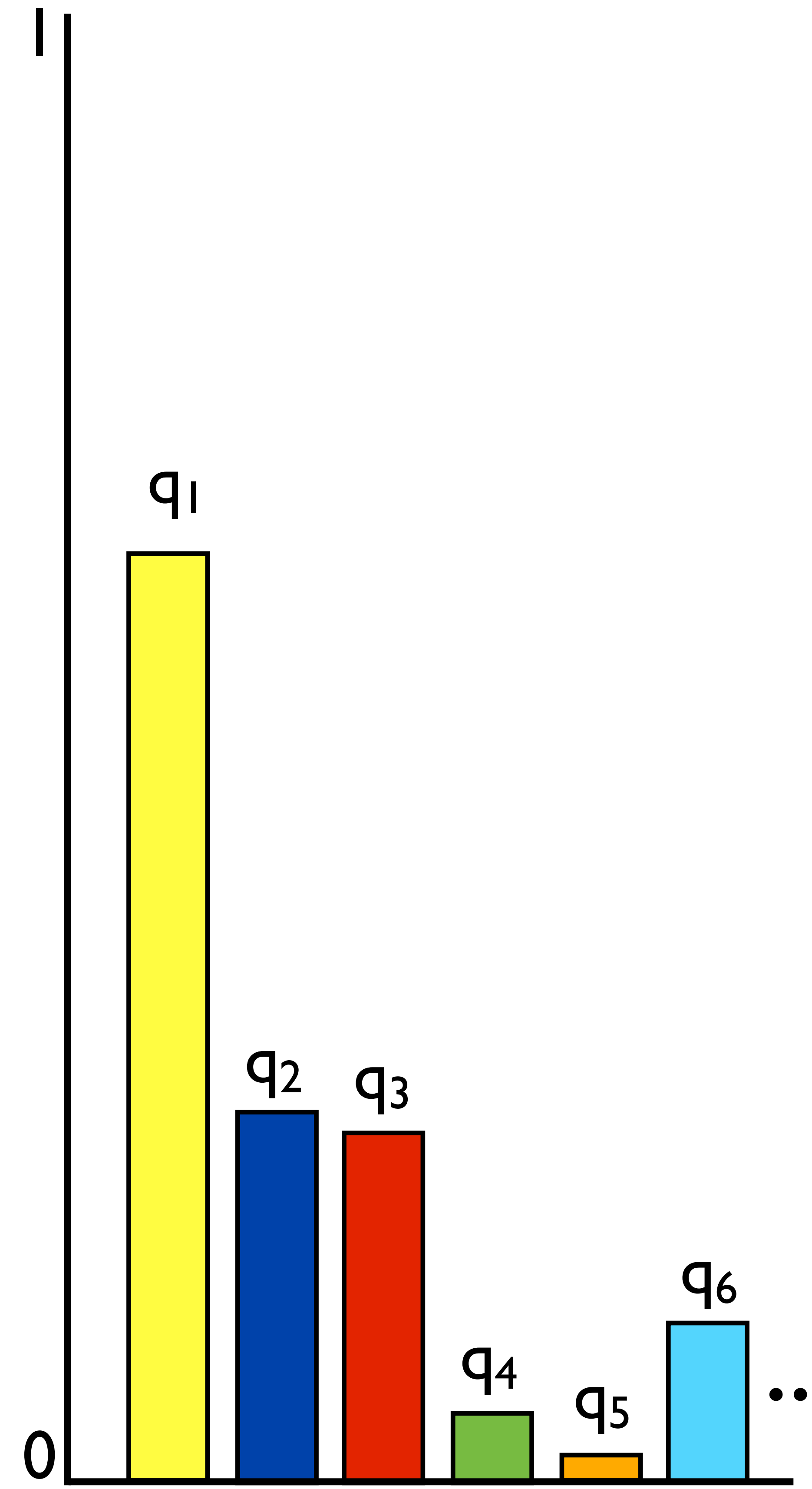
$$q_k^{S_{N,k}}(1-q_k)^{N-S_{N,k}}$$



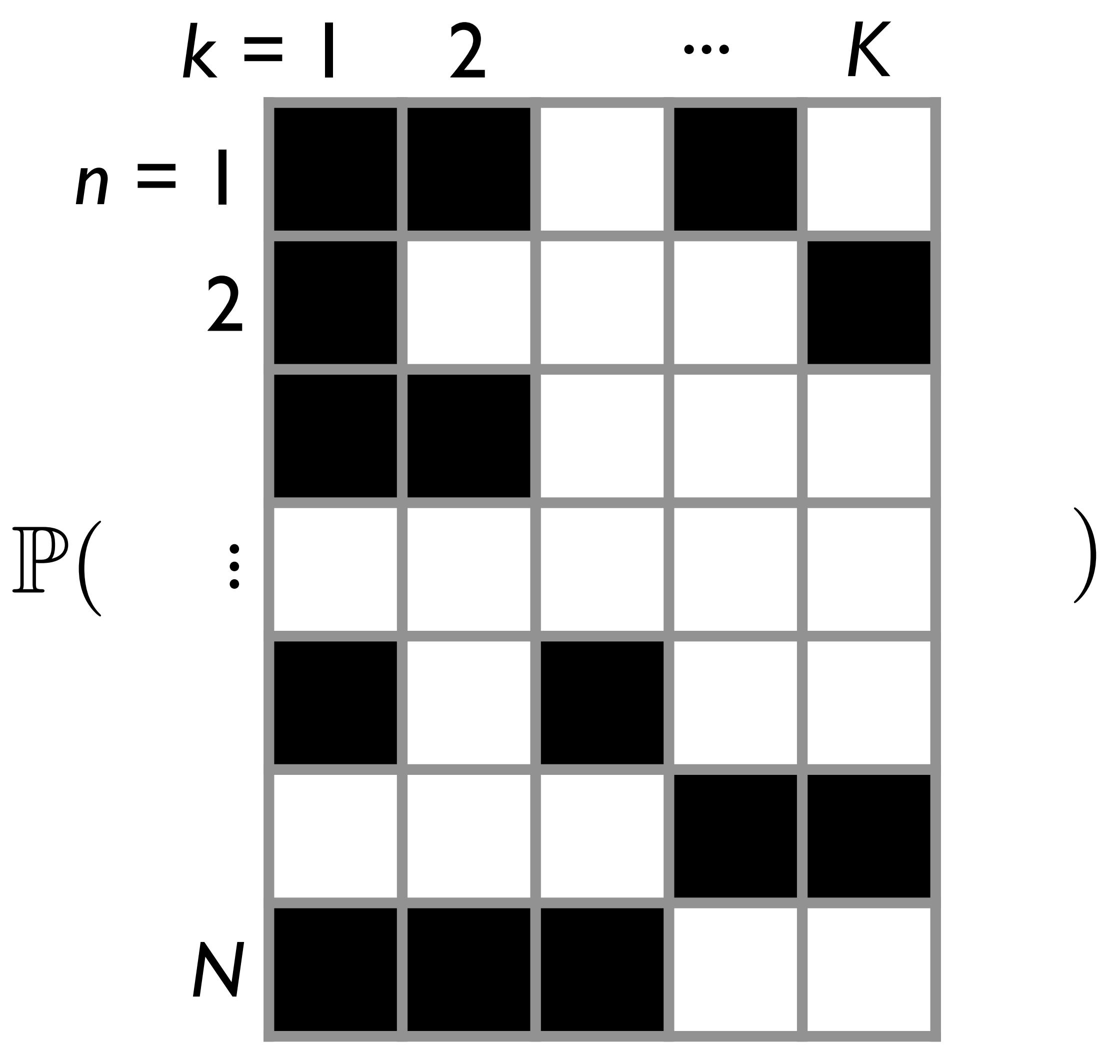
Feature frequency models: EFPFs?



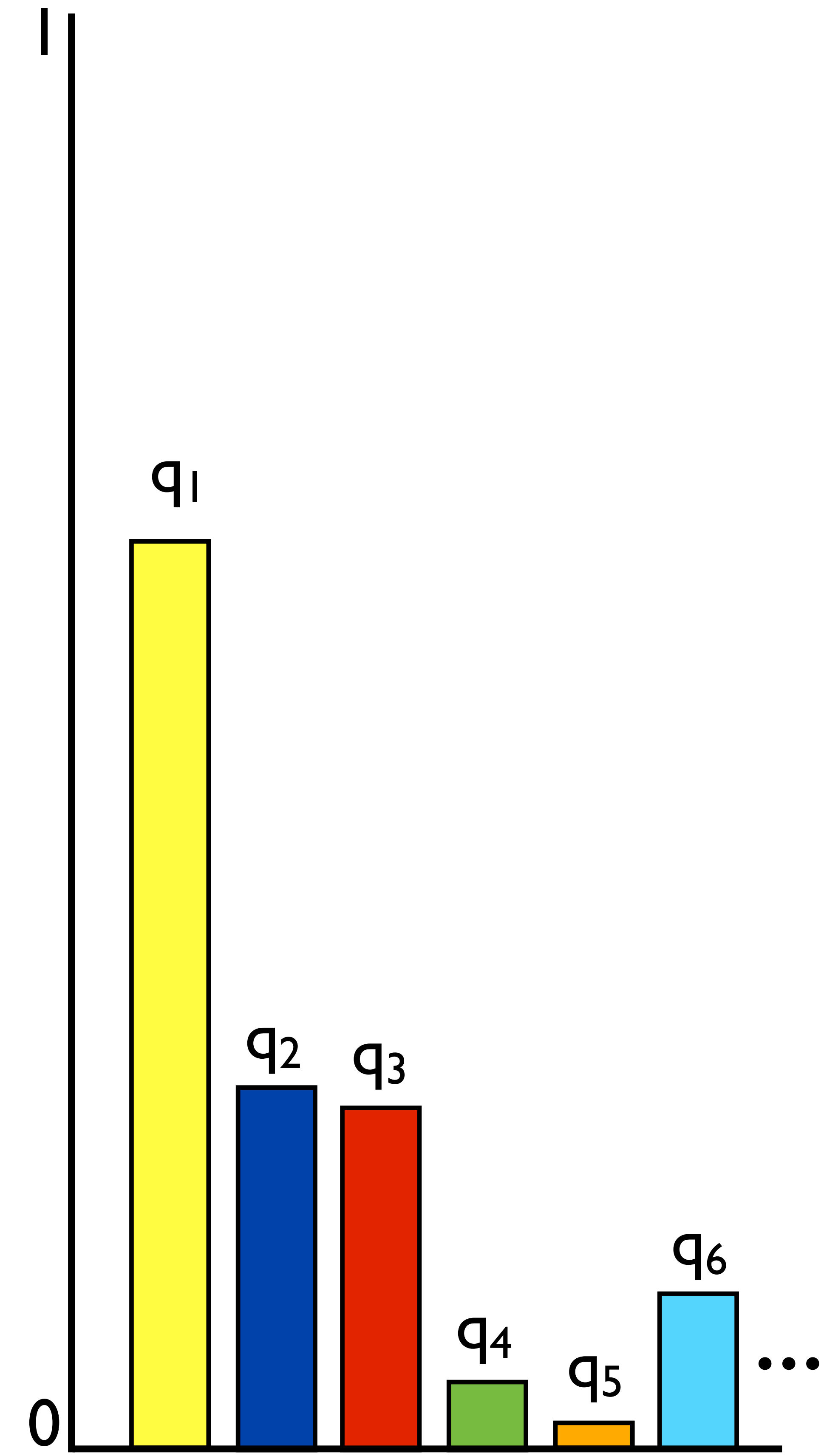
$$q_{i_k}^{S_{N,k}}(1 - q_{i_k})^{N - S_{N,k}}$$



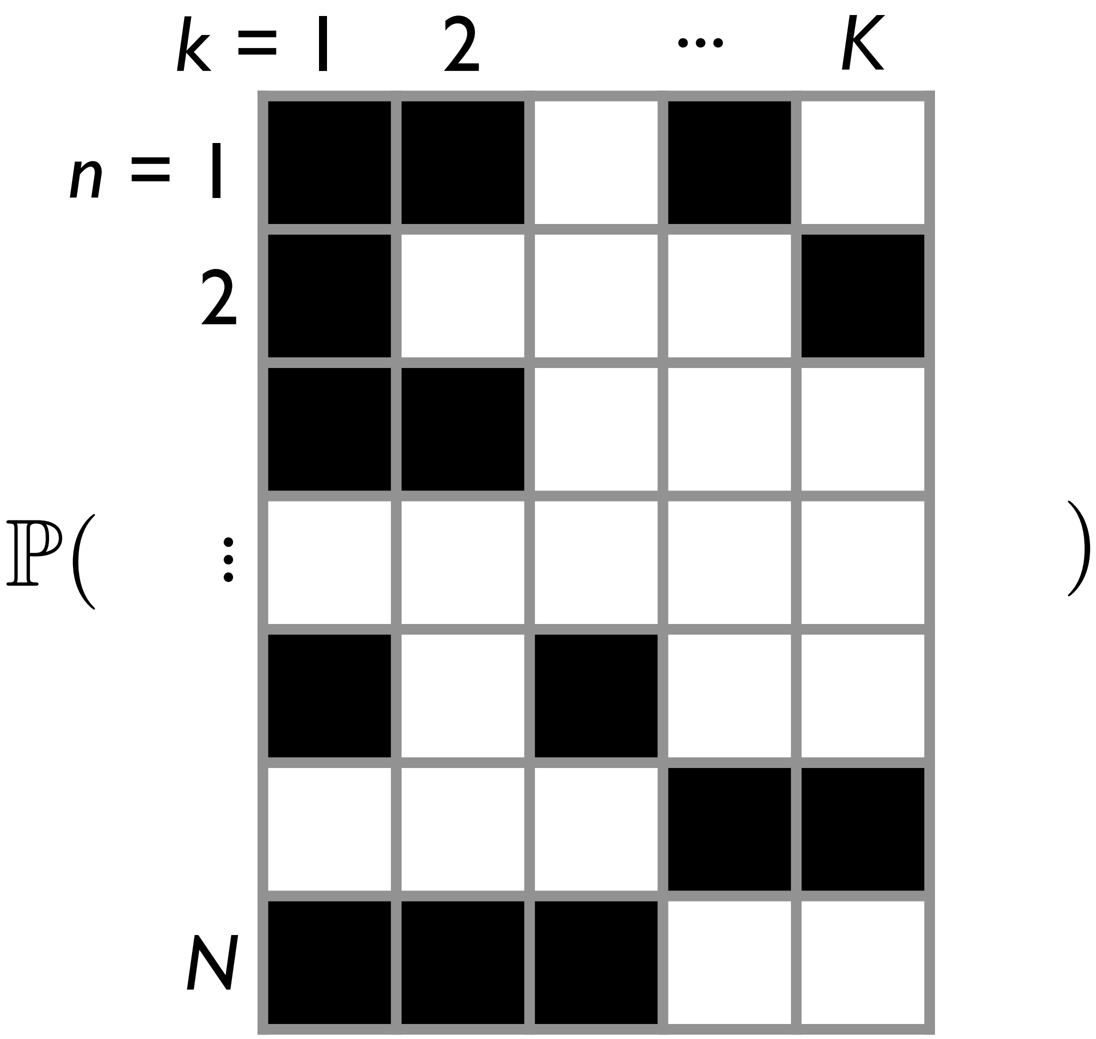
Feature frequency models: EFPFs?



$$\prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}}$$

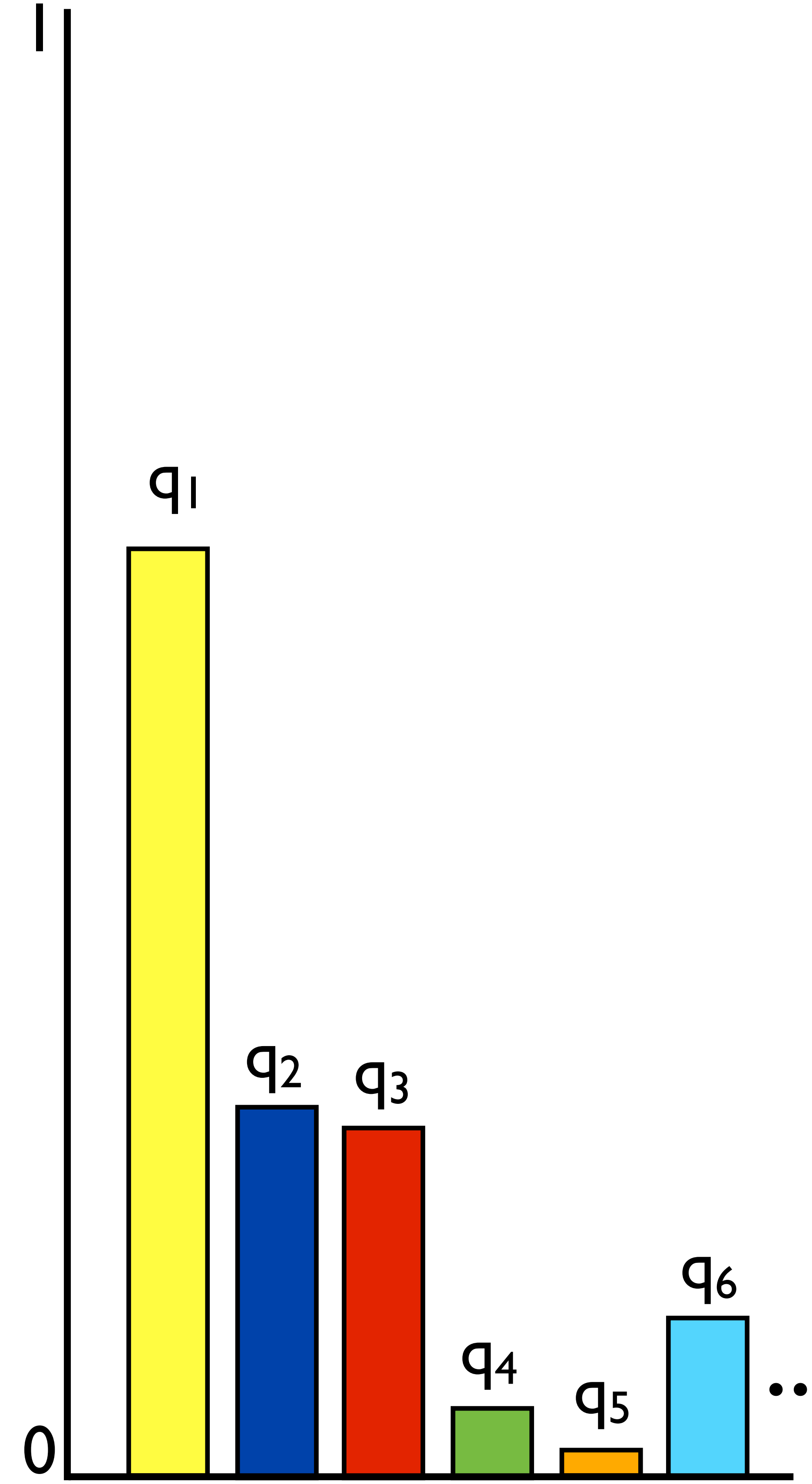


Feature frequency models: EFPFs?

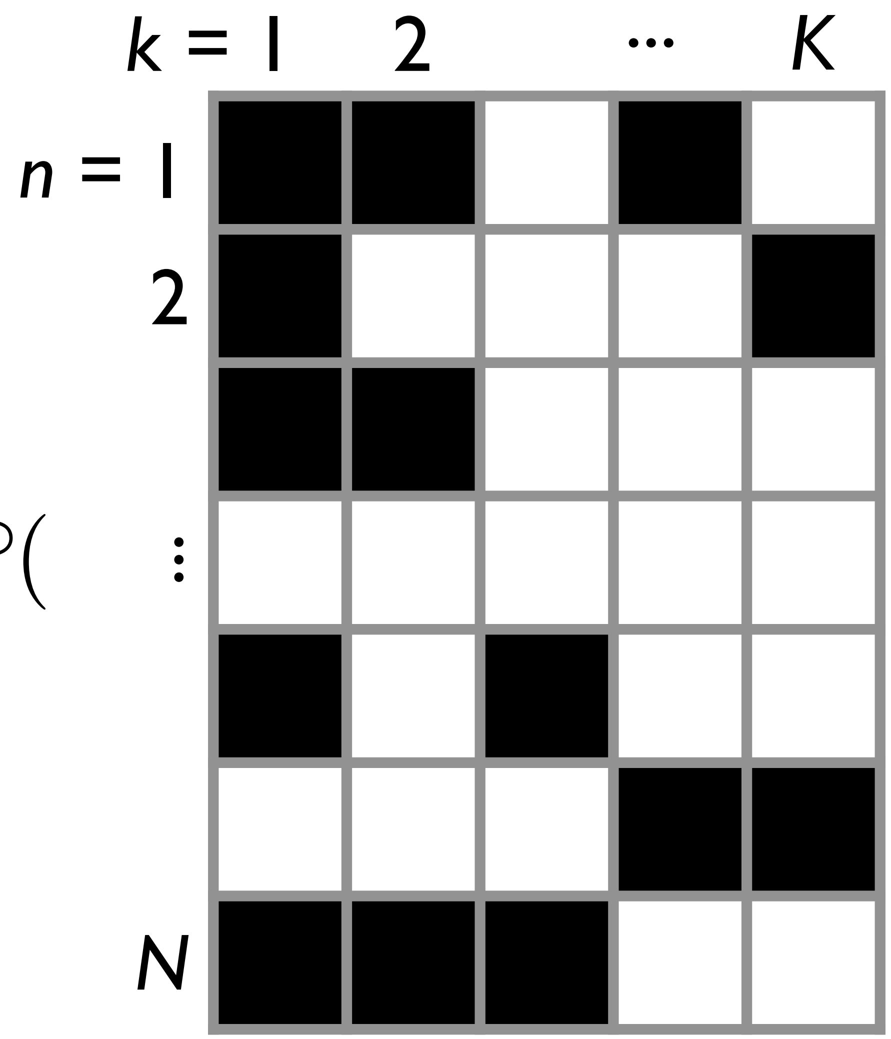


$$\prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}}$$

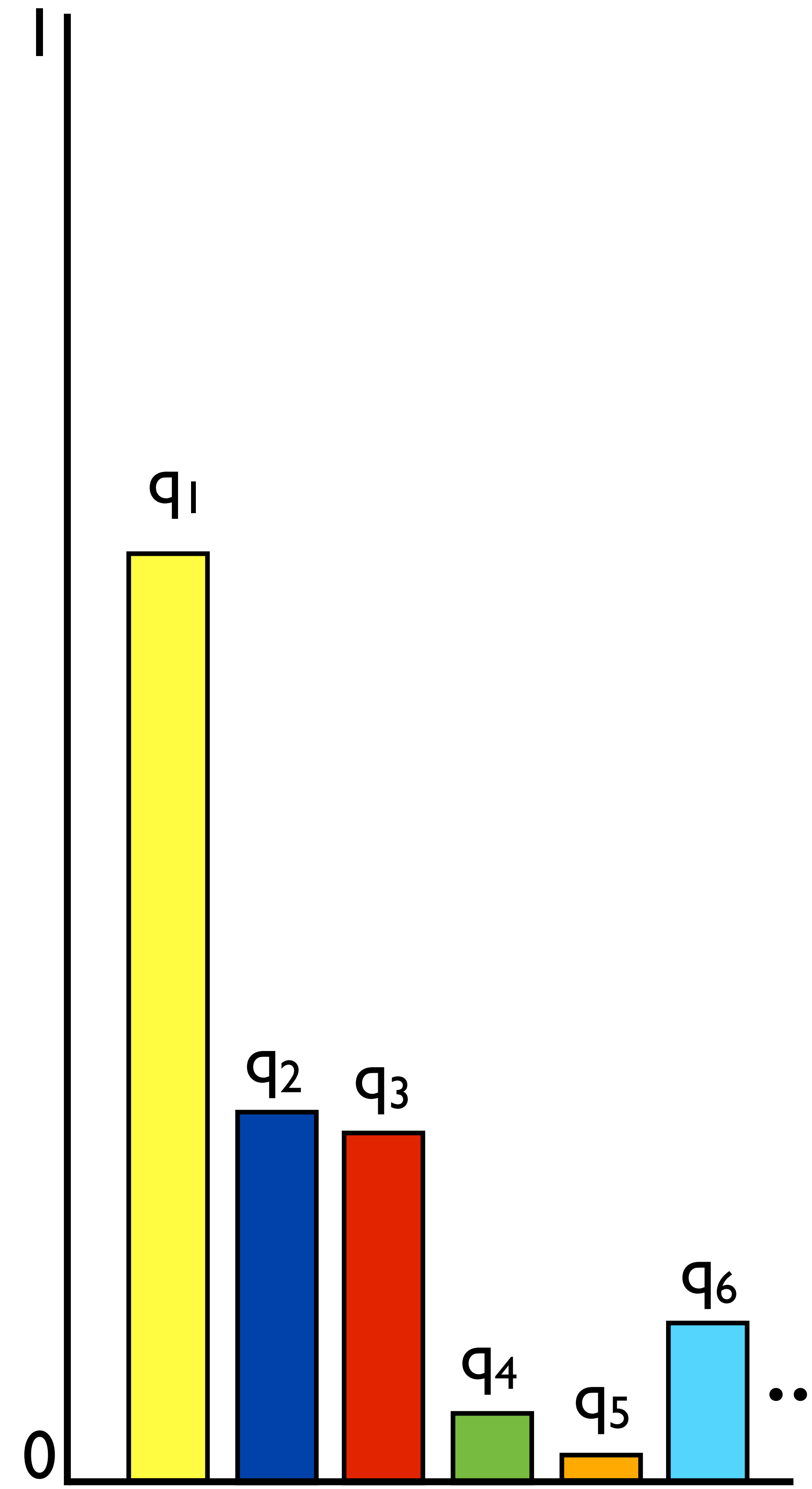
$$\cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N$$



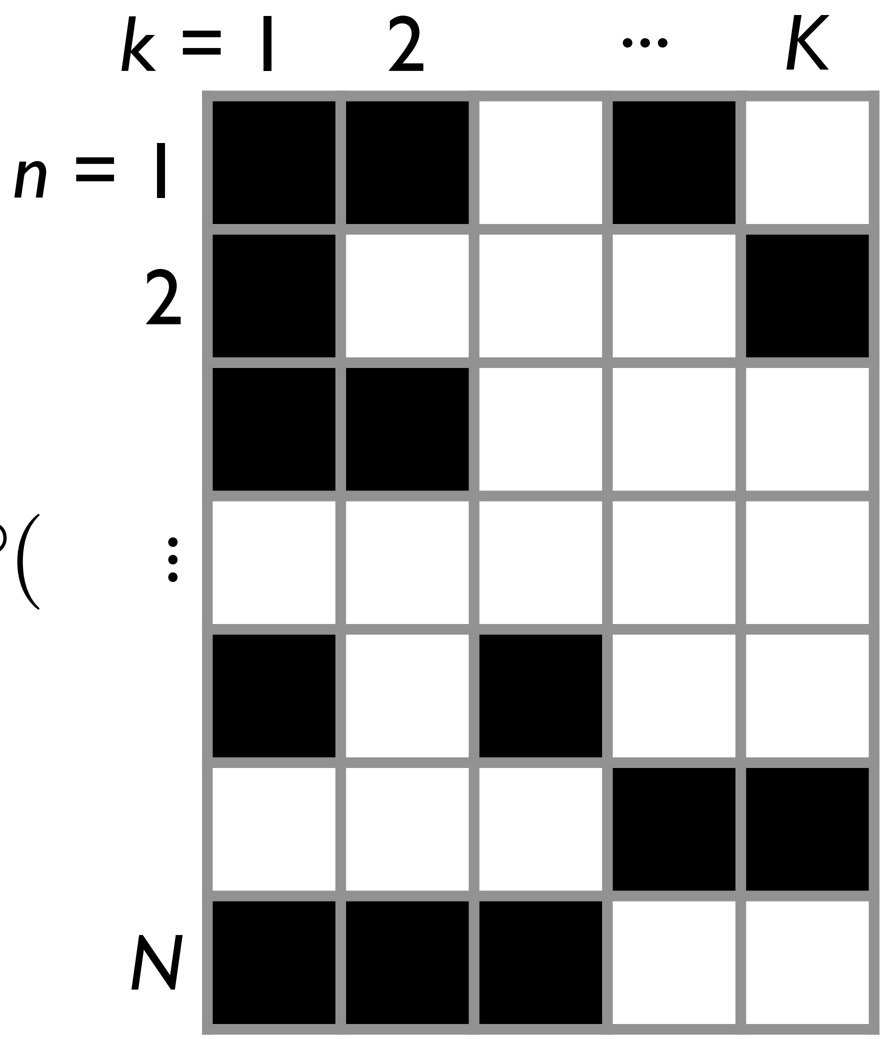
Feature frequency models: EFPFs?

$\mathbb{P}($  $)$

$$= \mathbb{E}\Big[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}} \cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N \Big]$$



Feature frequency models: EFPFs?

$\mathbb{P}($  $)$

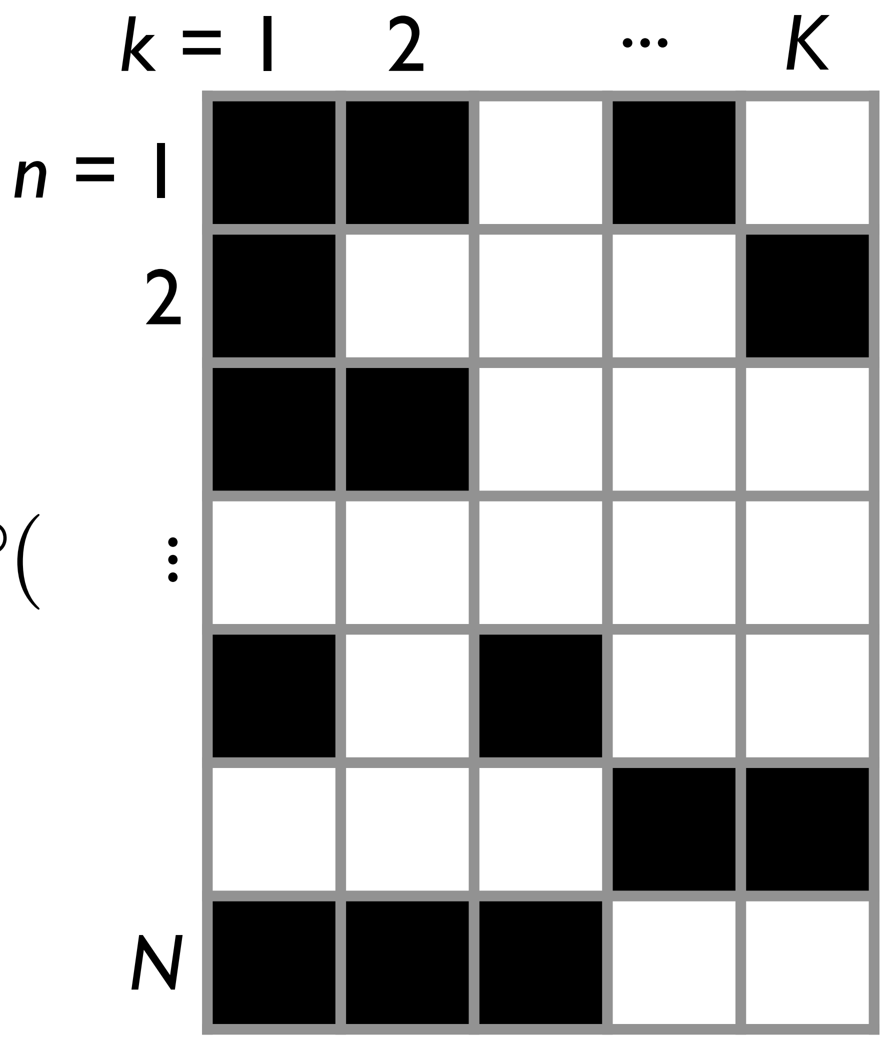
$$
= \mathbb{E}[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}}
$$

$$
\cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N]
$$

[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

$\mathbb{P}($



$)$

$$= \mathbb{E}\Big[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}}$$

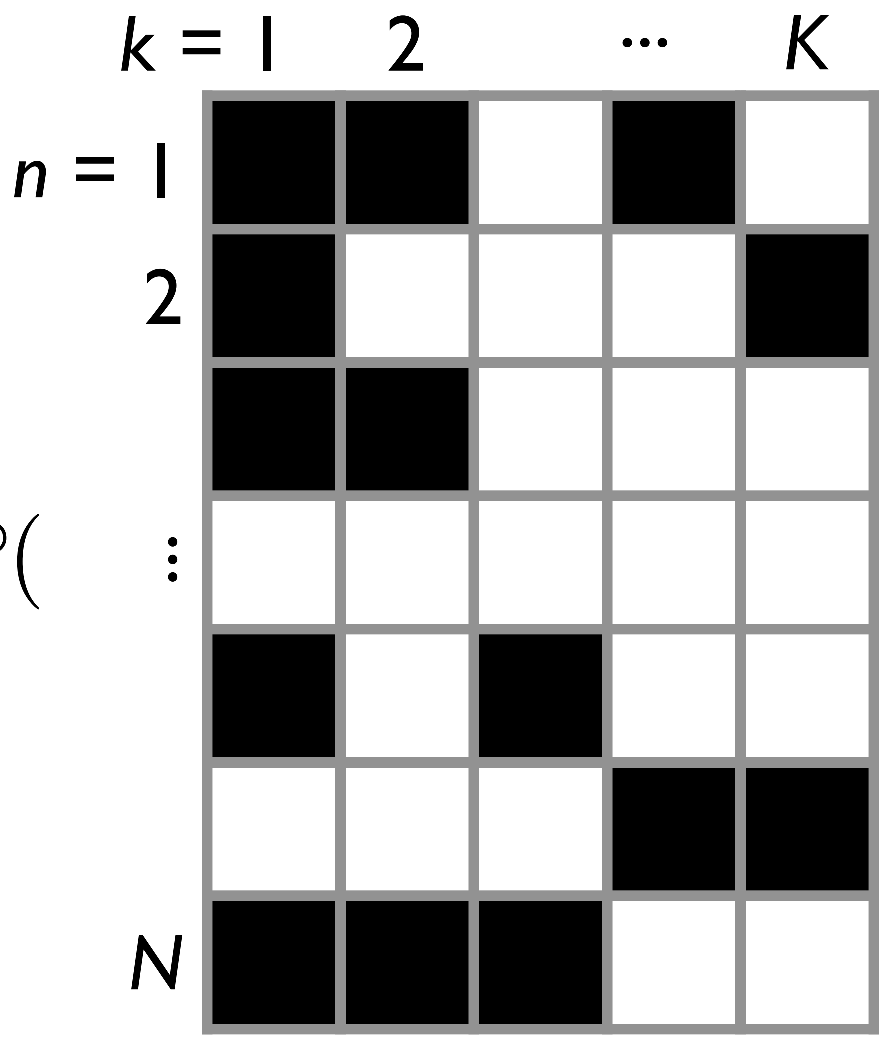
$$\cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^{N} \Big]$$

Size of *k*th feature

[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

$\mathbb{P}($



$)$

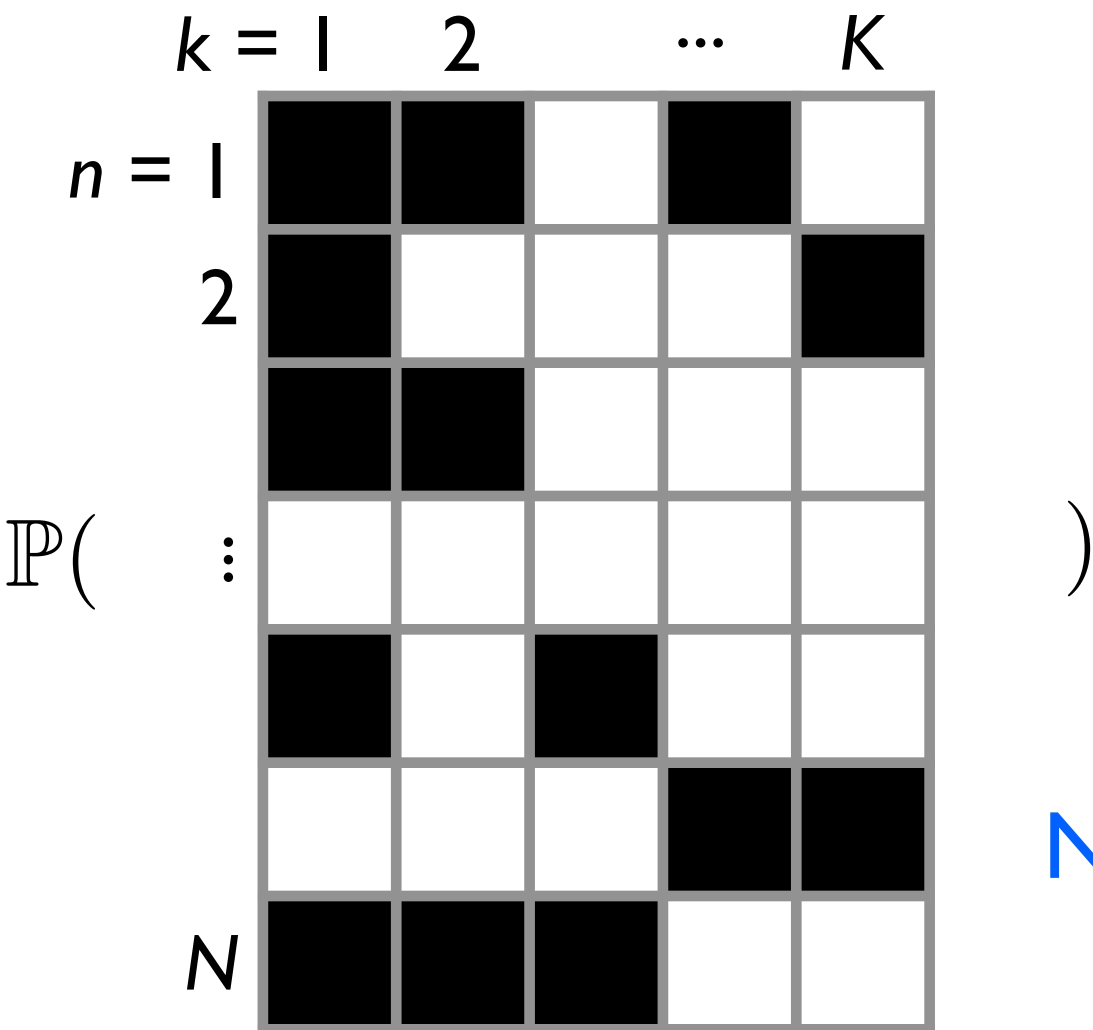
Number of features

Size of kth feature

$$= \mathbb{E}\Big[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}} \cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N \Big]$$

[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?



Number of features

Number of data points

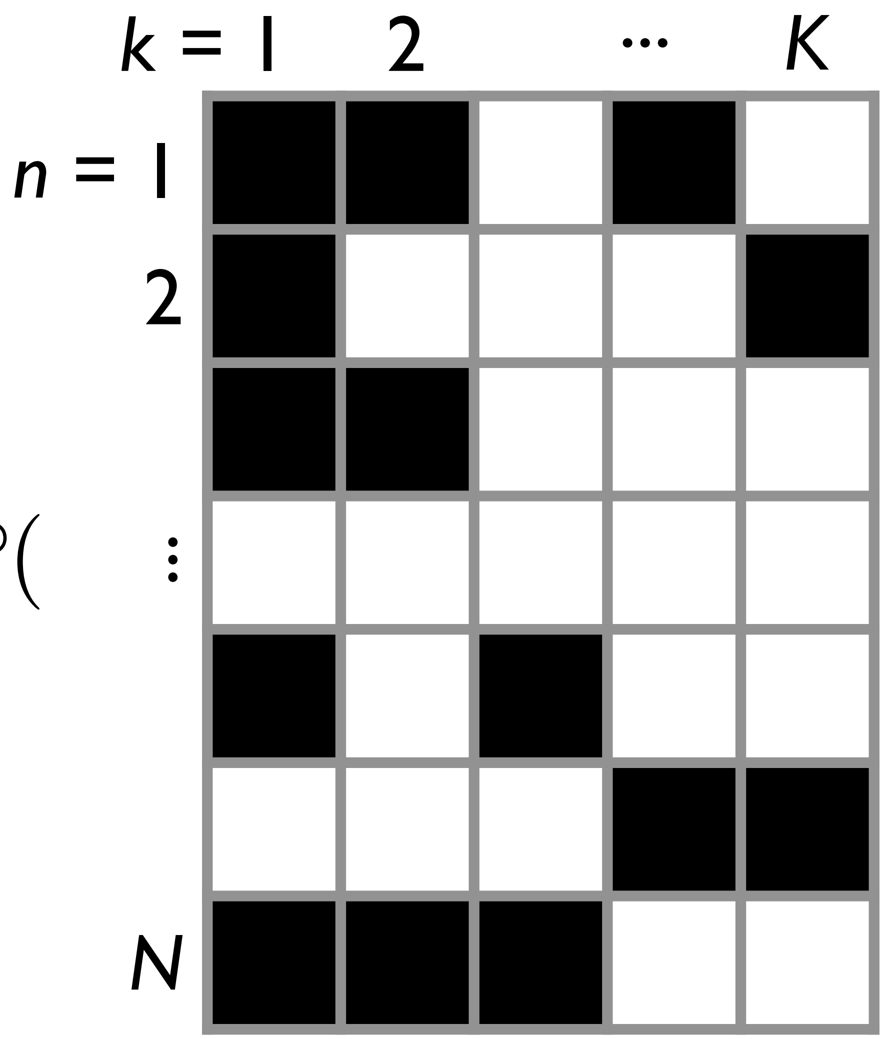
Size of kth feature

$$= \mathbb{E}\Big[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}} \cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N \Big]$$

[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

$\mathbb{P}($



$)$

Number of features

Number of data points

Size of kth feature

$$= \mathbb{E}\Big[\sum_{\text{distinct } i_k} \frac{1}{K!} \prod_{k=1}^{K} q_{i_k}^{S_{N,k}} (1 - q_{i_k})^{N - S_{N,k}}$$

$$\cdot \prod_{j \notin \{i_k\}_{k=1}^{K}} (1 - q_j)^N \Big] = p(N; S_{N,1}, S_{N,2}, \ldots, S_{N,K})$$

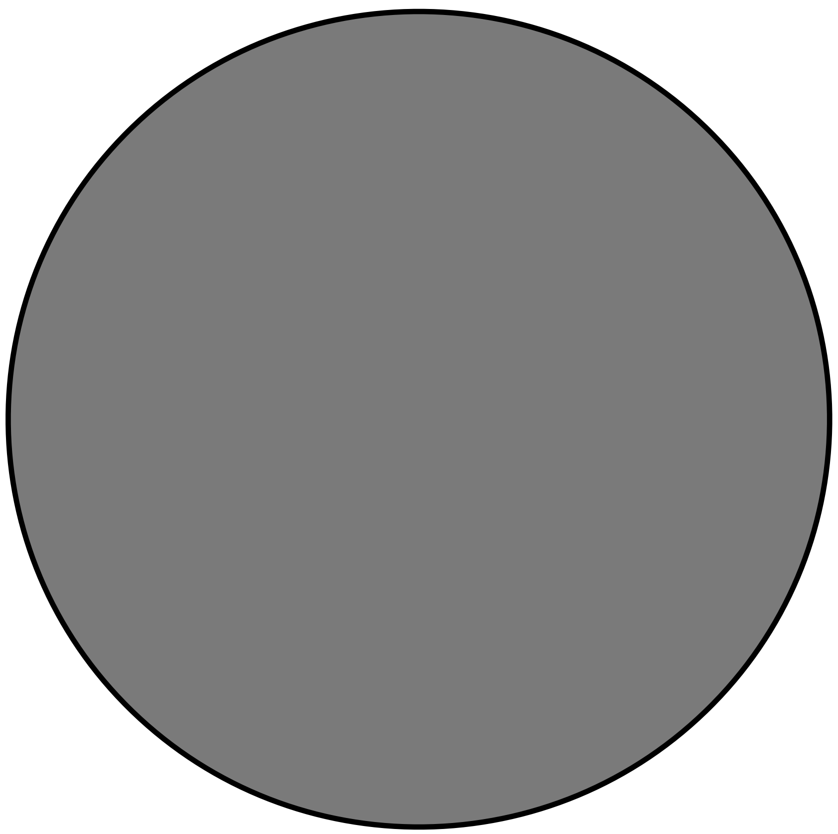
EFPF

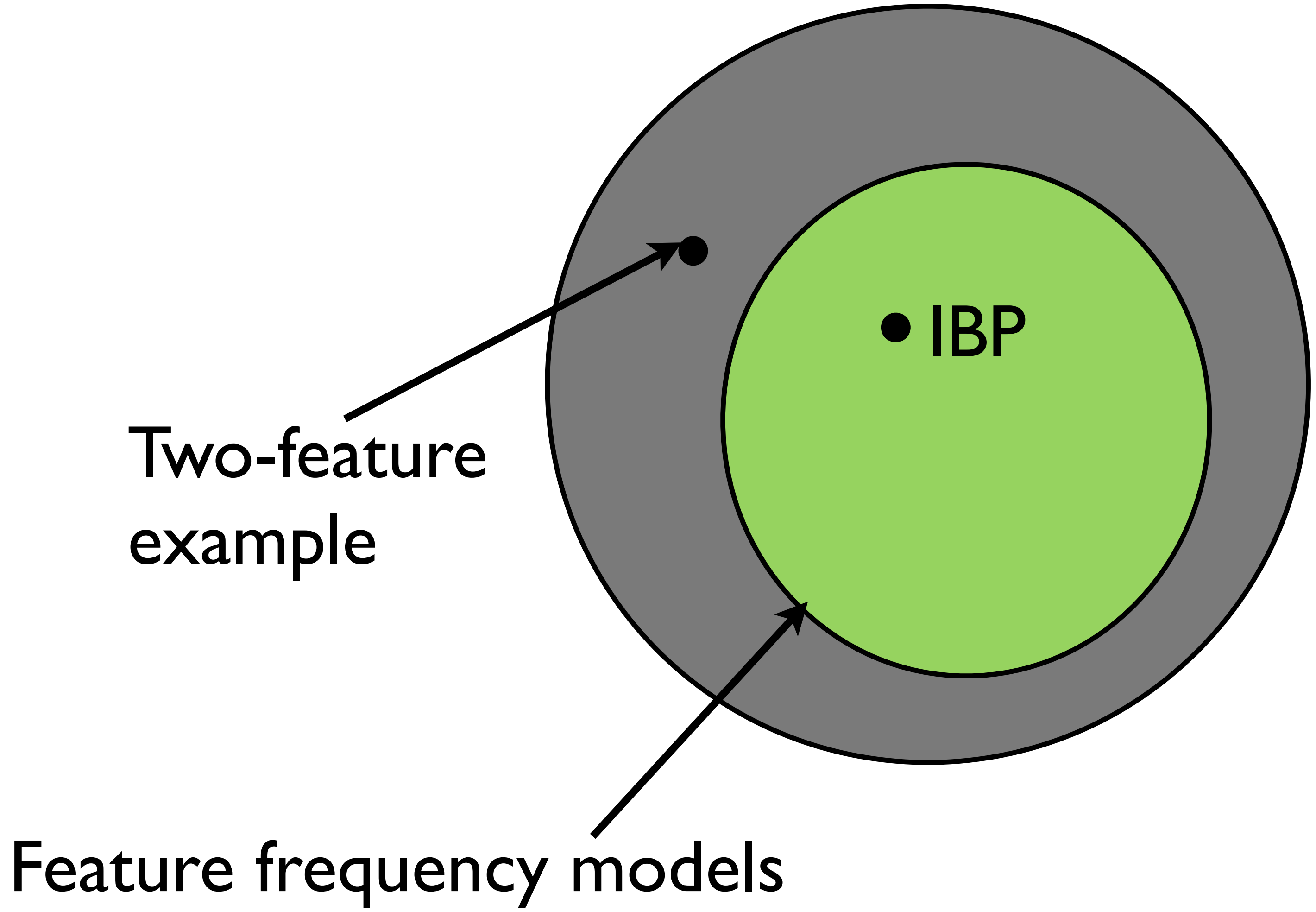
[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions

Exchangeable feature distributions
= Feature paintbox allocations

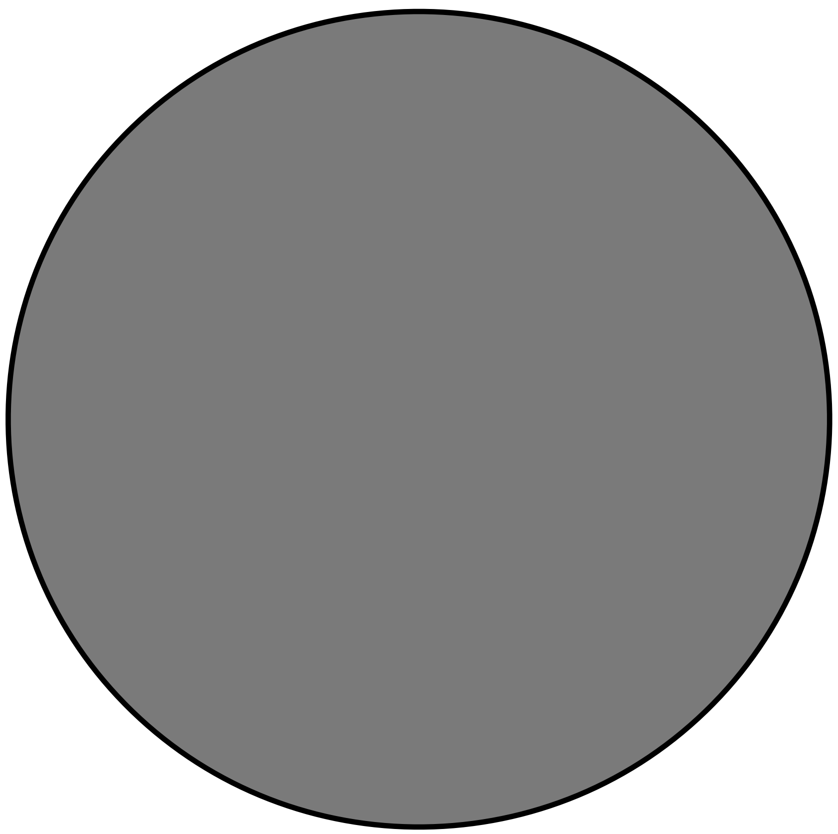




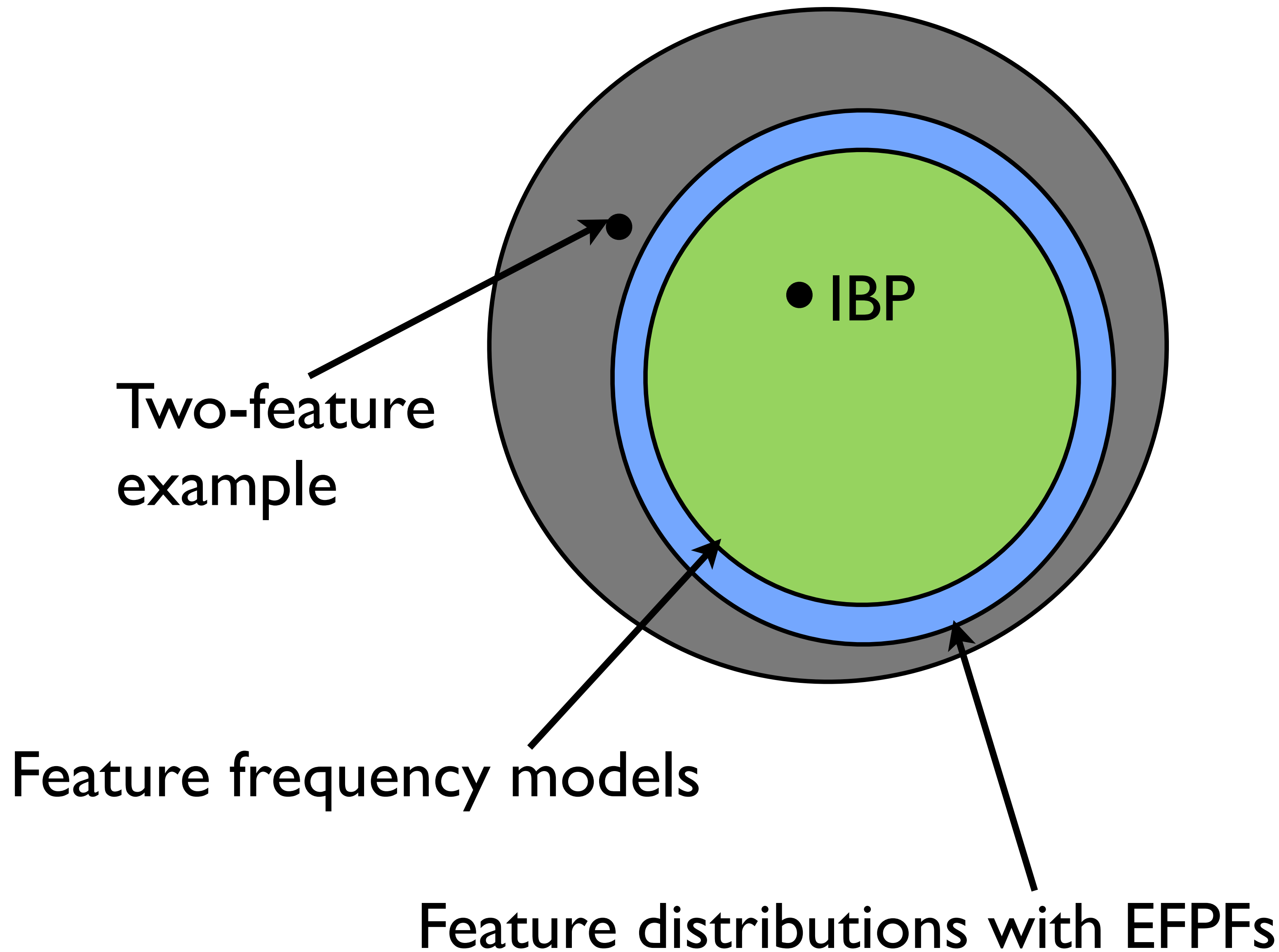
[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



Exchangeable feature distributions
= Feature paintbox allocations



[Broderick, Pitman, Jordan 2013]

Distributions with EFPFs: frequencies?

[Broderick, Pitman, Jordan 2013]

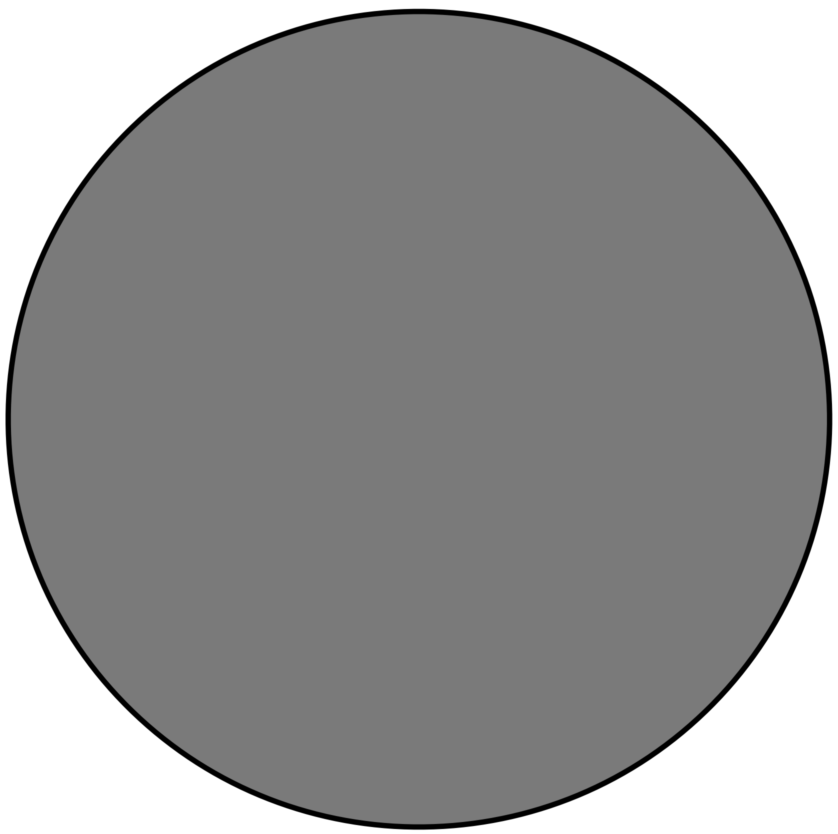
Distributions with EFPFs: frequencies?

✔ • Any number (+unbounded case) of features

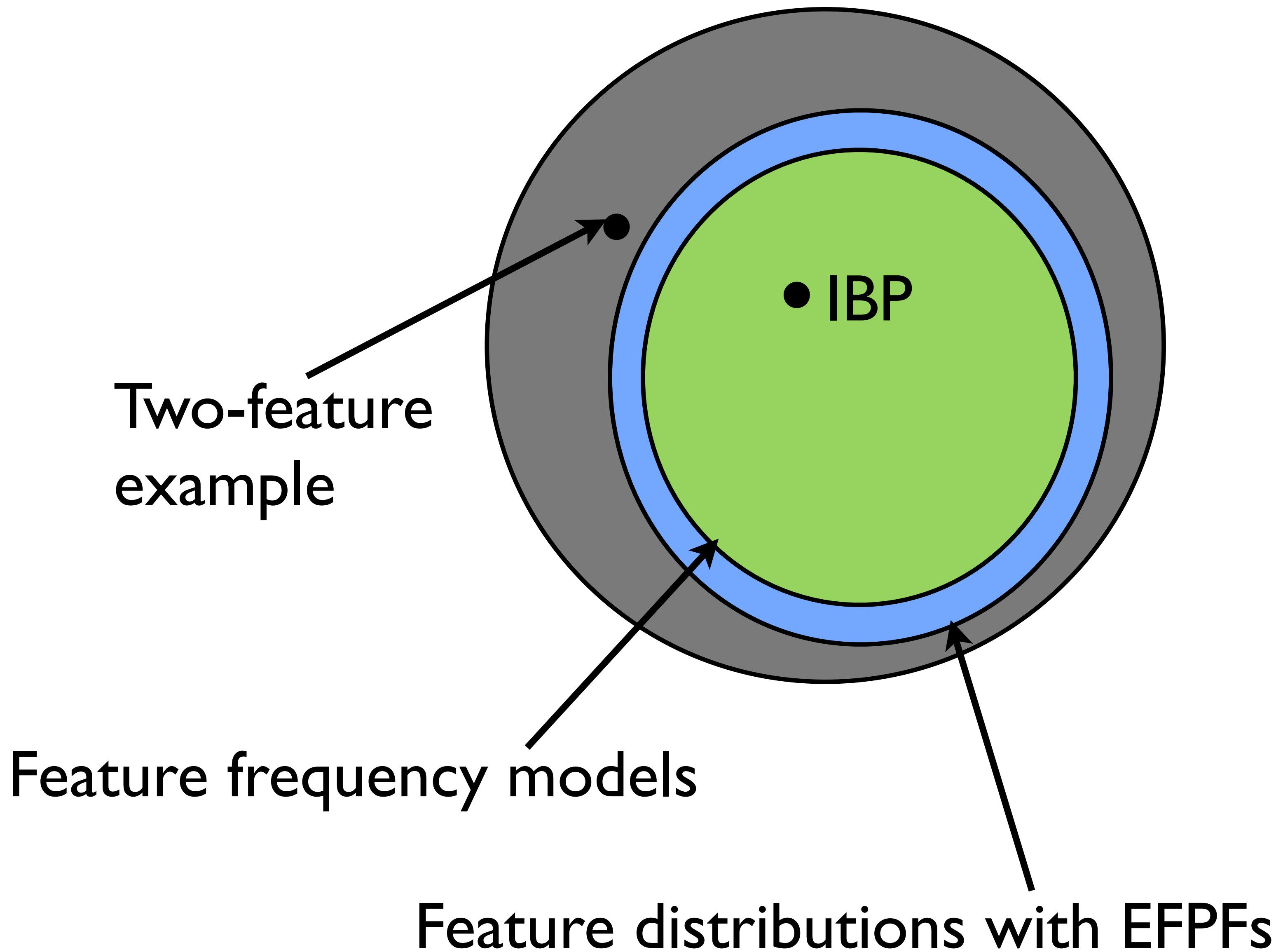
[Broderick, Pitman, Jordan 2013]

Distributions with EFPFs: frequencies?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



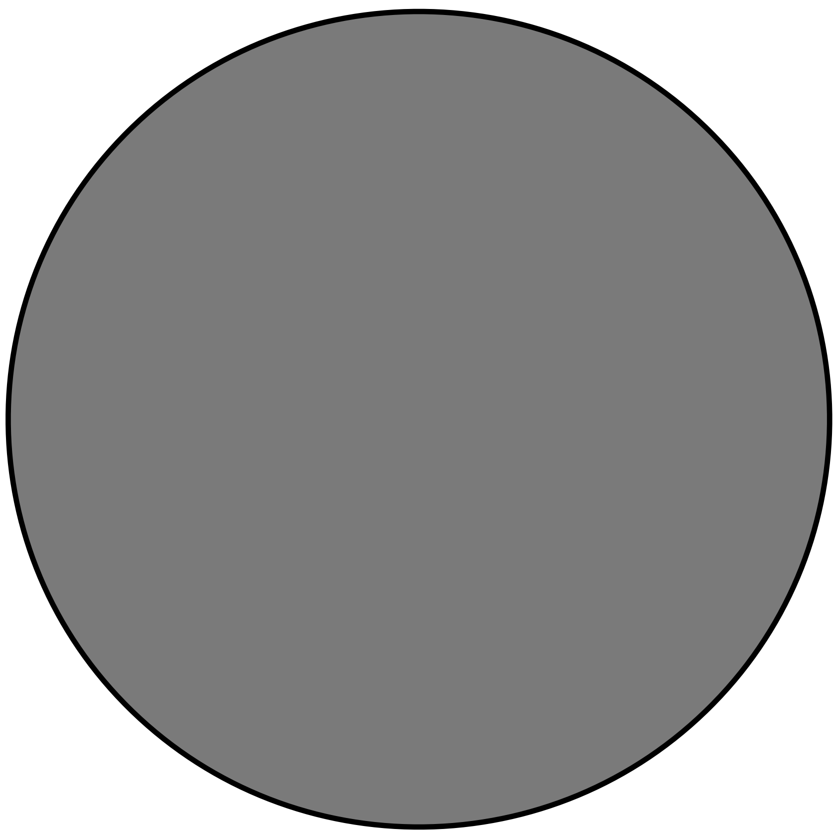
Exchangeable feature distributions
= Feature paintbox allocations



[Broderick, Pitman, Jordan 2013]

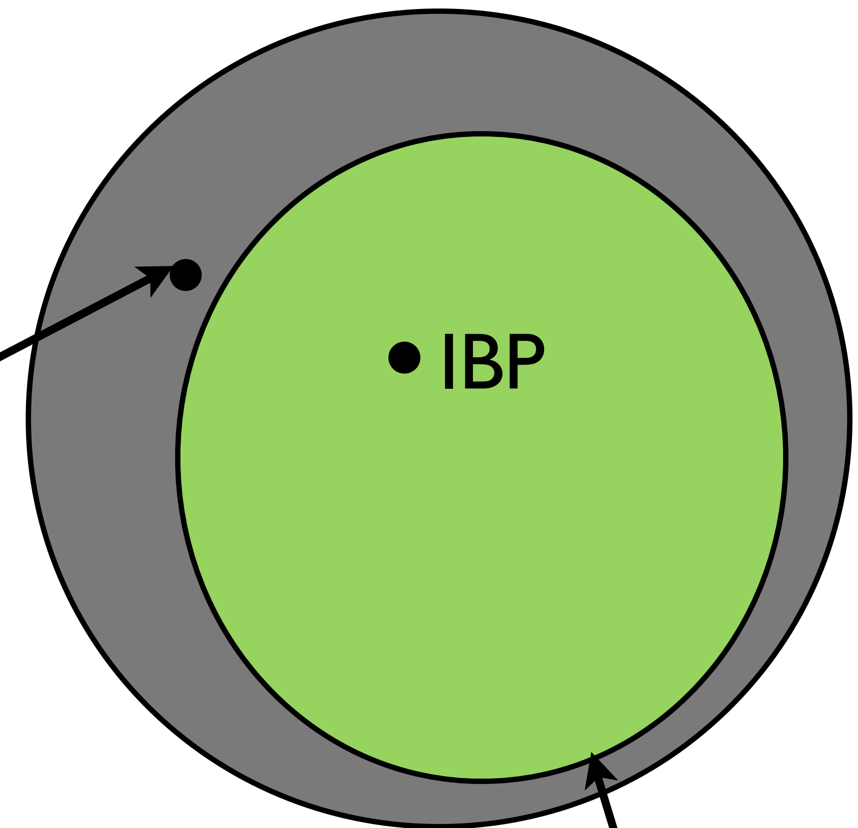
Distributions with EFPFs: frequencies?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



Exchangeable feature distributions
= Feature paintbox allocations

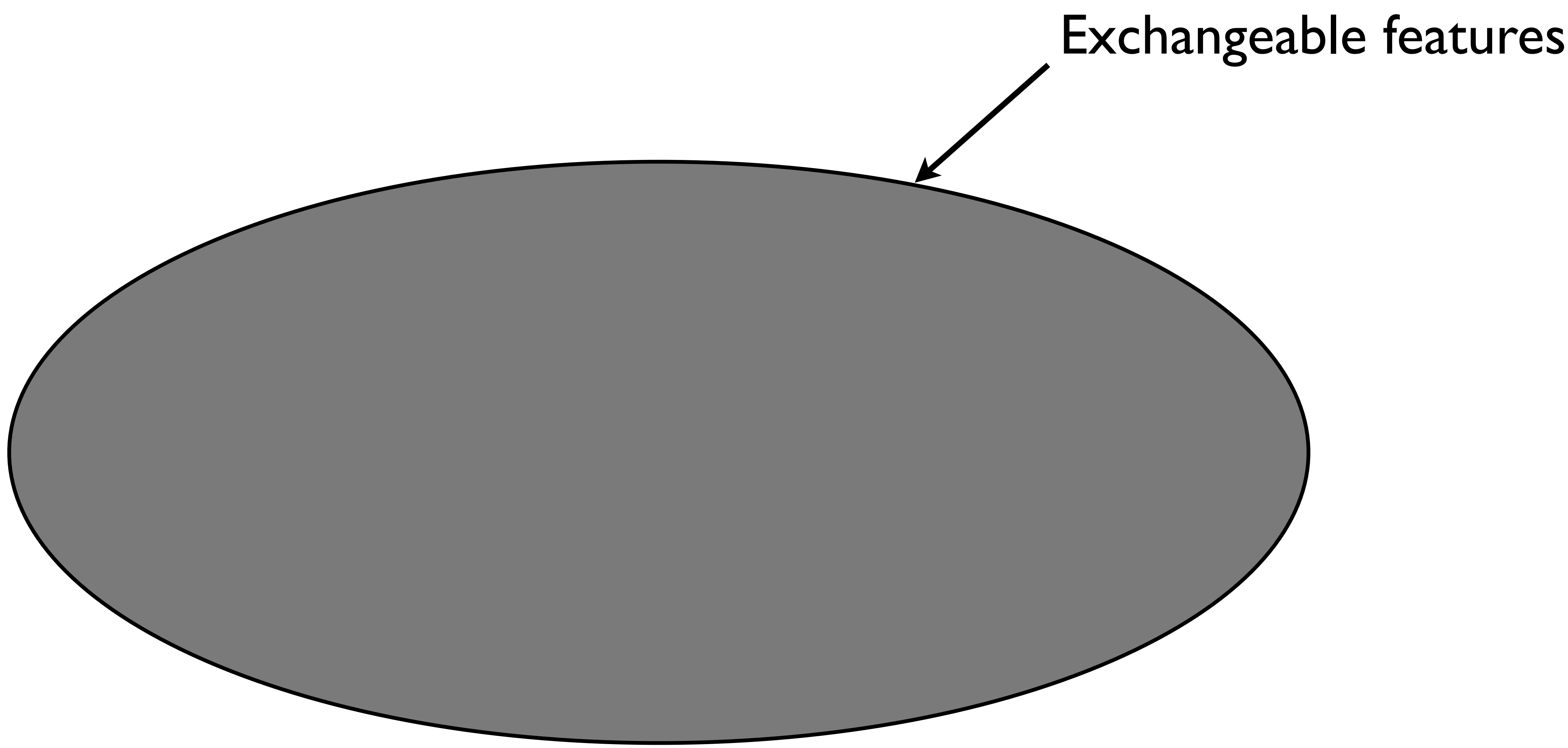
Two-feature example



Feature distributions with EFPFs
= Feature frequency models

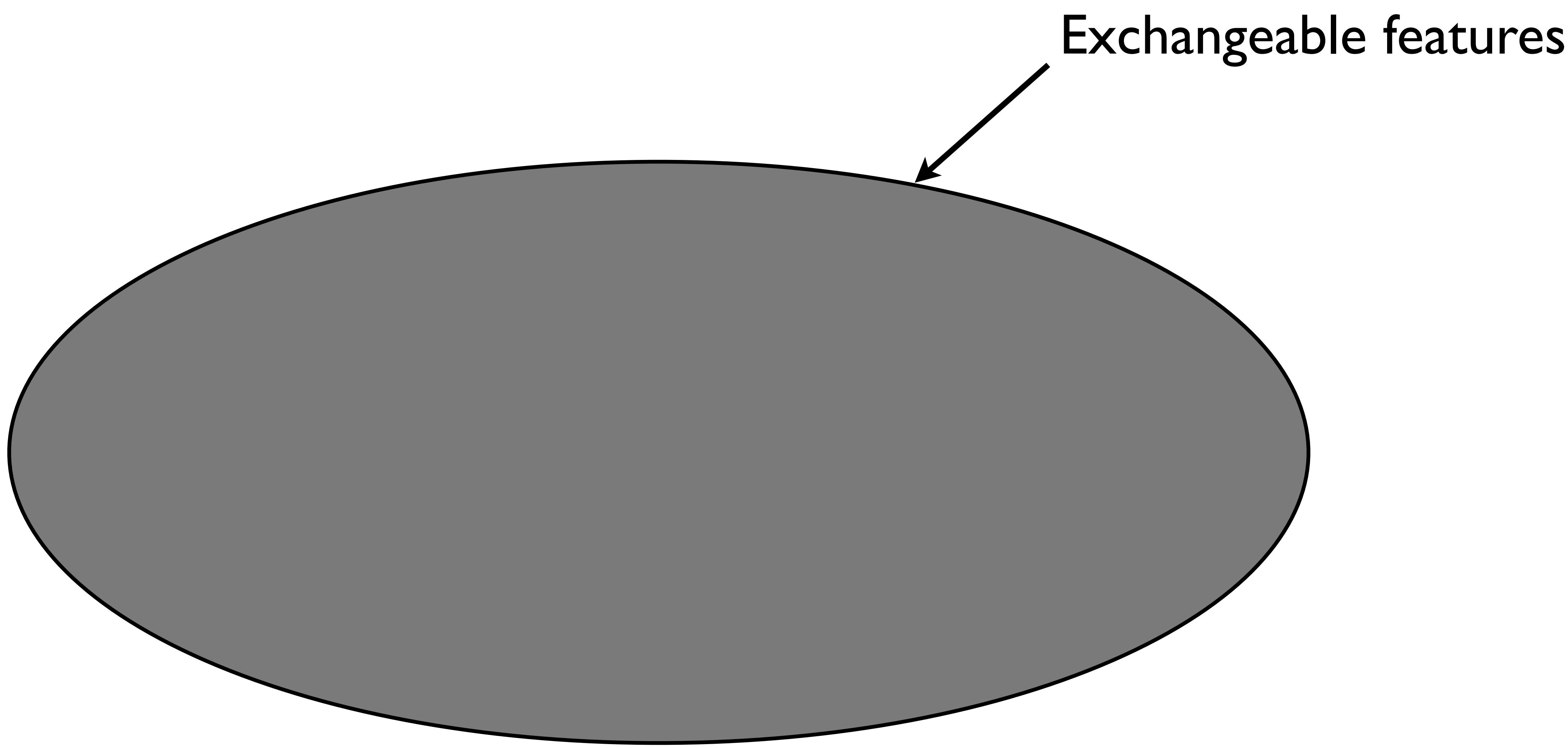
[Broderick, Pitman, Jordan 2013]

Theory conclusions



Theory conclusions

- Feature paintbox: characterization of exchangeable feature models



Theory conclusions

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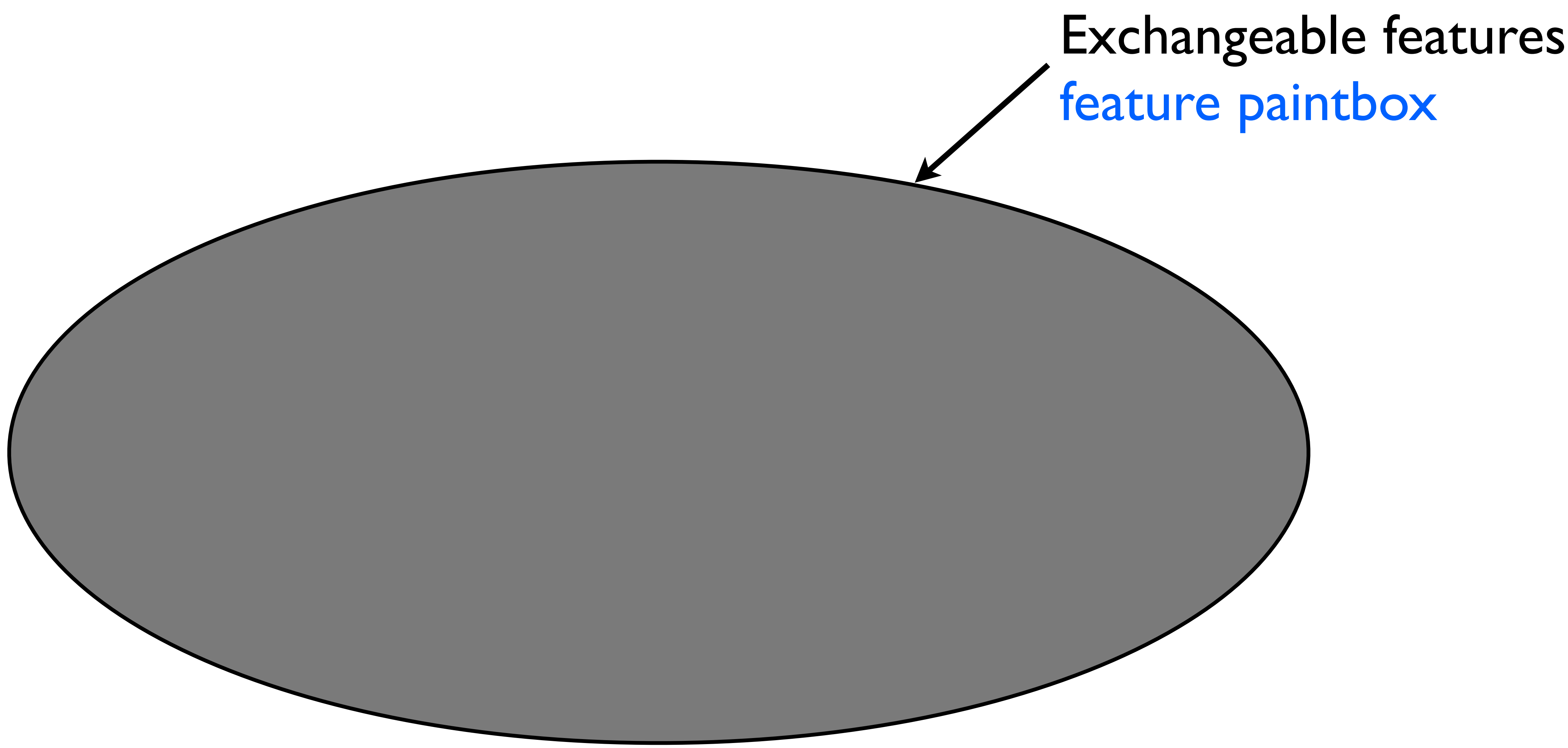
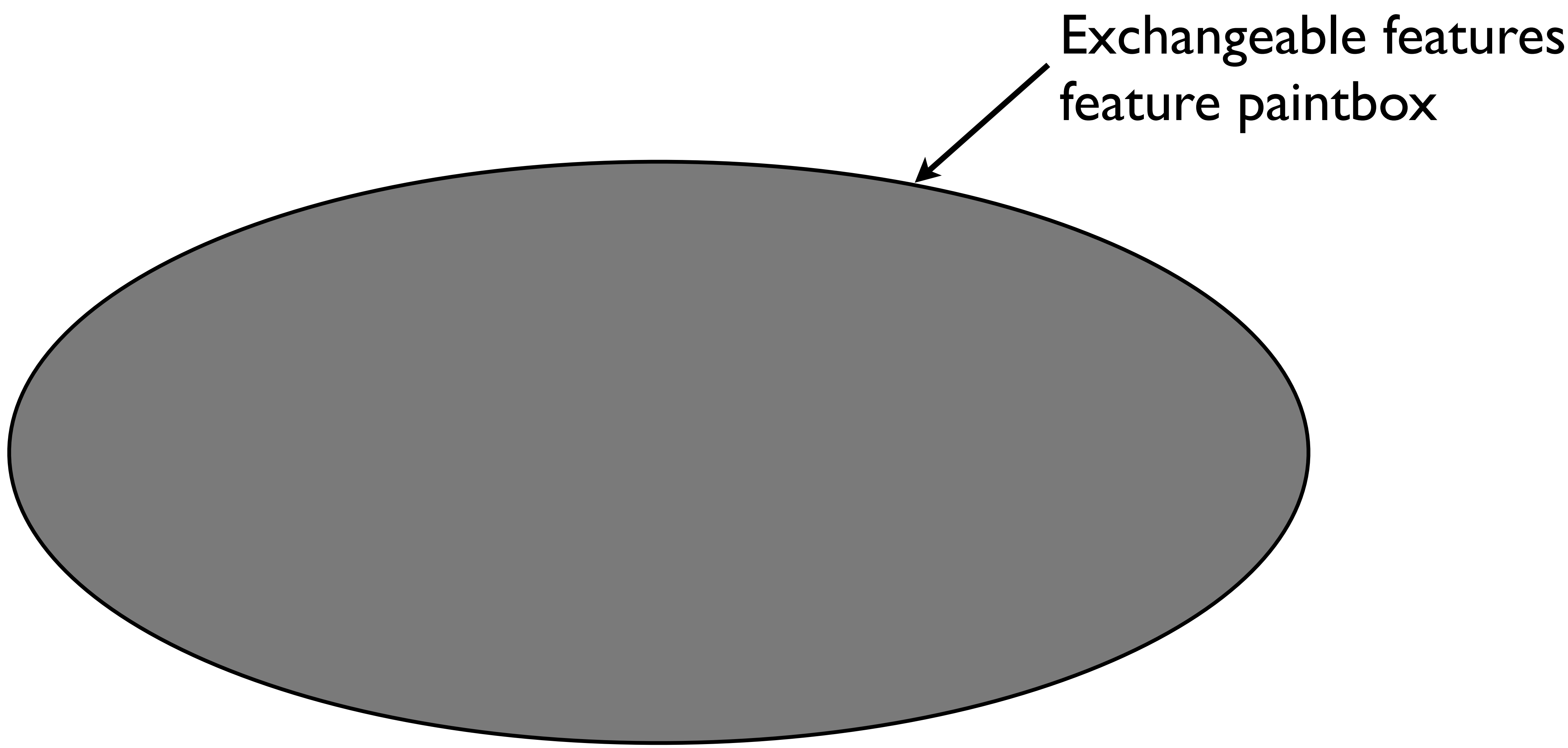


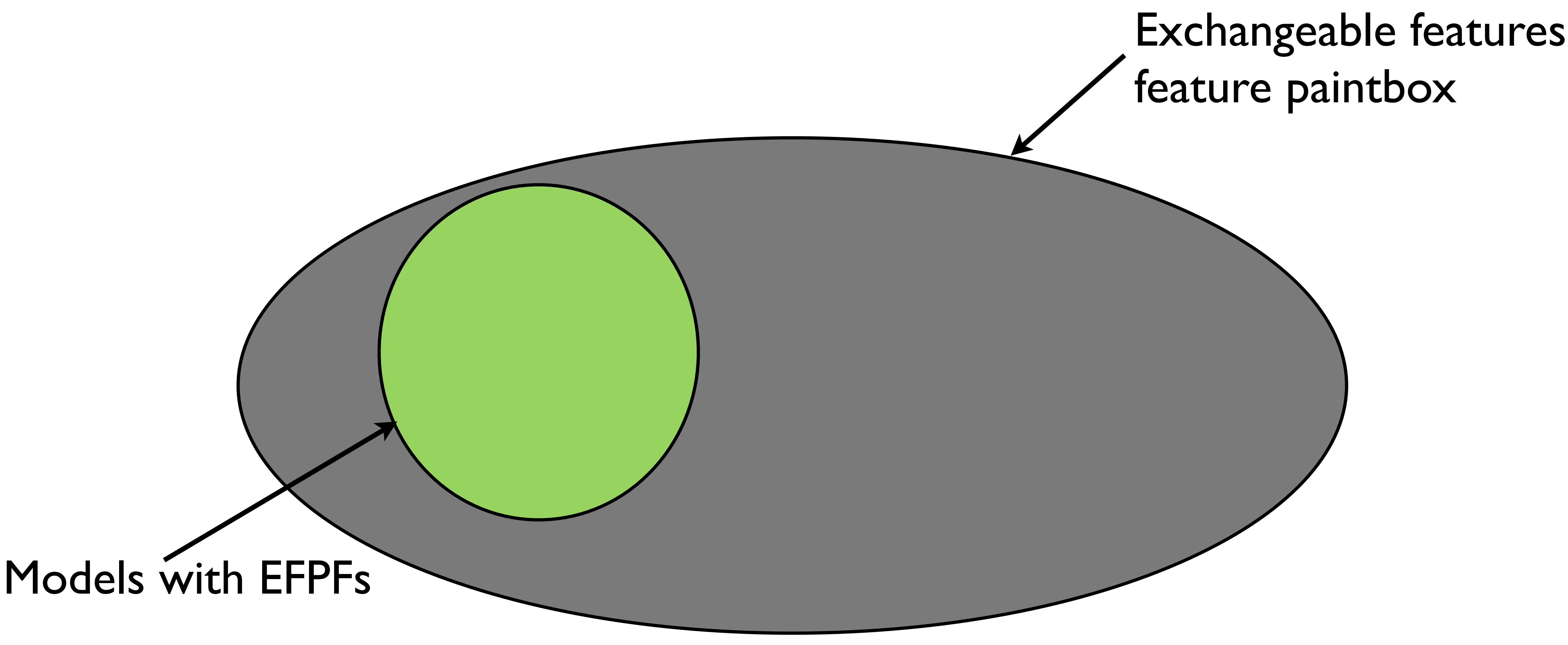
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



Theory conclusions

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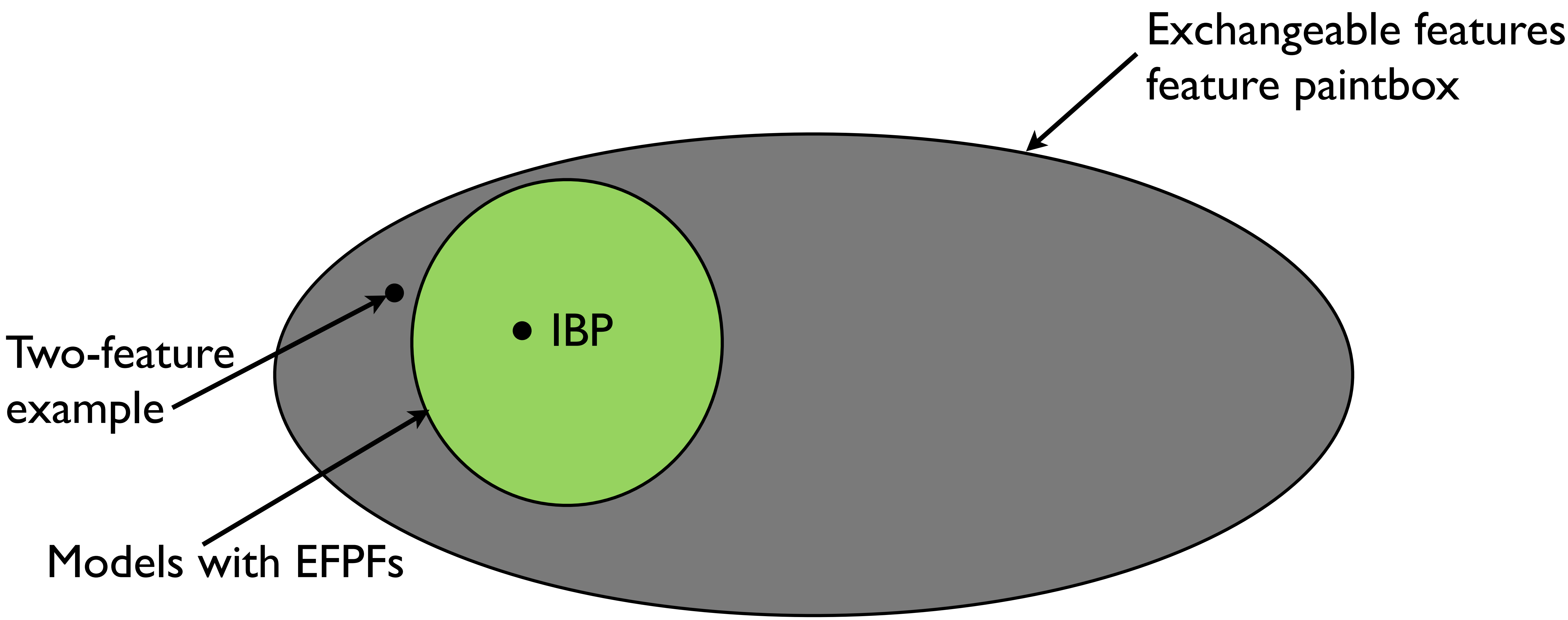

Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
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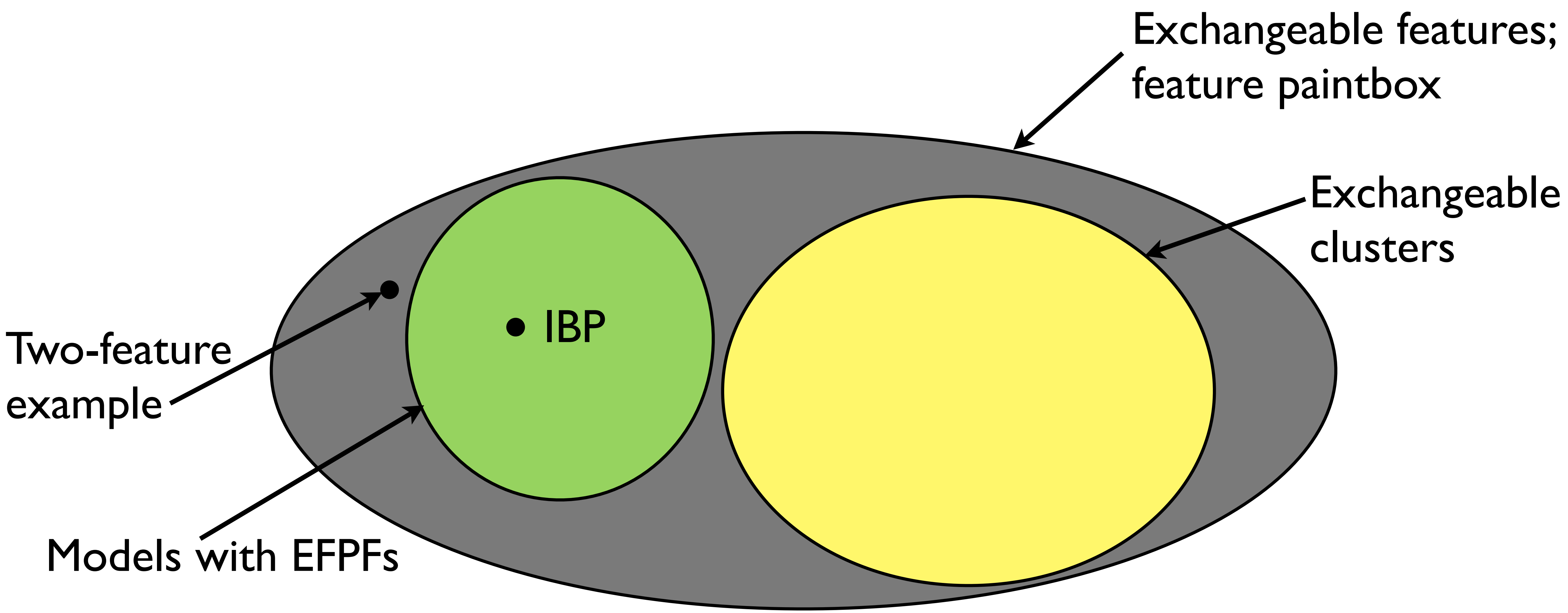
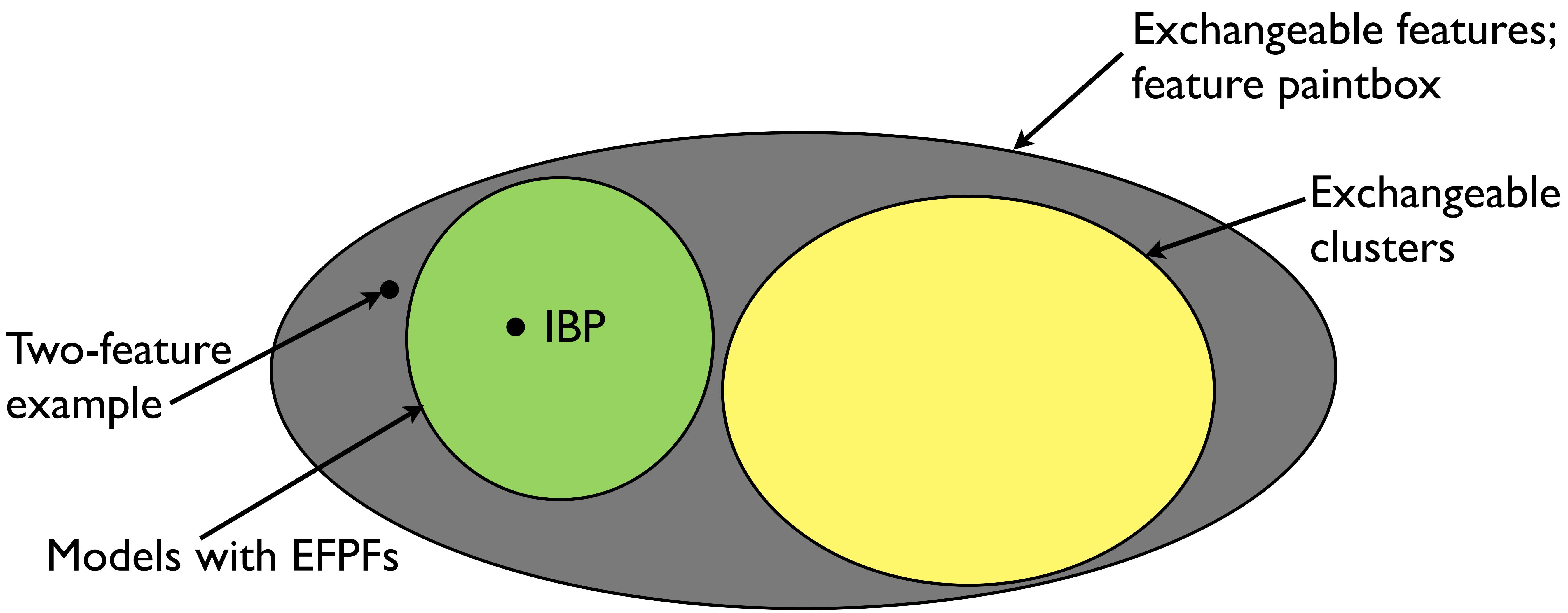


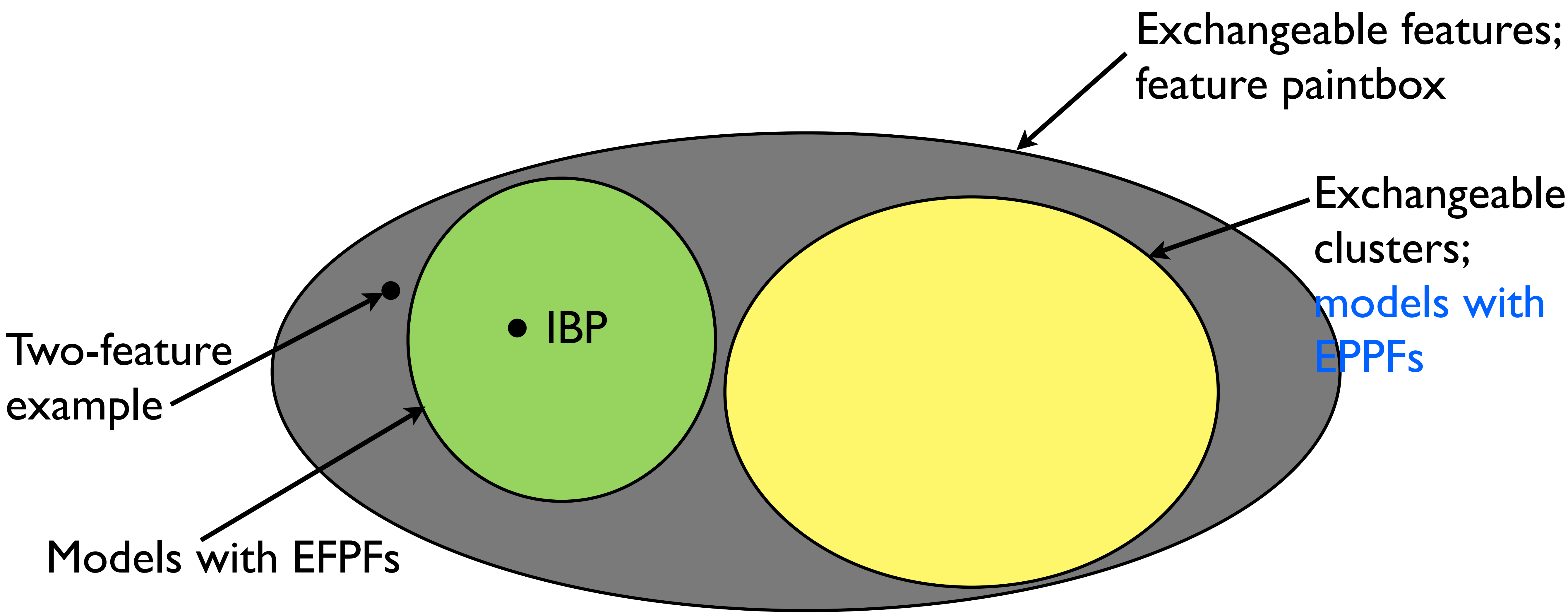
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



Theory conclusions

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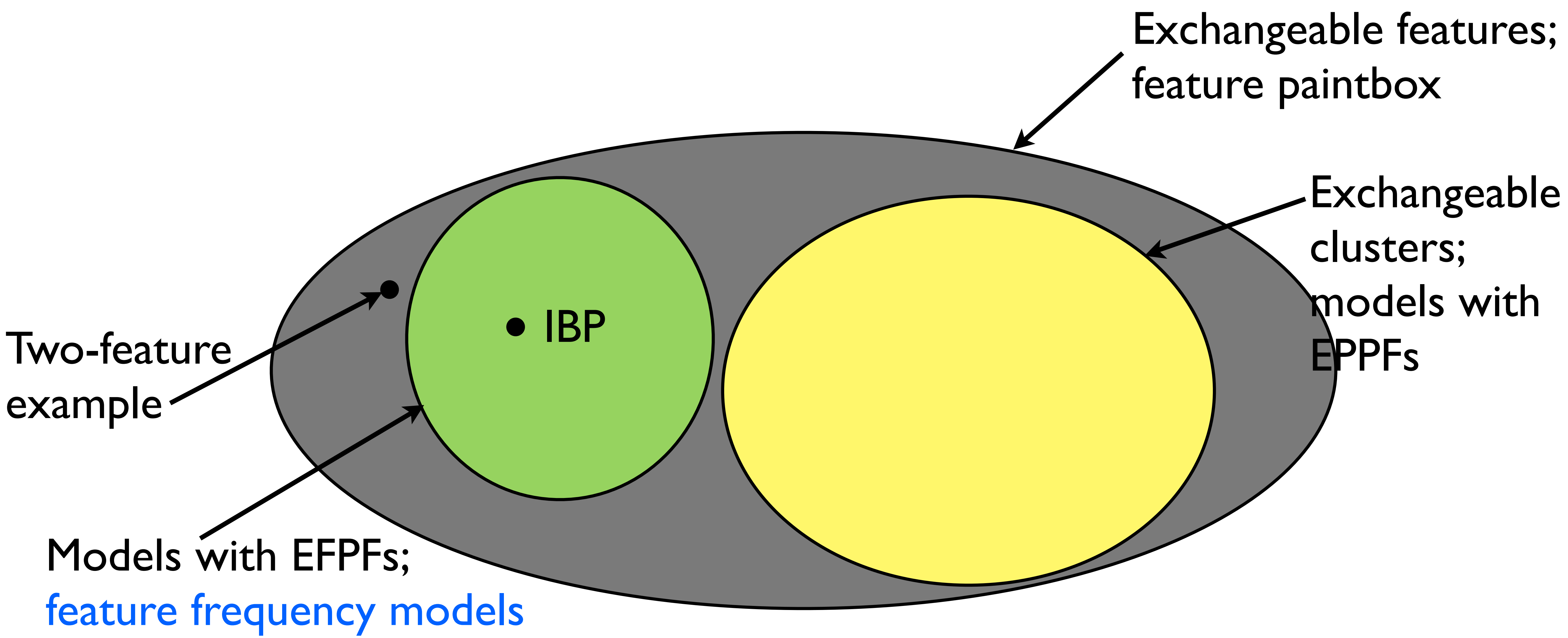
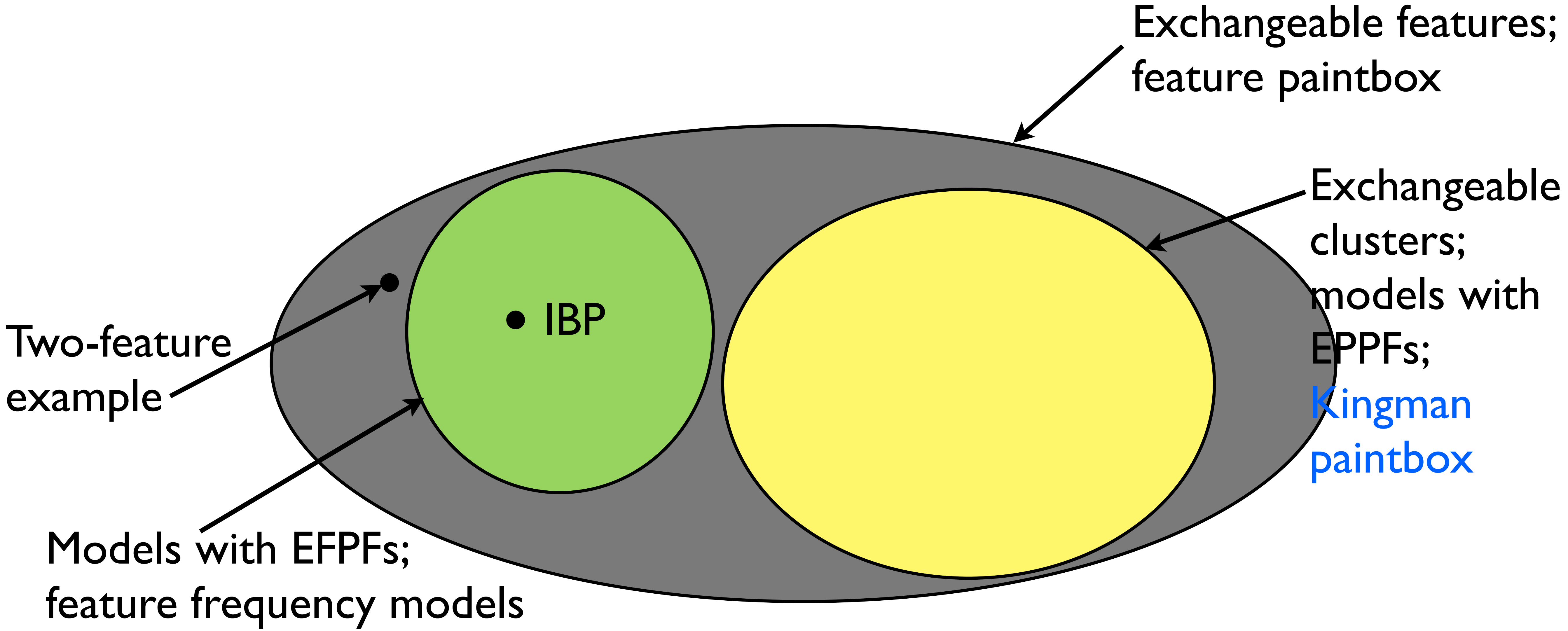


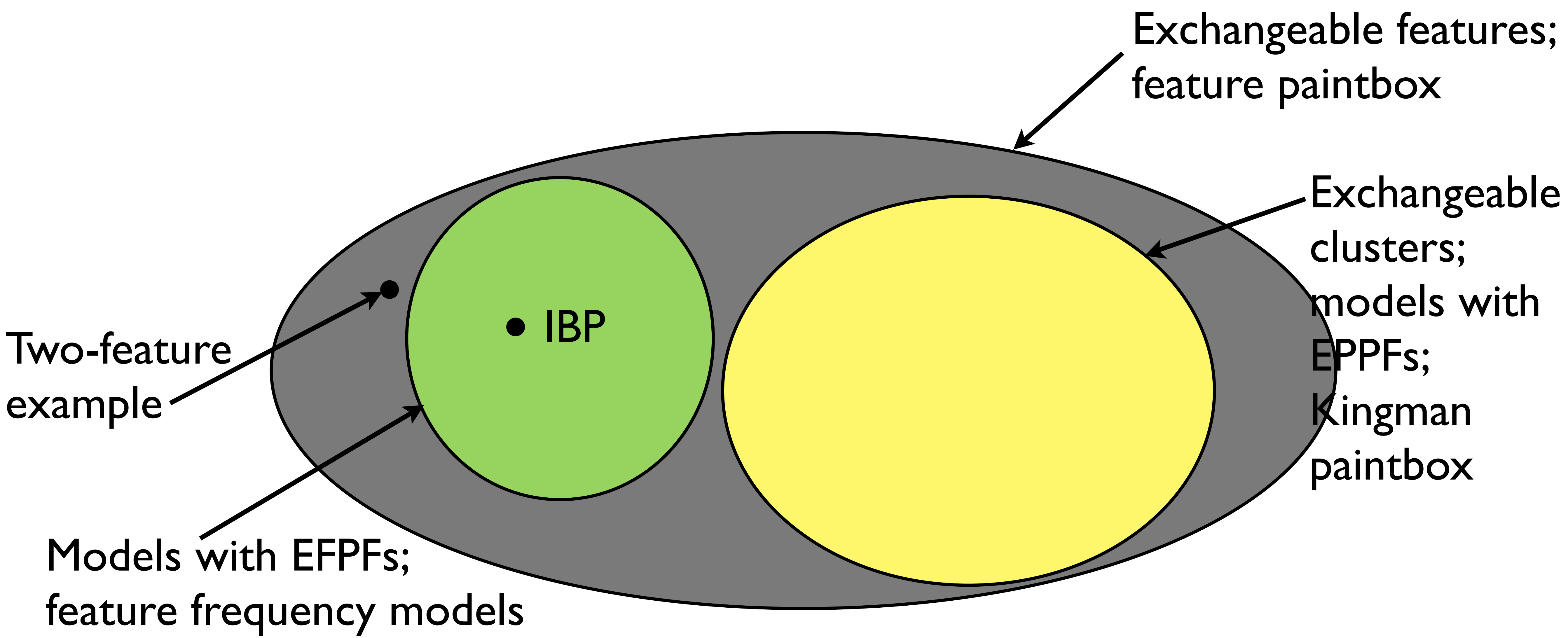
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



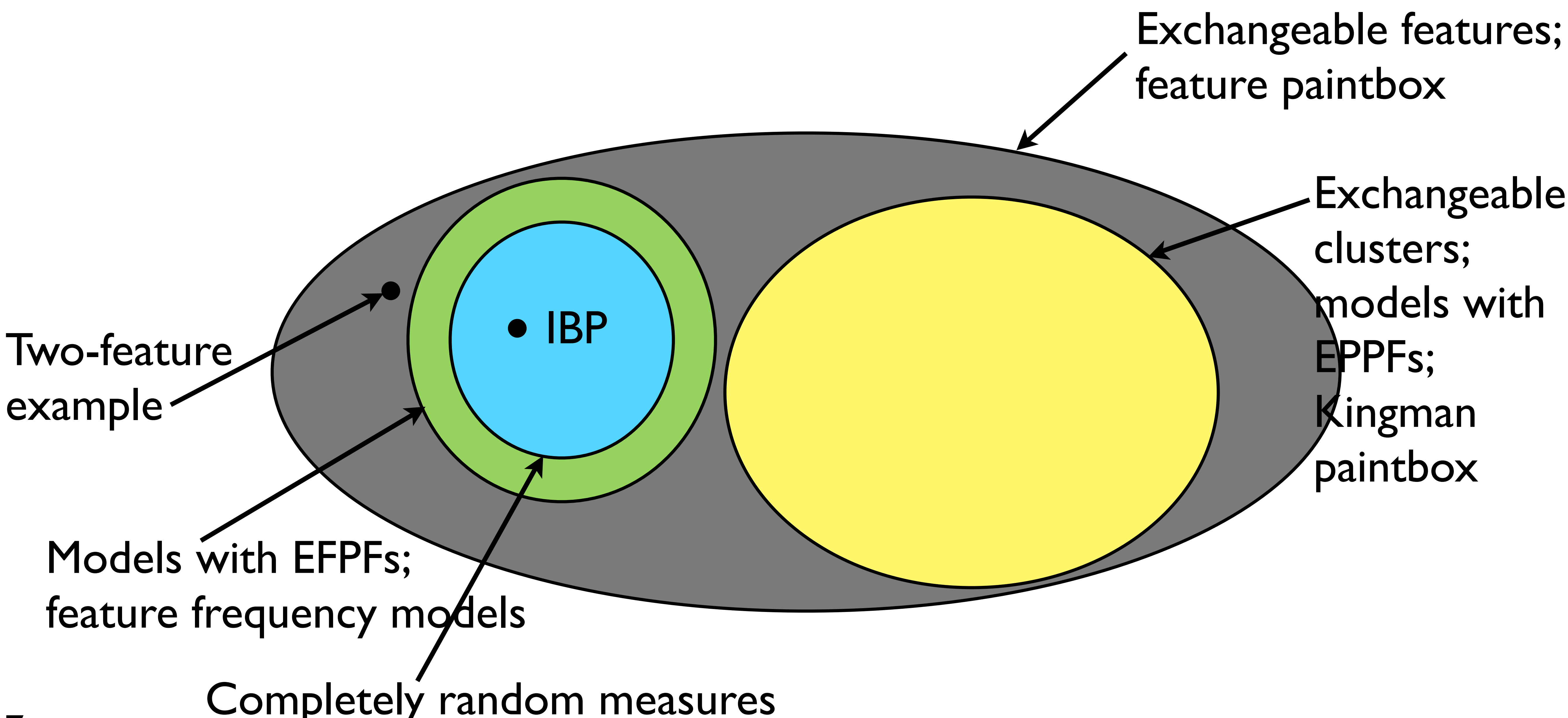
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections



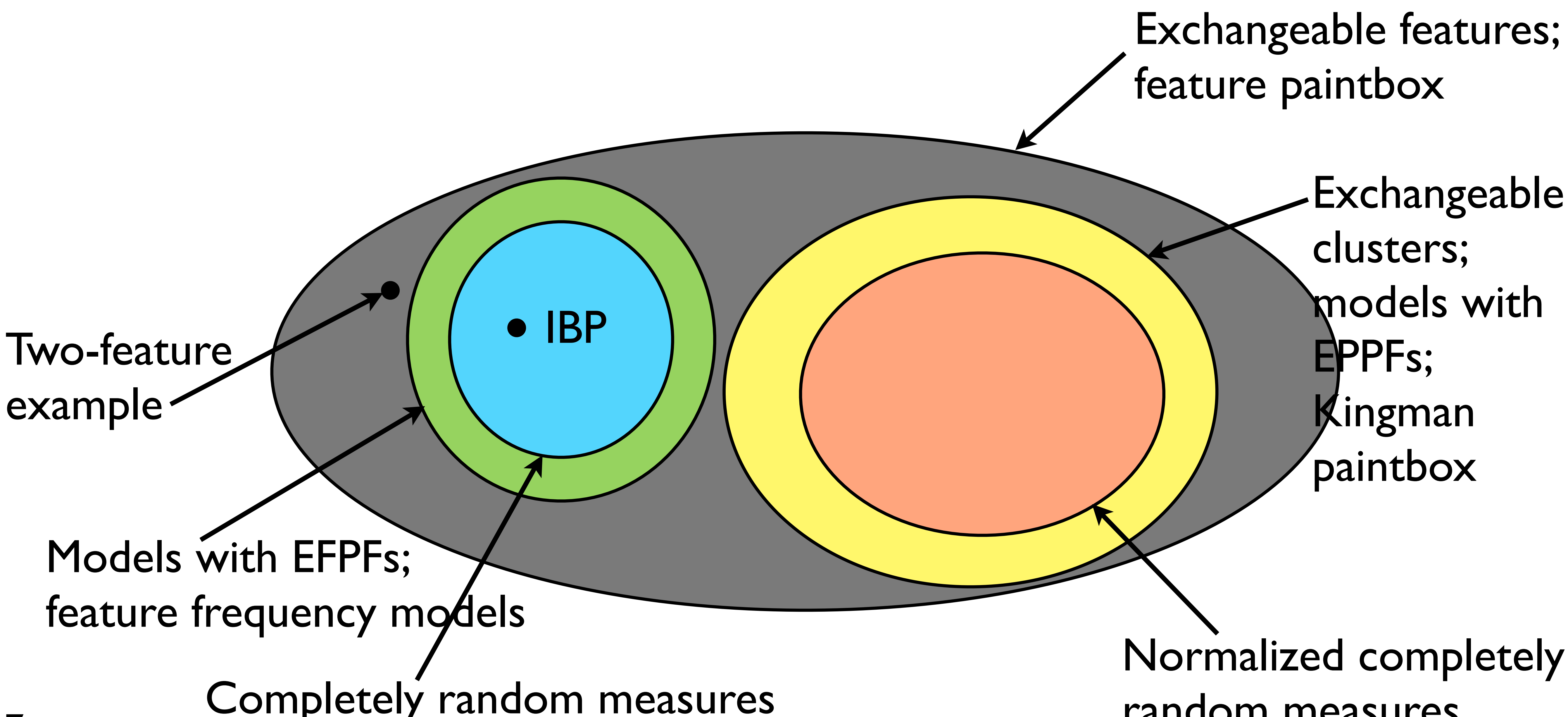
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



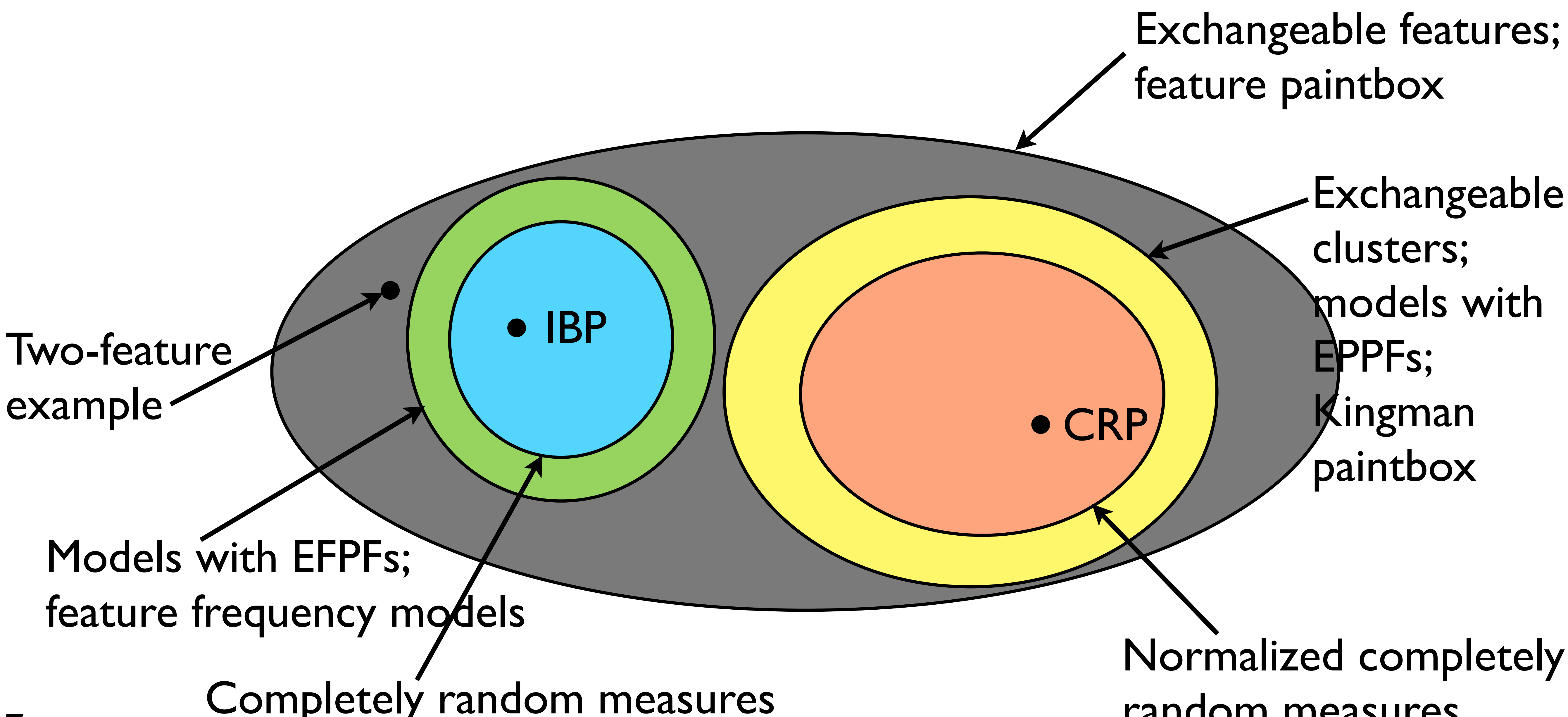
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



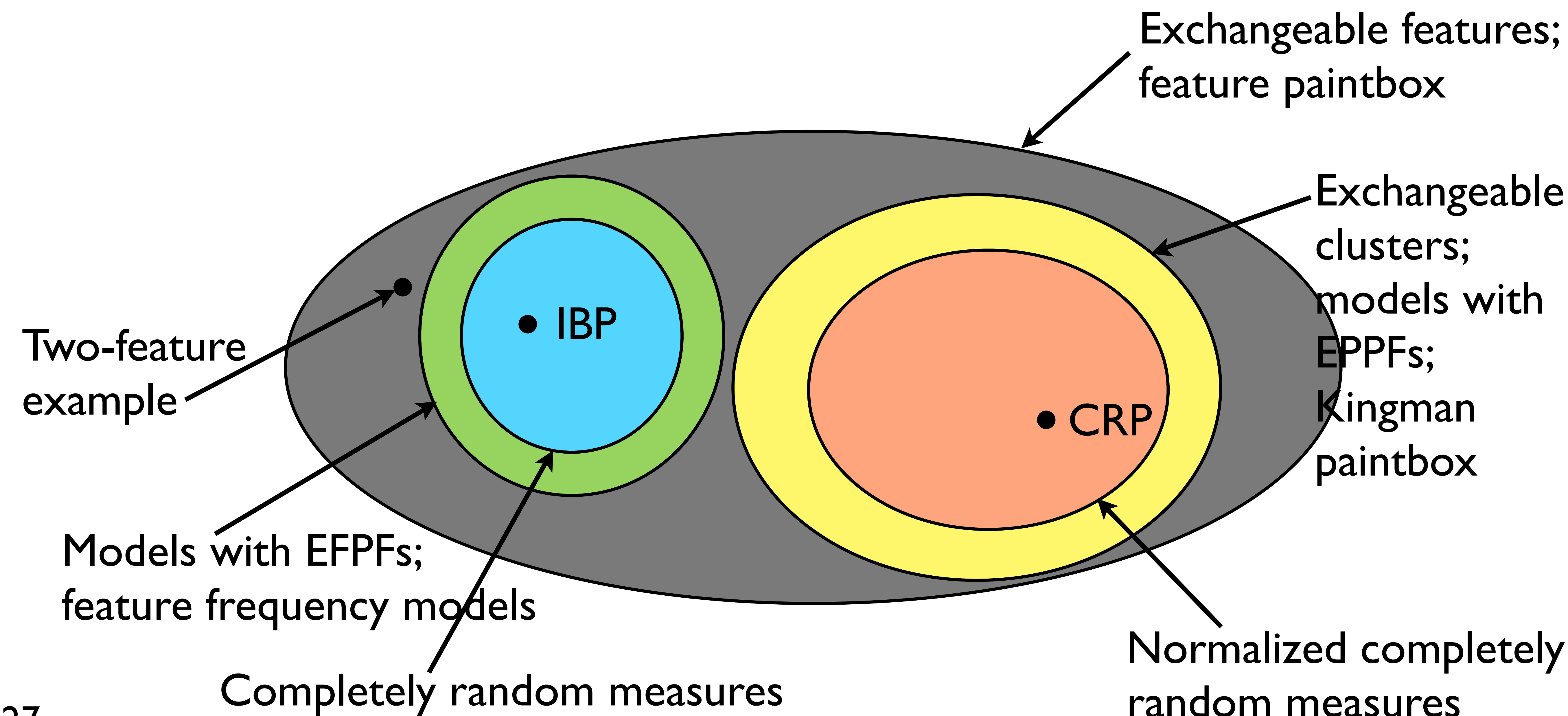
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



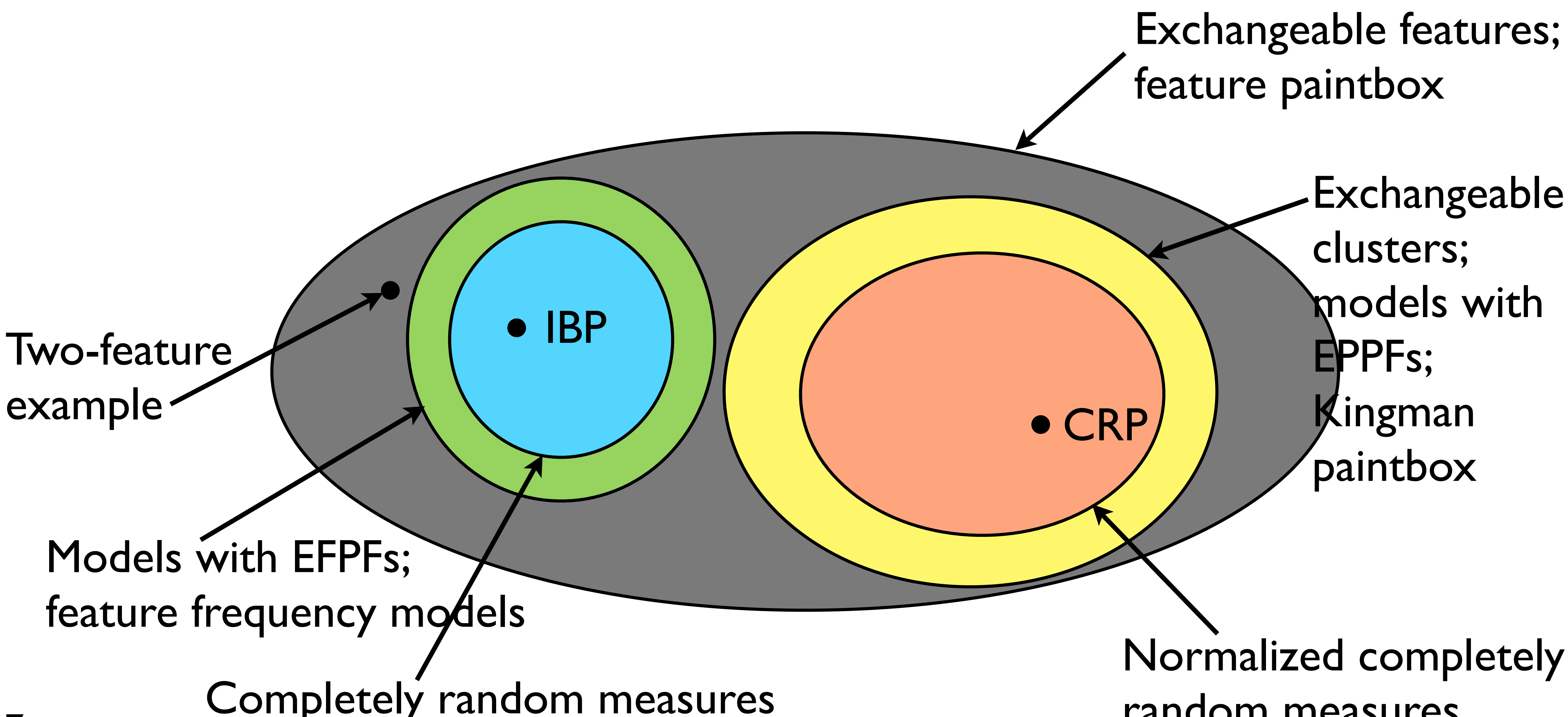
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust)



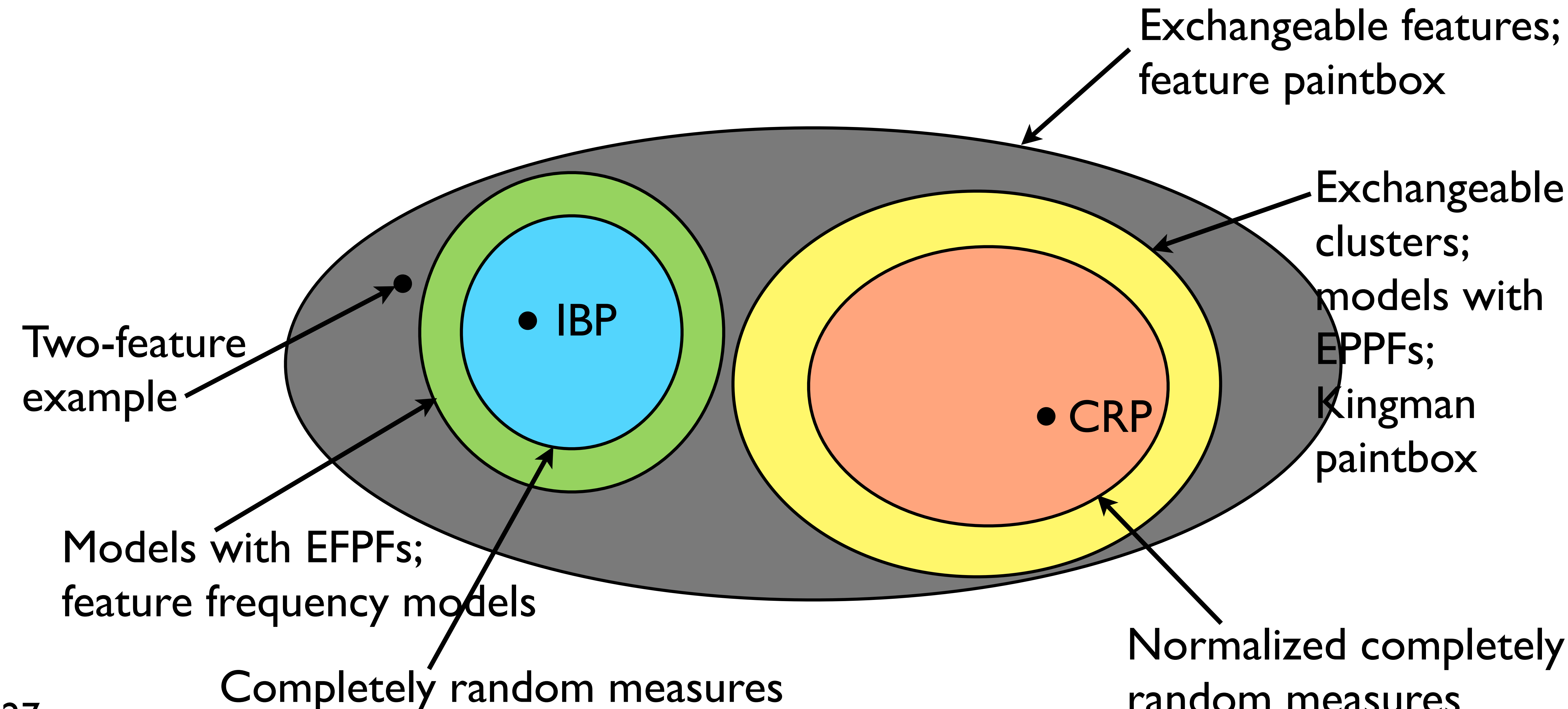
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)



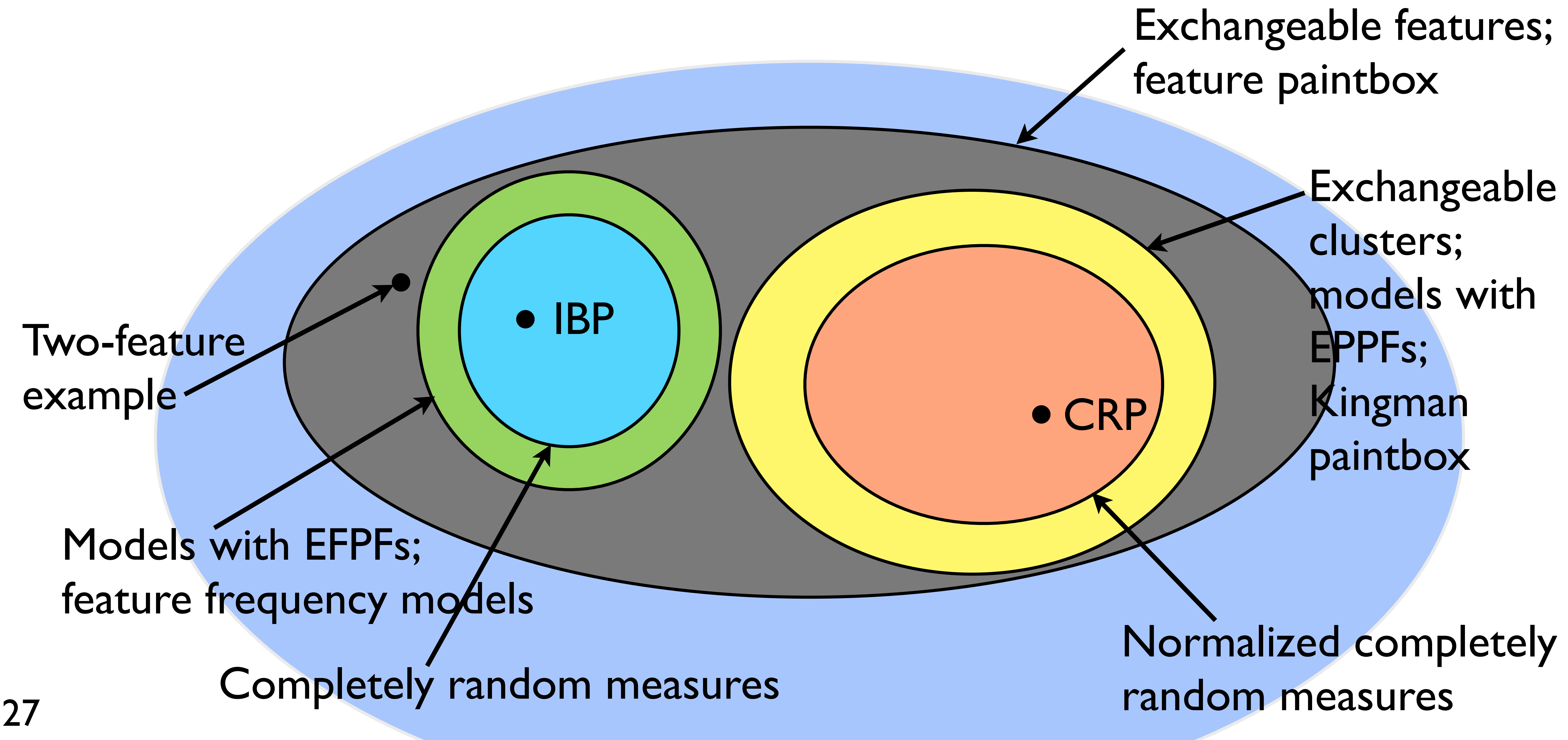
Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)
- Other combinatorial structures



Theory conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)
- Other combinatorial structures



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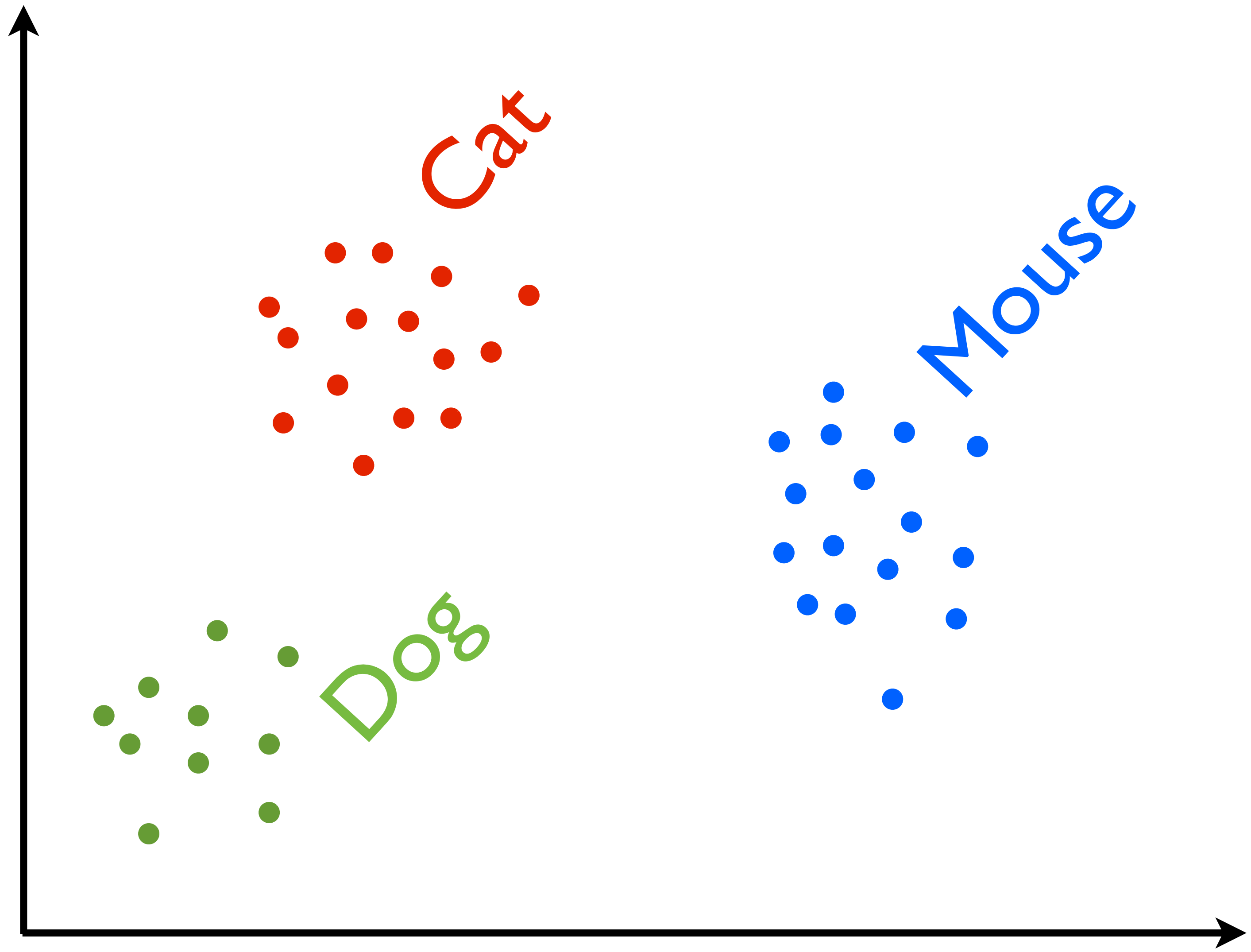
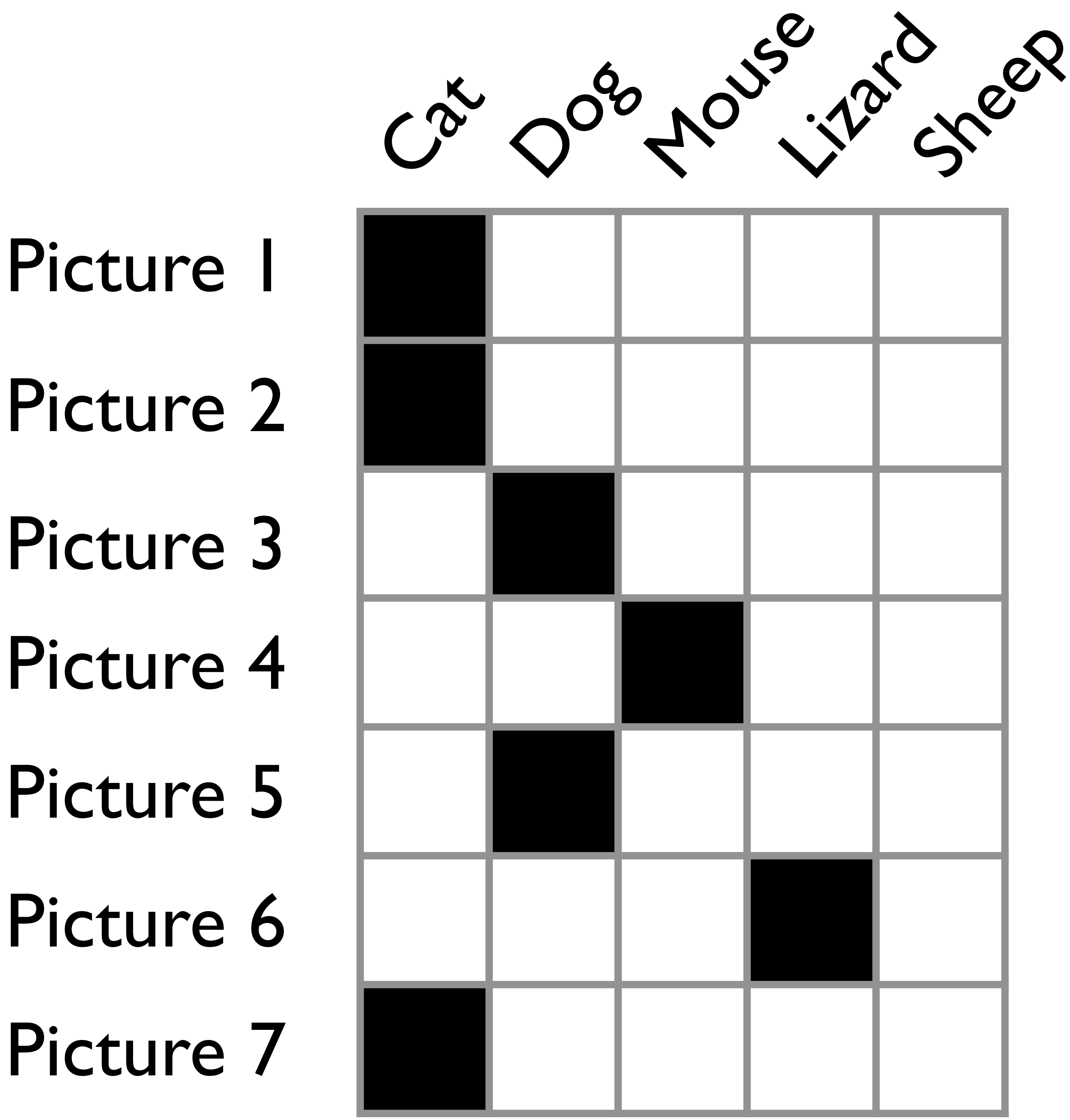
Clusters

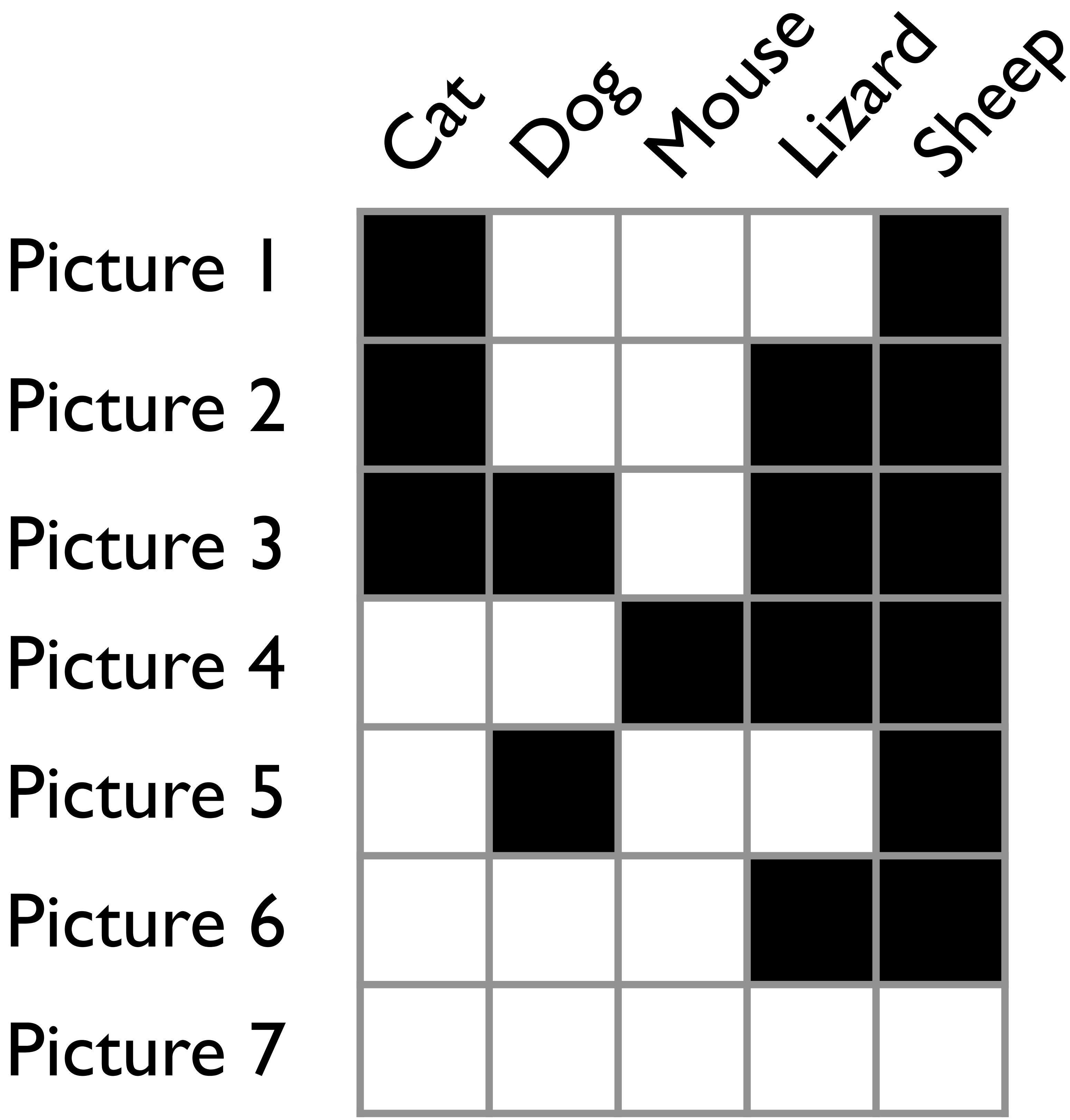


"clusters"

Clusters


Cat
Dog
Mouse
Lizard
Sheep
Picture 1
Picture 2
Picture 3
Picture 4
Picture 5
Picture 6
Picture 7

Features


Cat
Dog
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Picture 1
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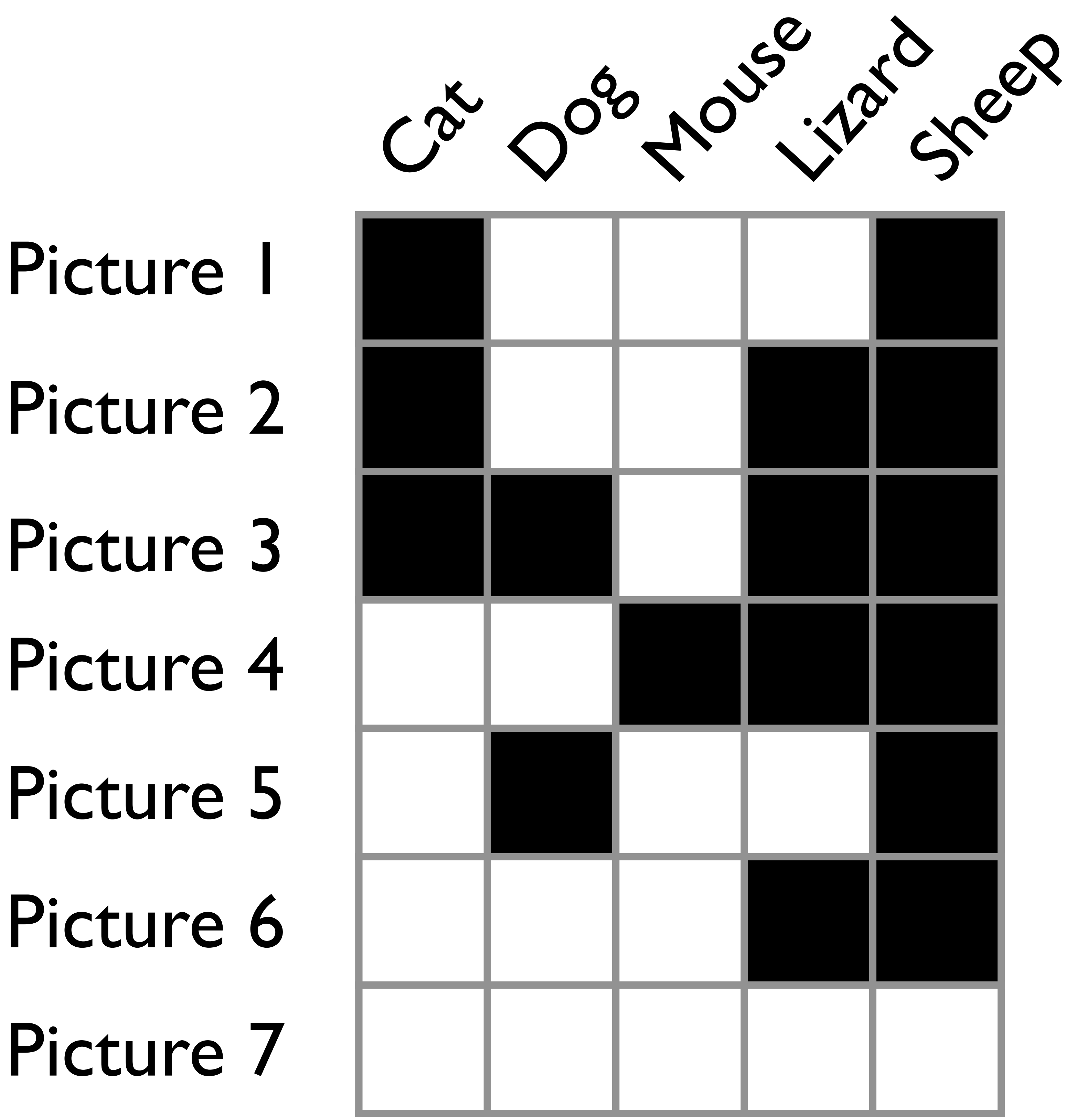
Picture 7

Features



Many other possible latent structures in data

How do we learn latent structure?

How do we learn latent structure?

K-means

How do we learn latent structure?

K-means

- Fast

How do we learn latent structure?

K-means

- Fast
- Can parallelize

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- Straightforward

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Nonparametric Bayes

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Nonparametric Bayes

- Modular (general latent structure)

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- Modular (general latent structure)
- Flexible (K can grow as data grows)

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Nonparametric Bayes

- Modular (general latent structure)
- Flexible (K can grow as data grows)
- Coherent treatment of uncertainty

How do we learn latent structure?

K-means
- Fast
- Can parallelize
- Straightforward
- Only works for K clusters

Nonparametric Bayes
- Modular (general latent structure)
- Flexible (K can grow as data grows)
- Coherent treatment of uncertainty

But...
- E.g., Silicon Valley: can have petabytes of data
- Practitioners turn to what runs

MAD-Bayes Perspectives

- Bayesian nonparametrics assists the optimization-based inference community

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  - New, modular, flexible, nonparametric objectives & regularizers

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Inspiration

- Consider a finite Gaussian mixture model

MAD-Bayes Perspectives

- Bayesian nonparametrics assists the optimization-based inference community
  - New, modular, flexible, nonparametric objectives & regularizers
  - Alternative perspective: fast initialization

Inspiration

- Consider a finite Gaussian mixture model
- The steps of the EM algorithm limit to the steps of the K-means algorithm as the Gaussian variance is taken to 0

MAD-Bayes

The MAD-Bayes idea

- Start with nonparametric Bayes model
- Take a similar limit to get a K-means-like objective

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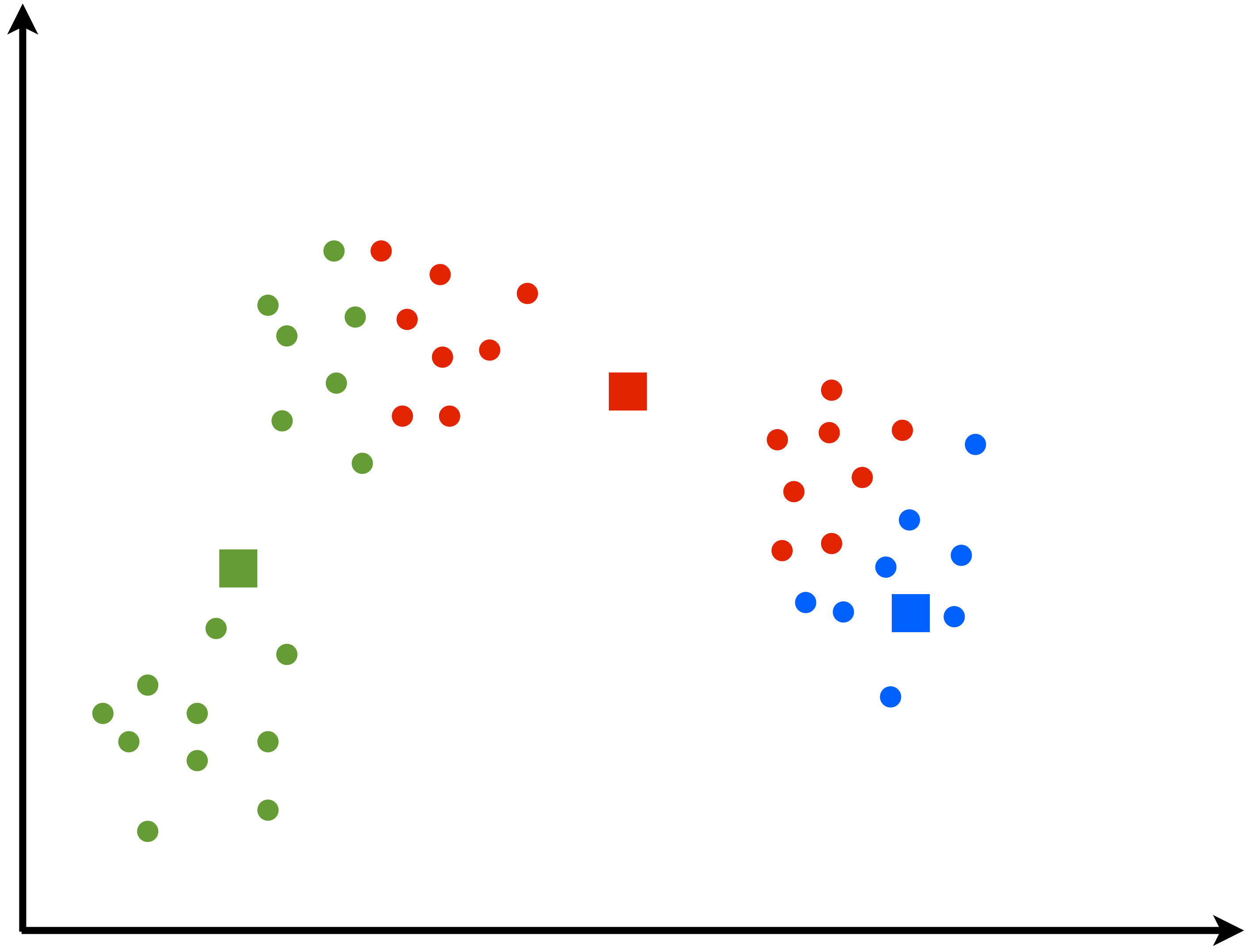
K-means

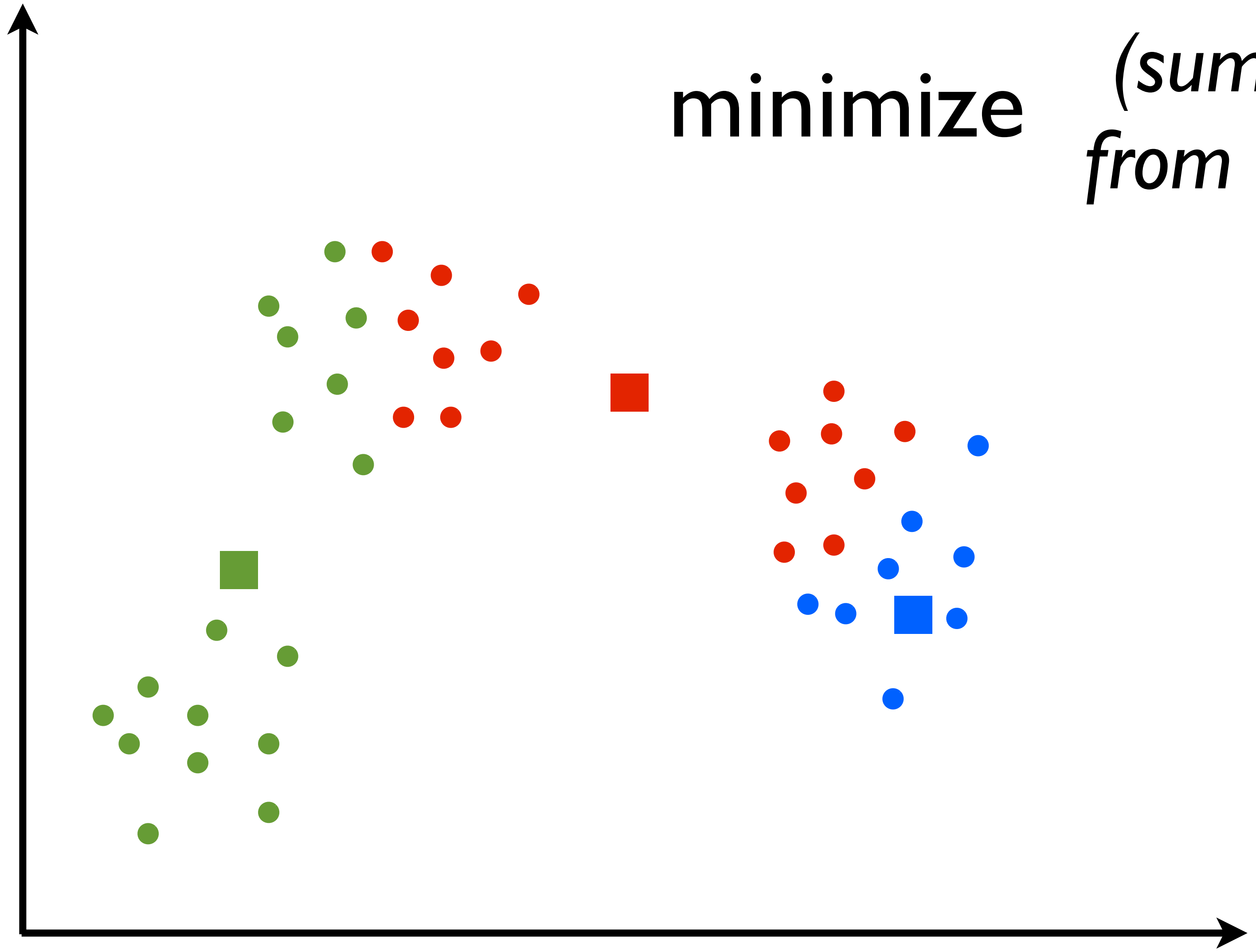
K-means clustering problem



K-means

K-means clustering problem

minimize *(sum of square distances from data points to cluster centers)*



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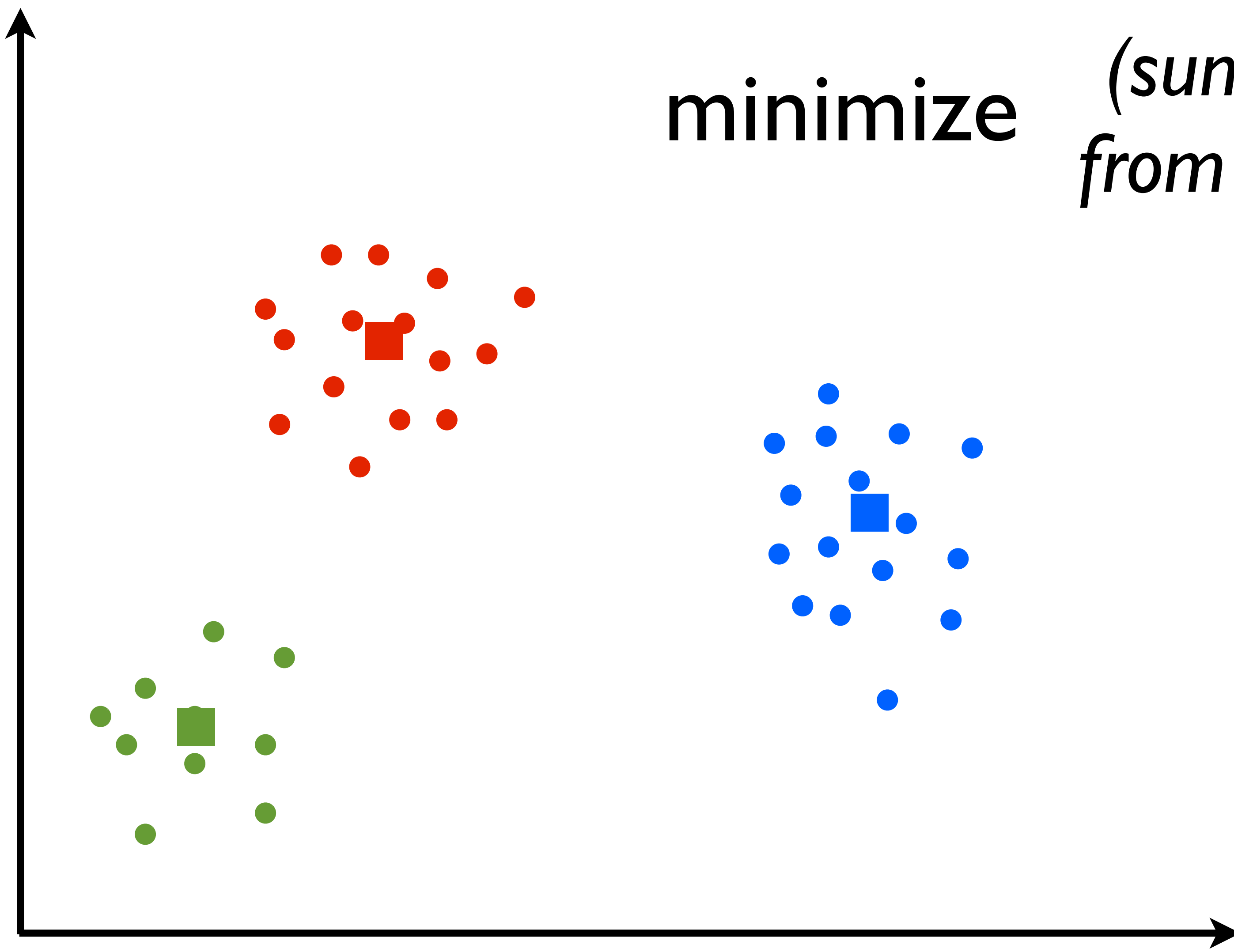

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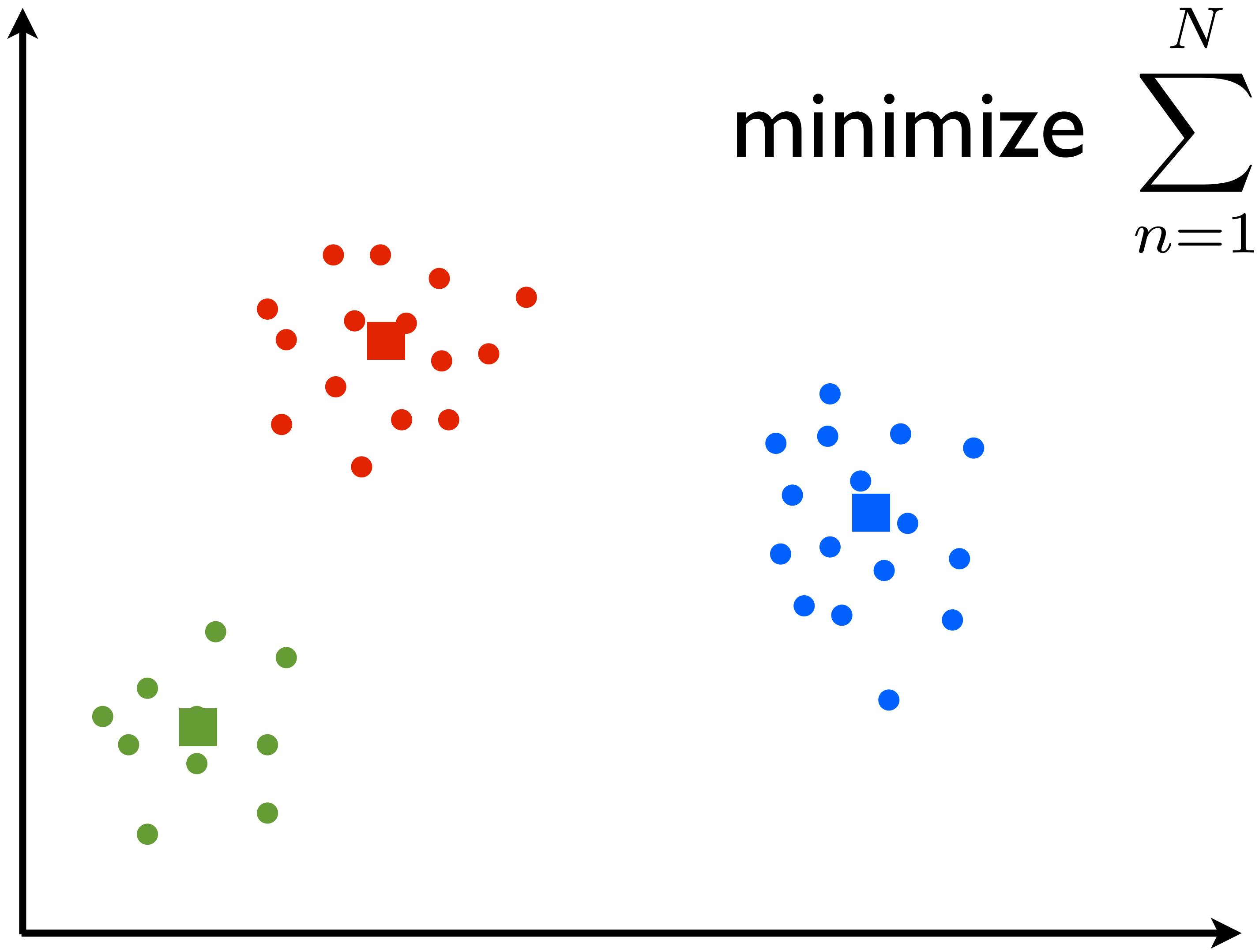
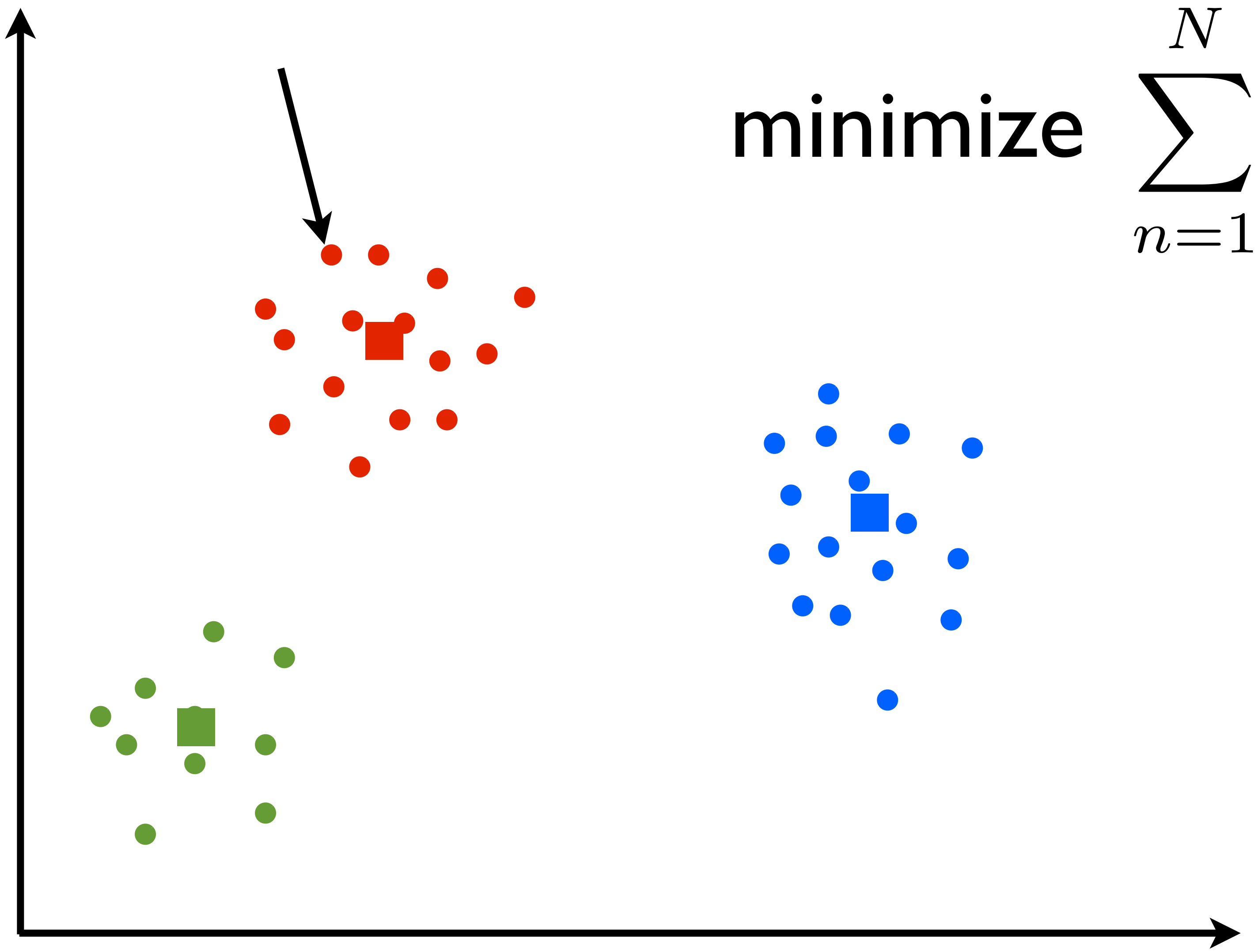
K-means

K-means clustering problem

minimize $\sum_{n=1}^{N} \|x_n - \text{center}_n\|^2$



K-means



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K-means

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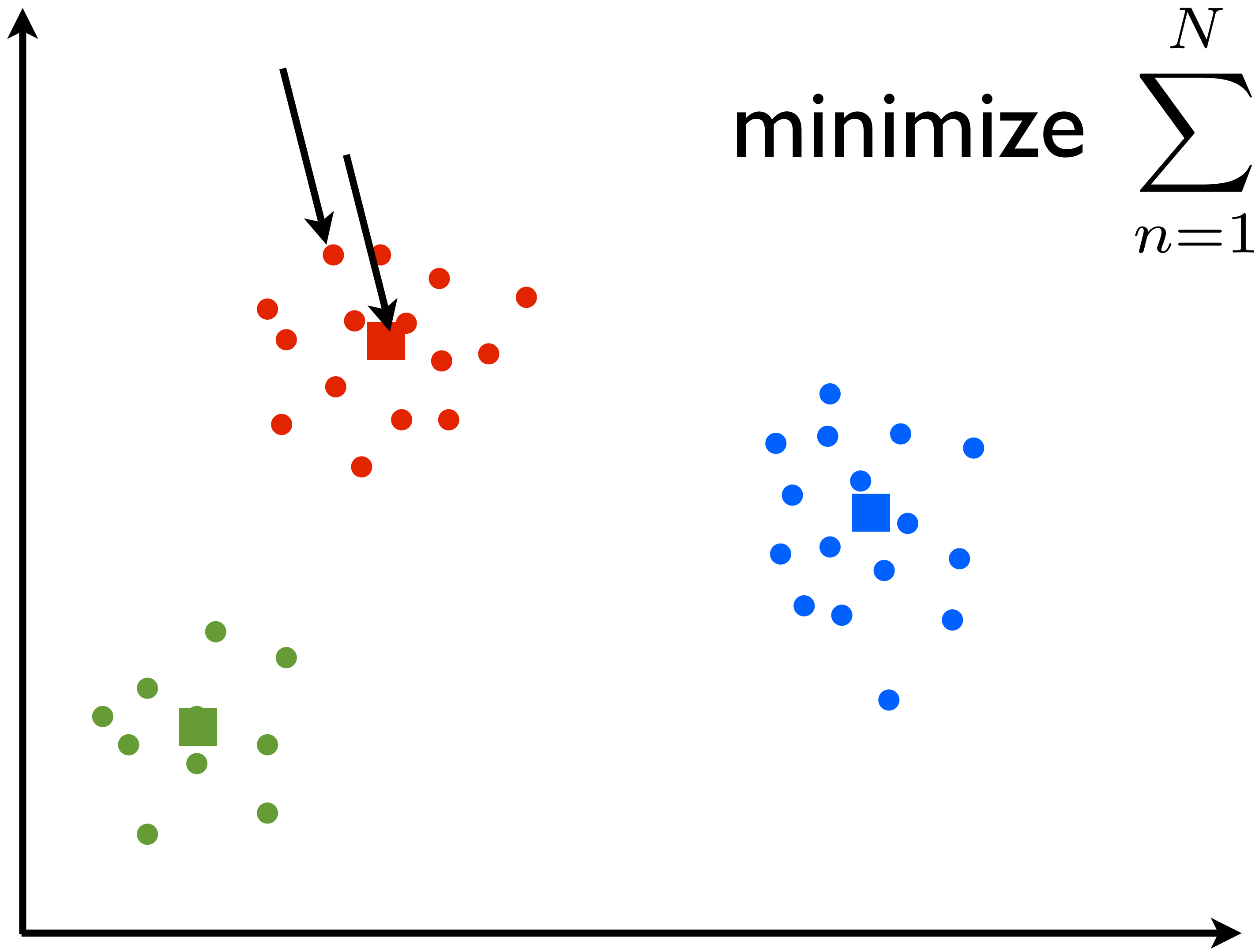

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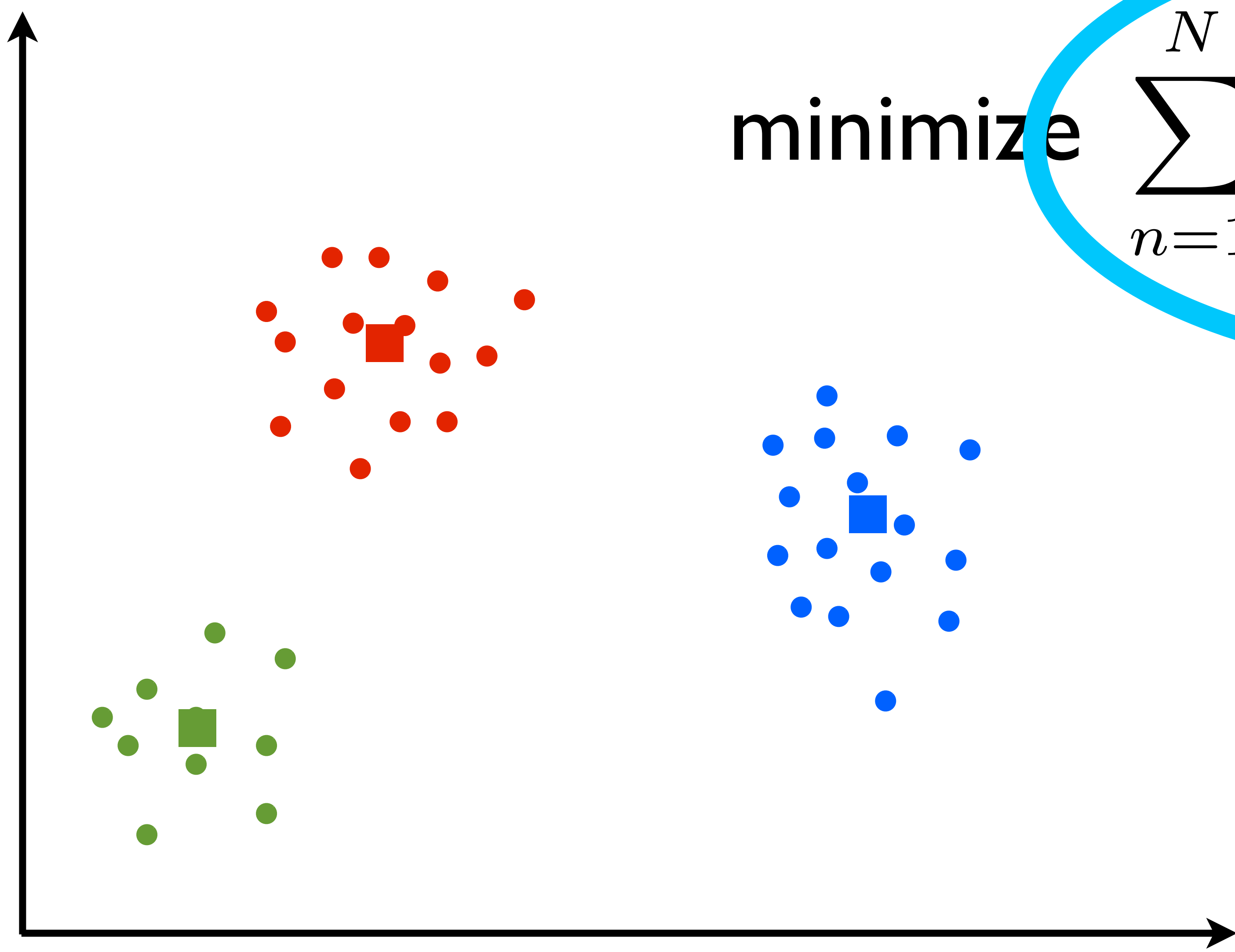
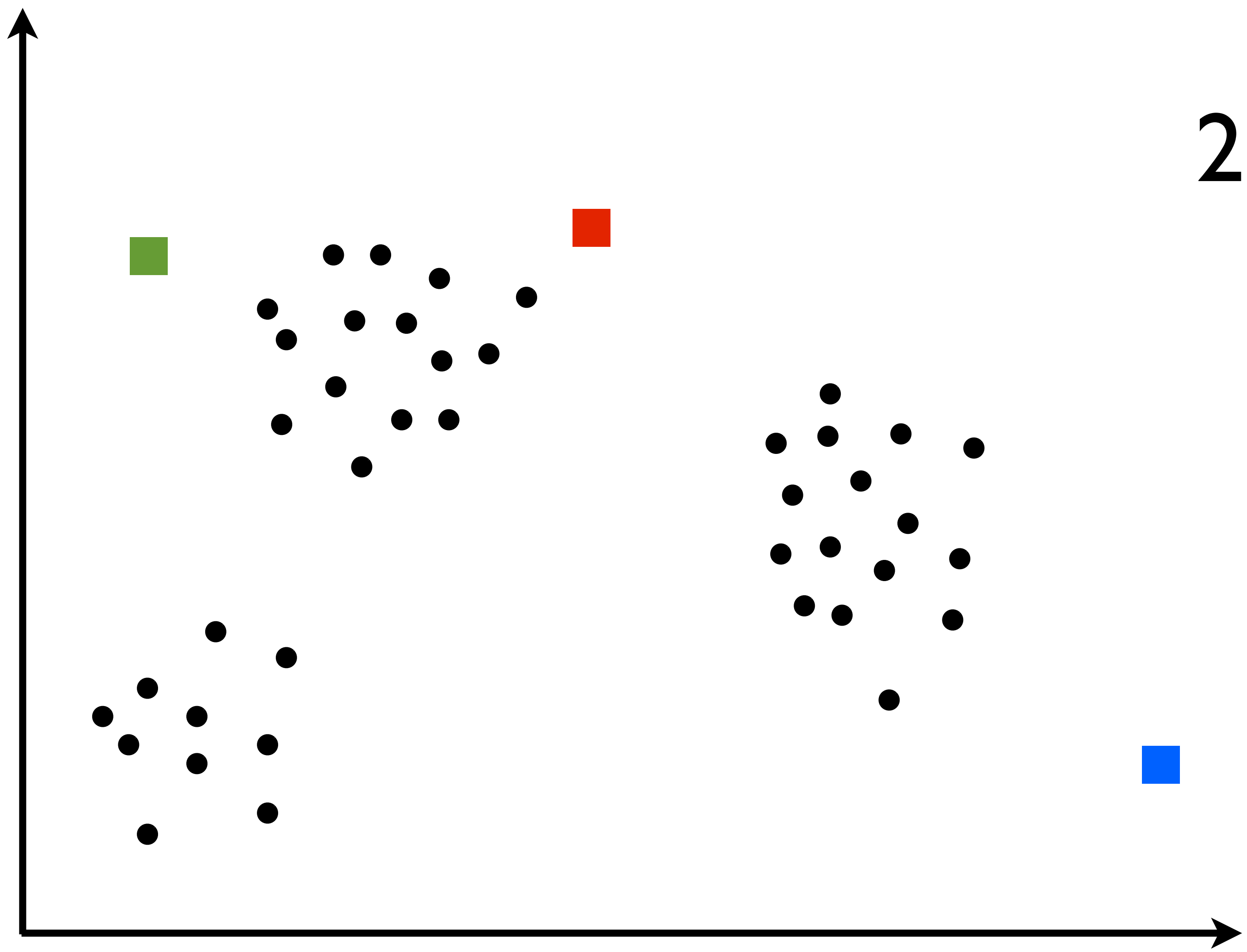
K-means

K-means objective

minimize $\sum_{n=1}^{N} \|x_n - \text{center}_n\|^2$



Lloyd's algorithm



Iterate until no changes:

1. For n = 1, ..., N
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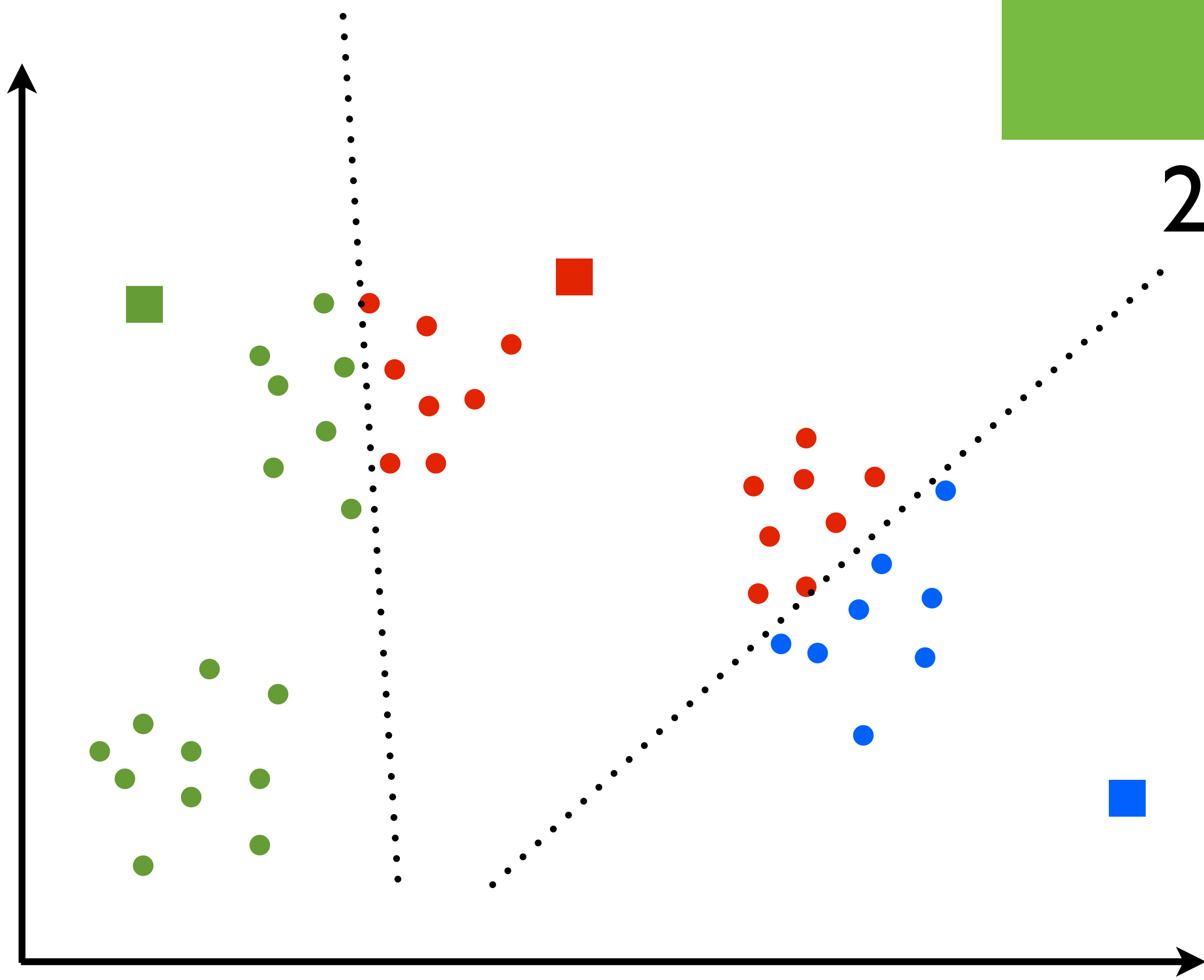
Lloyd's algorithm



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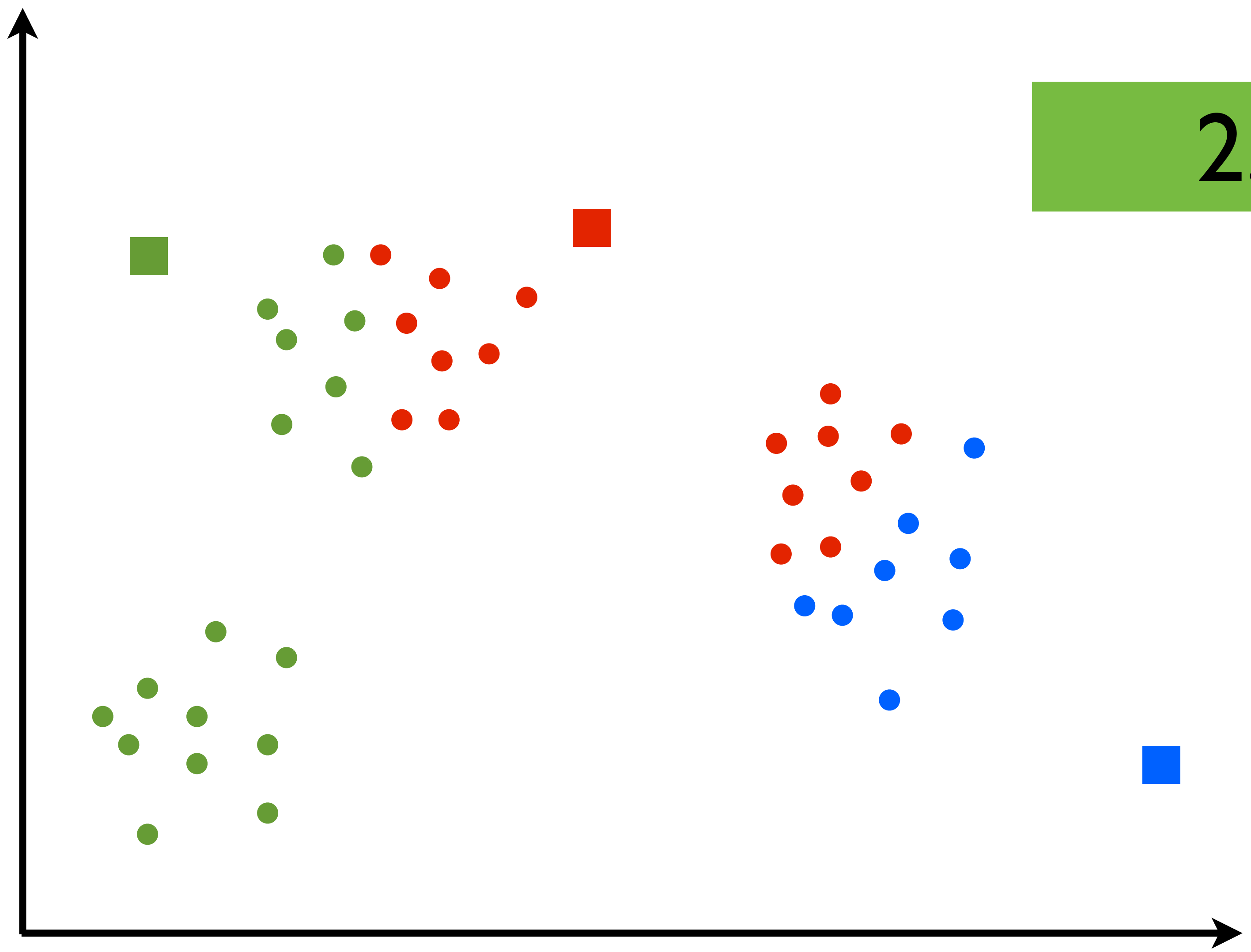

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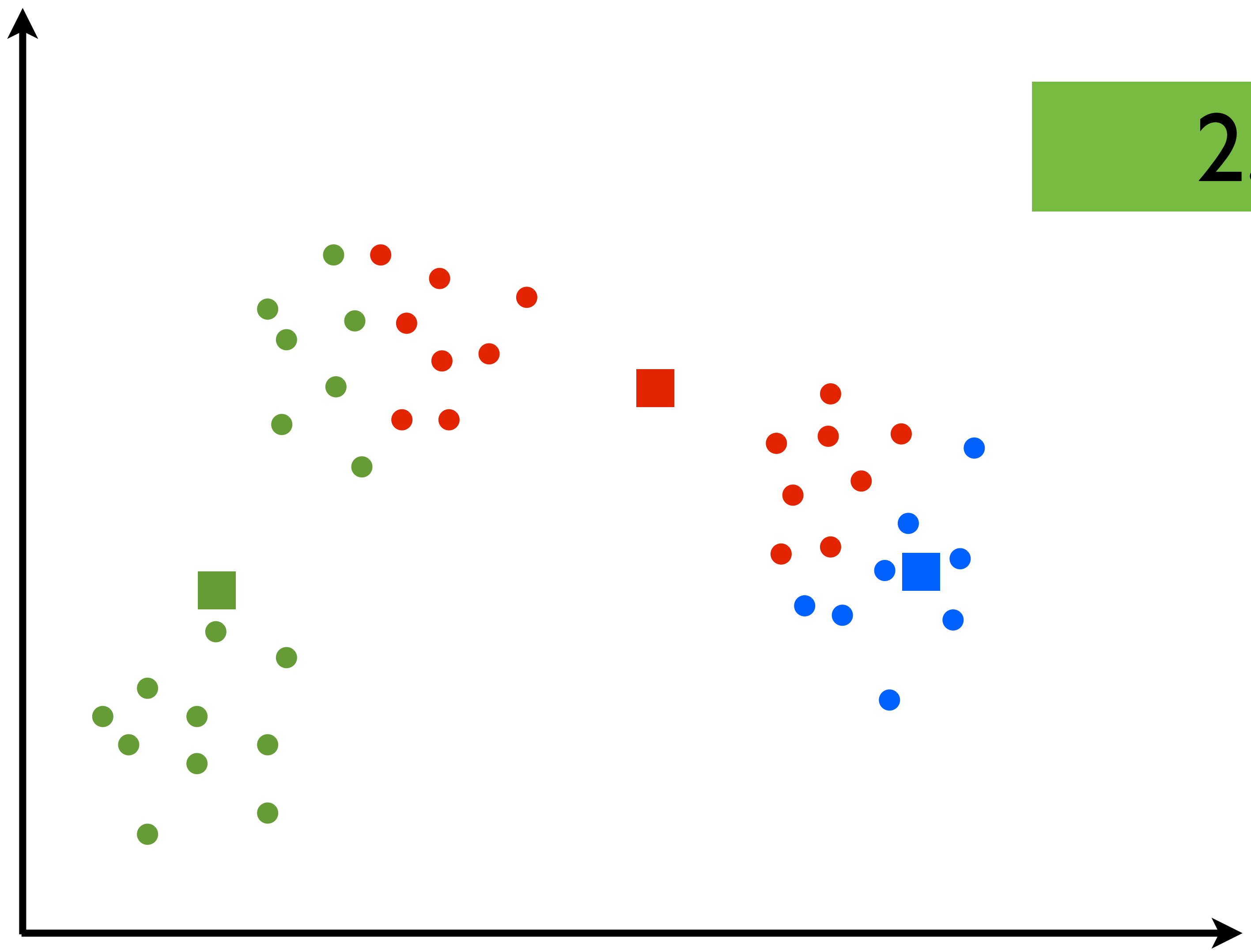
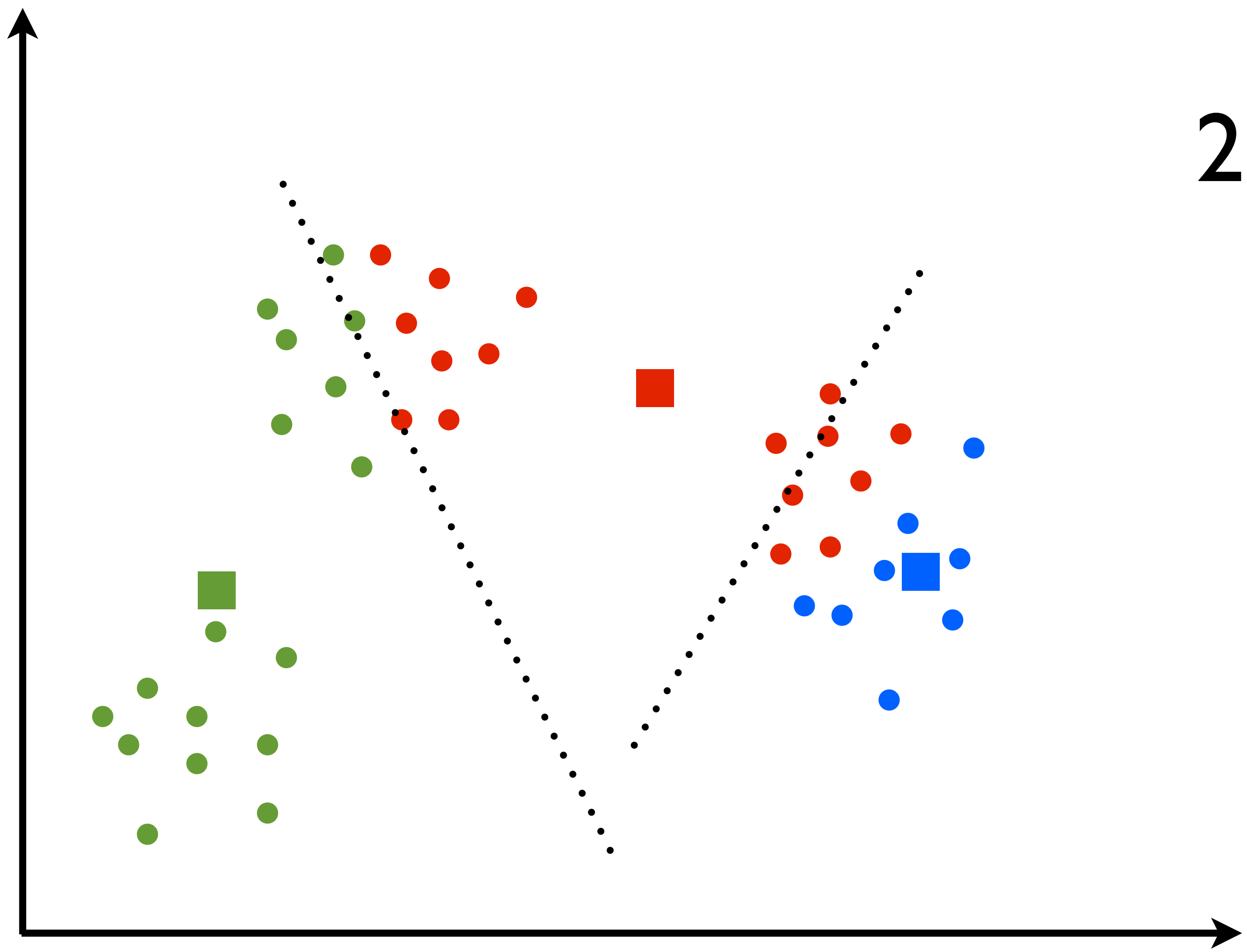


Lloyd's algorithm



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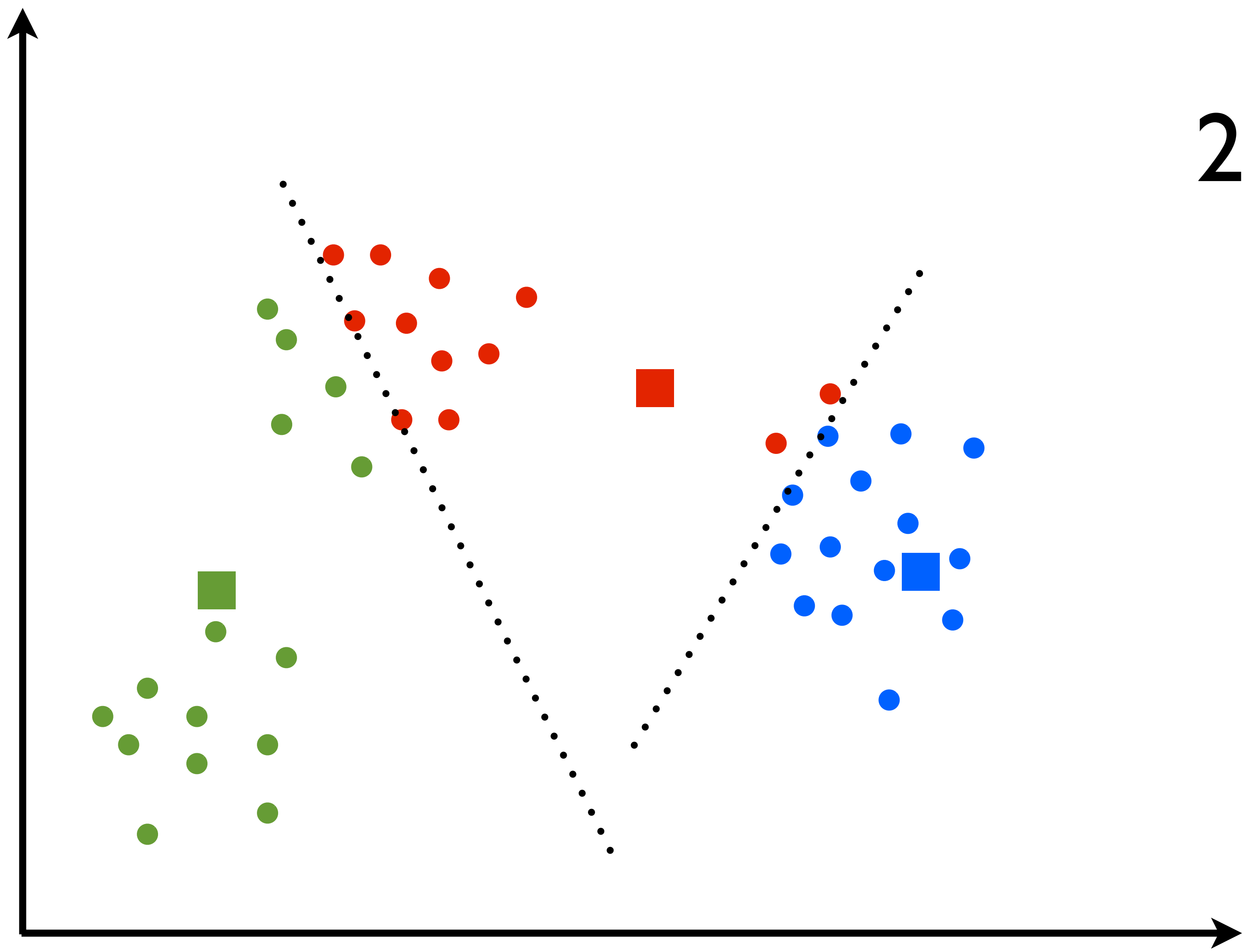

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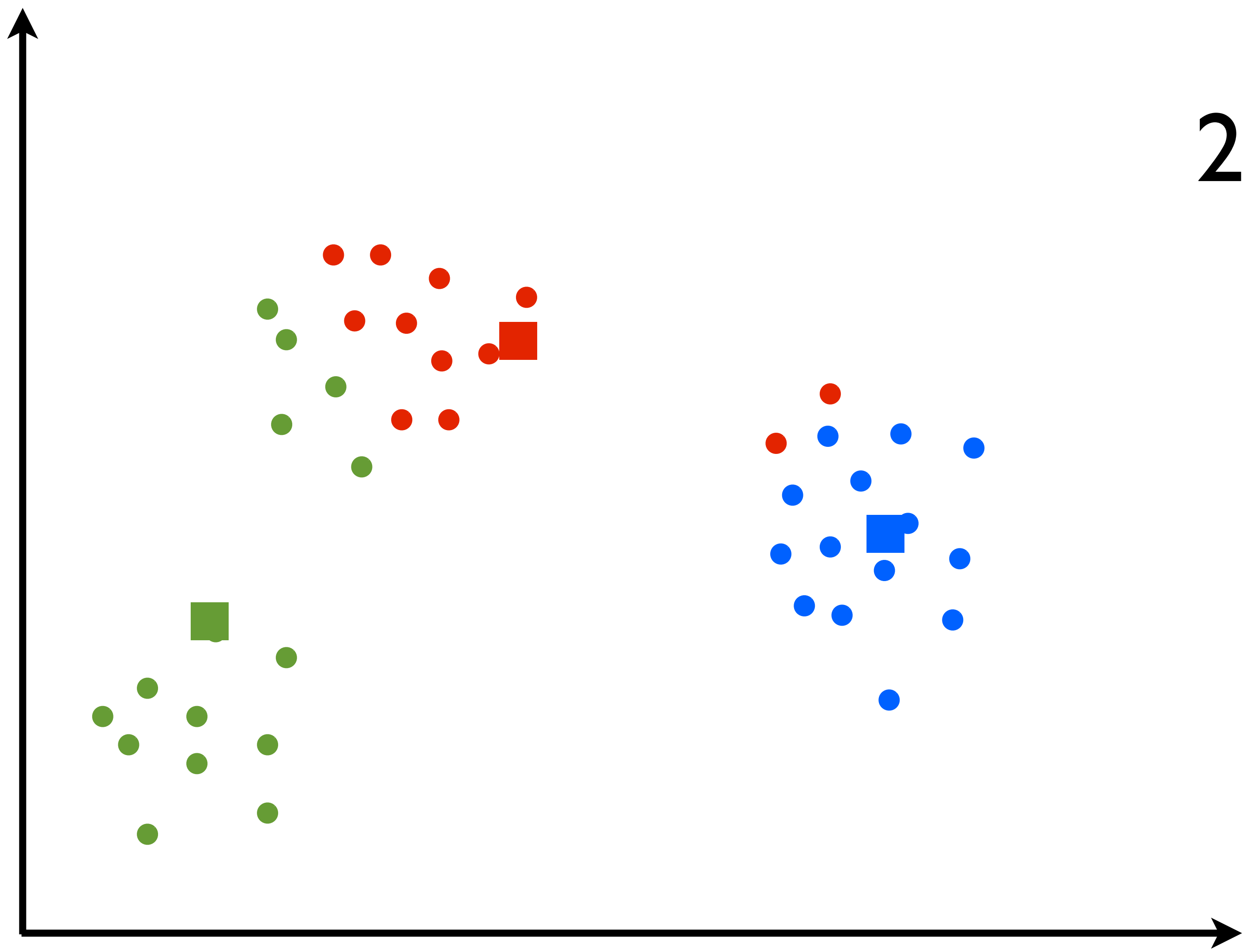

Lloyd's algorithm

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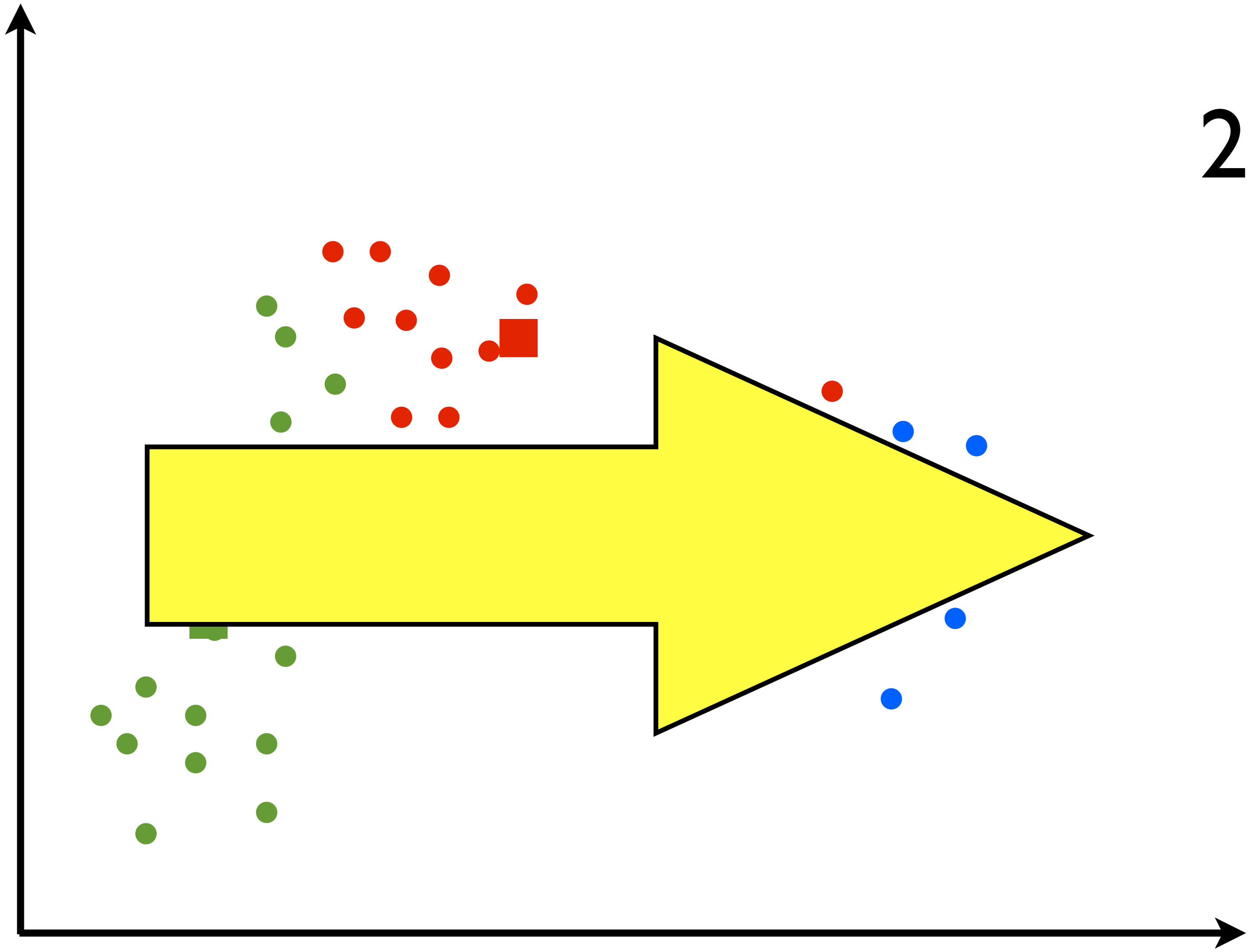
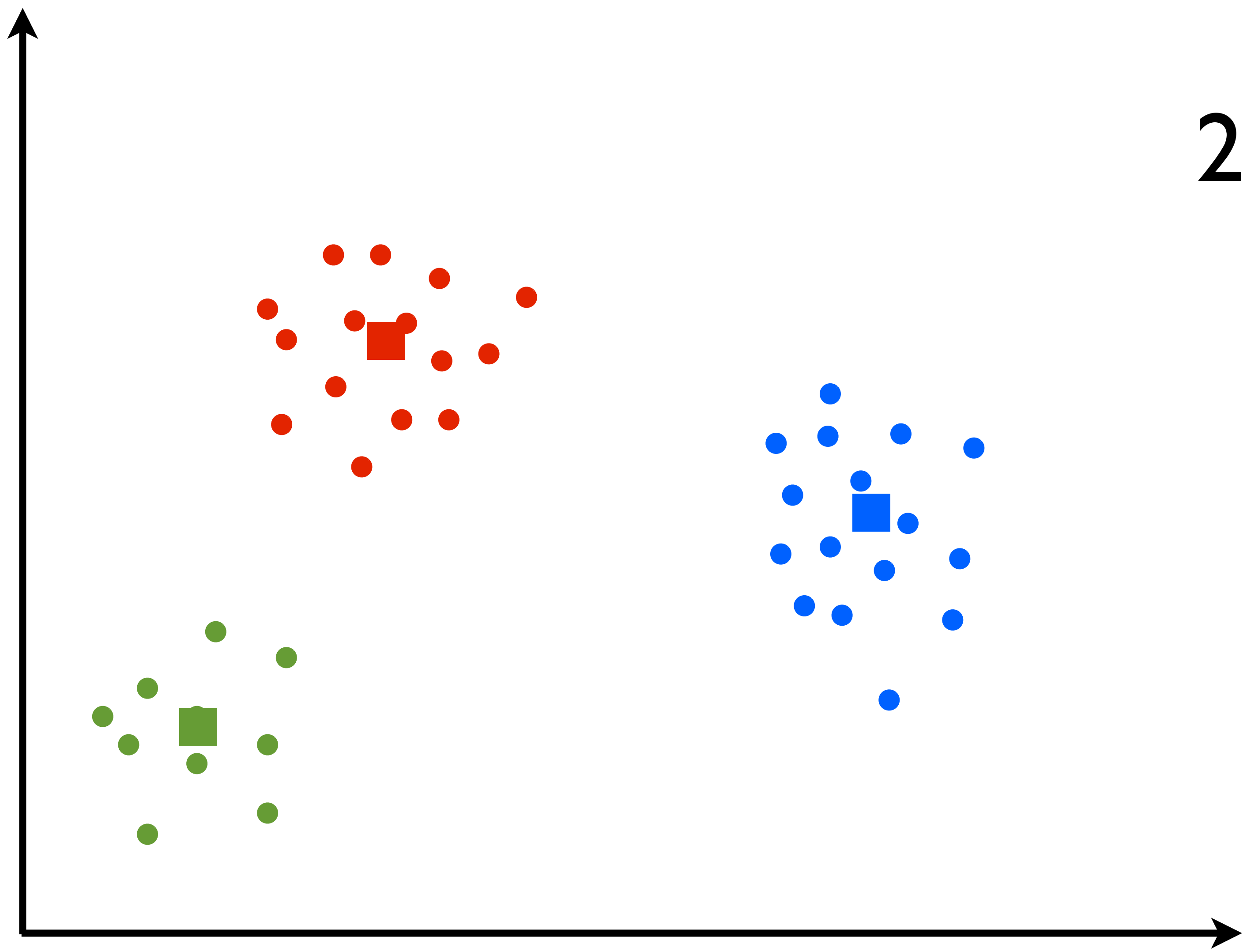


Lloyd's algorithm



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MAD-Bayes

The MAD-Bayes idea

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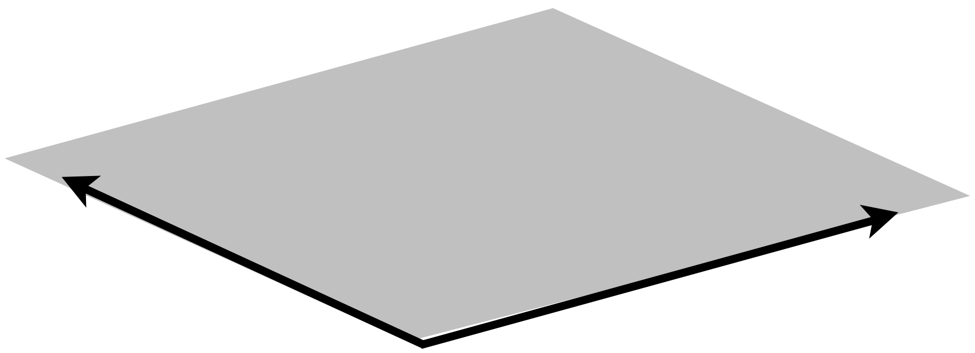
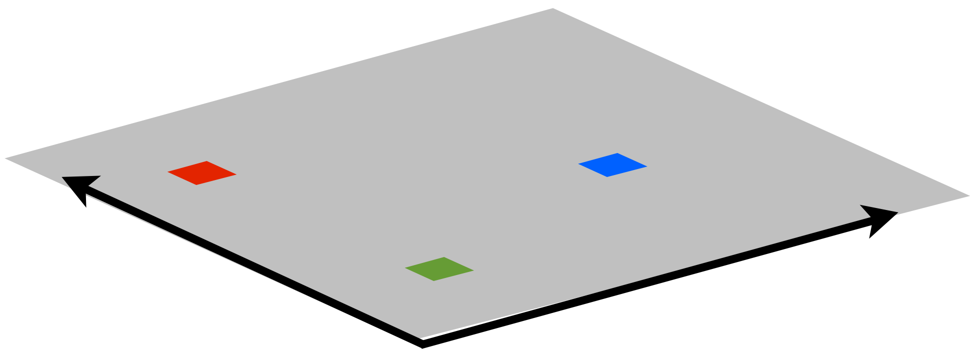
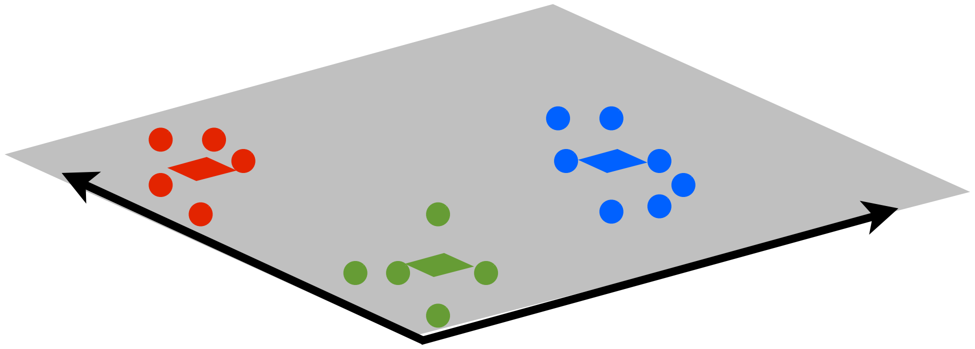
Bayesian model



Bayesian model



Bayesian model



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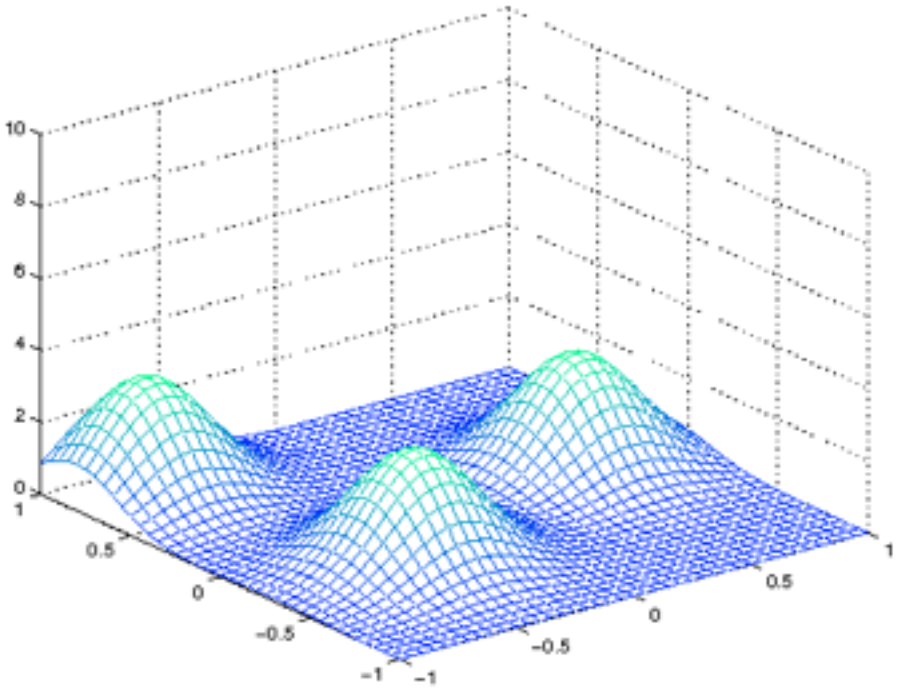
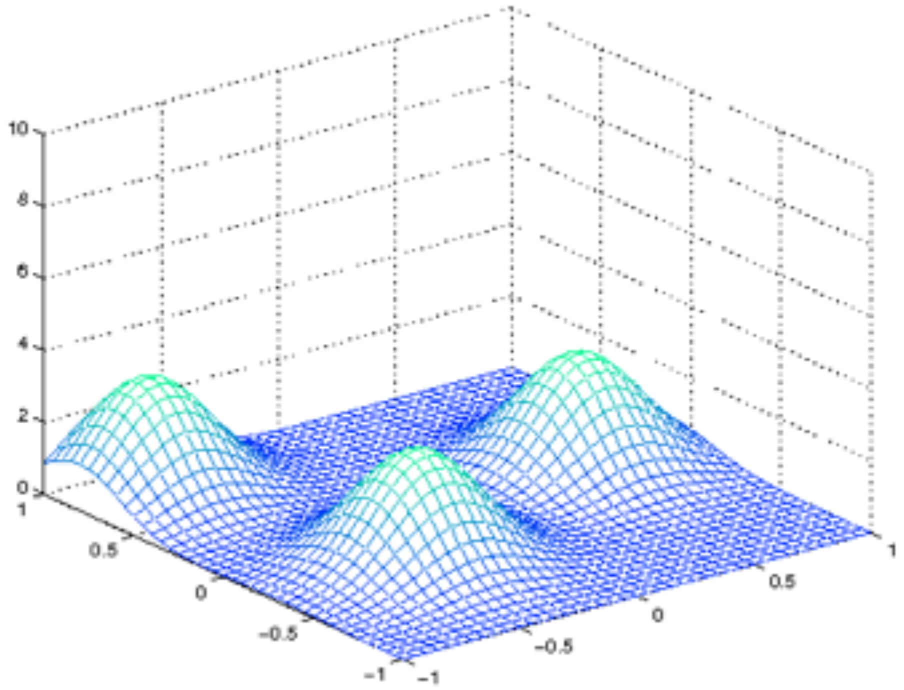


Bayesian model



Nonparametric

- number of parameters can grow with the number of data points

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- *Maximum a Posteriori* (MAP) is an optimization problem

$$\text{argmax}_{\text{parameters}} \mathbb{P}(\text{parameters}|\text{data})$$

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- We take a limit of the objective (posterior) and get one like K-means

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- *Maximum a Posteriori* (MAP) is an optimization problem

$$\mathrm{argmax}_{\mathrm{parameters}} \mathbb{P}(\mathrm{parameters}|\mathrm{data})$$

- We take a limit of the objective (posterior) and get one like K-means
  - ◇ "Small-variance asymptotics"

MAD-Bayes

Bayesian posterior

K-means-like objectives

MAD-Bayes

Bayesian posterior		K-means-like objectives
Mixture of K Gaussians	$\Rightarrow$	K-means

MAD-Bayes

Bayesian posterior		K-means-like objectives
Mixture of K Gaussians	⇒	K-means
Dirichlet process mixture	⇒	Unbounded number of clusters

MAD-Bayes

<u>Bayesian posterior</u>	<u>K-means-like objectives</u>
Mixture of K Gaussians $\Rightarrow$	K-means
Dirichlet process mixture $\Rightarrow$	Unbounded number of clusters
Hierarchical Dirichlet process $\Rightarrow$	Multiple data sets share cluster centers

MAD-Bayes

Bayesian posterior		K-means-like objectives
Mixture of K Gaussians	→	K-means
Dirichlet process mixture	→	Unbounded number of clusters
Hierarchical Dirichlet process	→	Multiple data sets share cluster centers
⋮		⋮

MAD-Bayes

Bayesian posterior		K-means-like objectives
Mixture of K Gaussians	→	K-means
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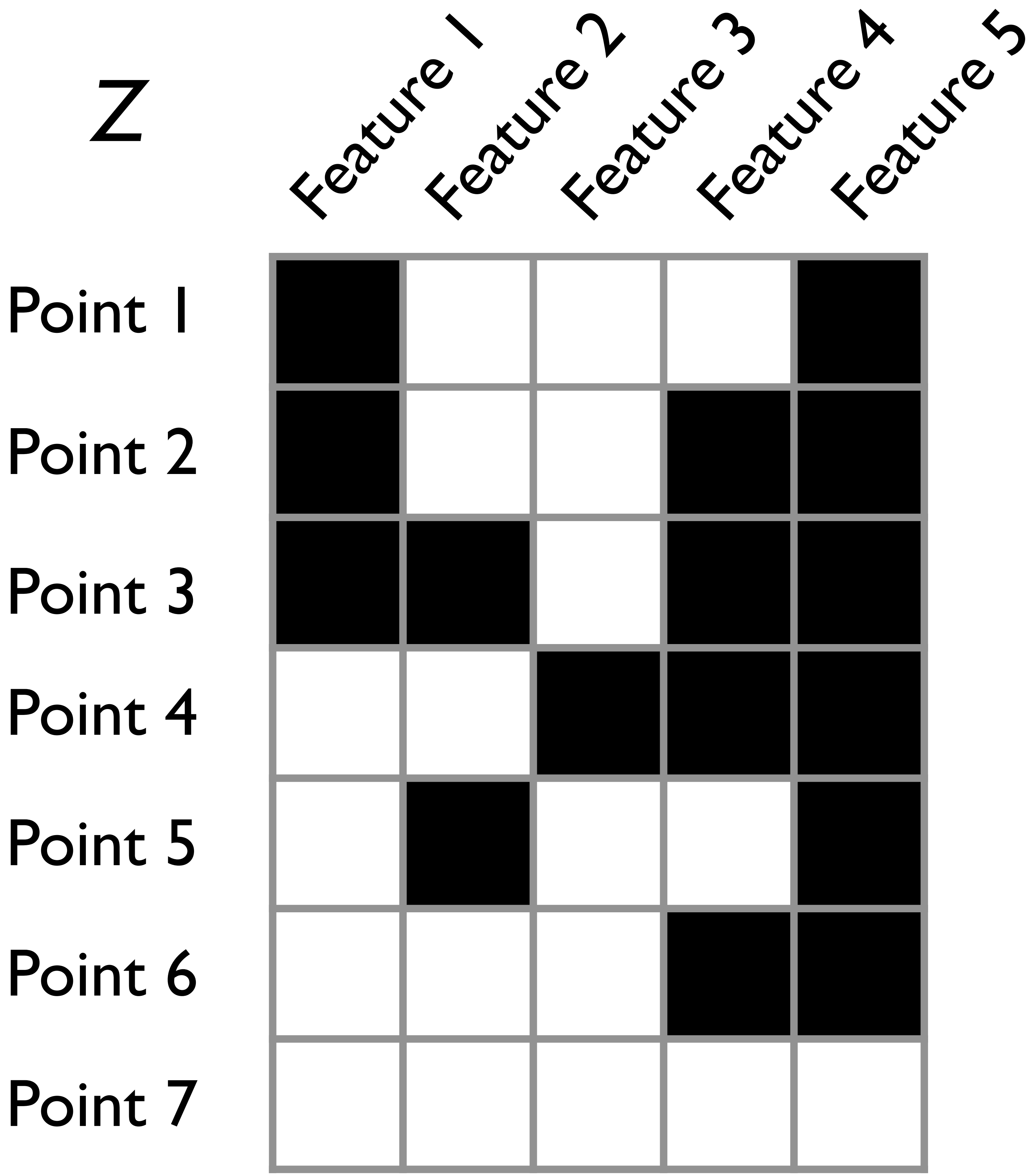
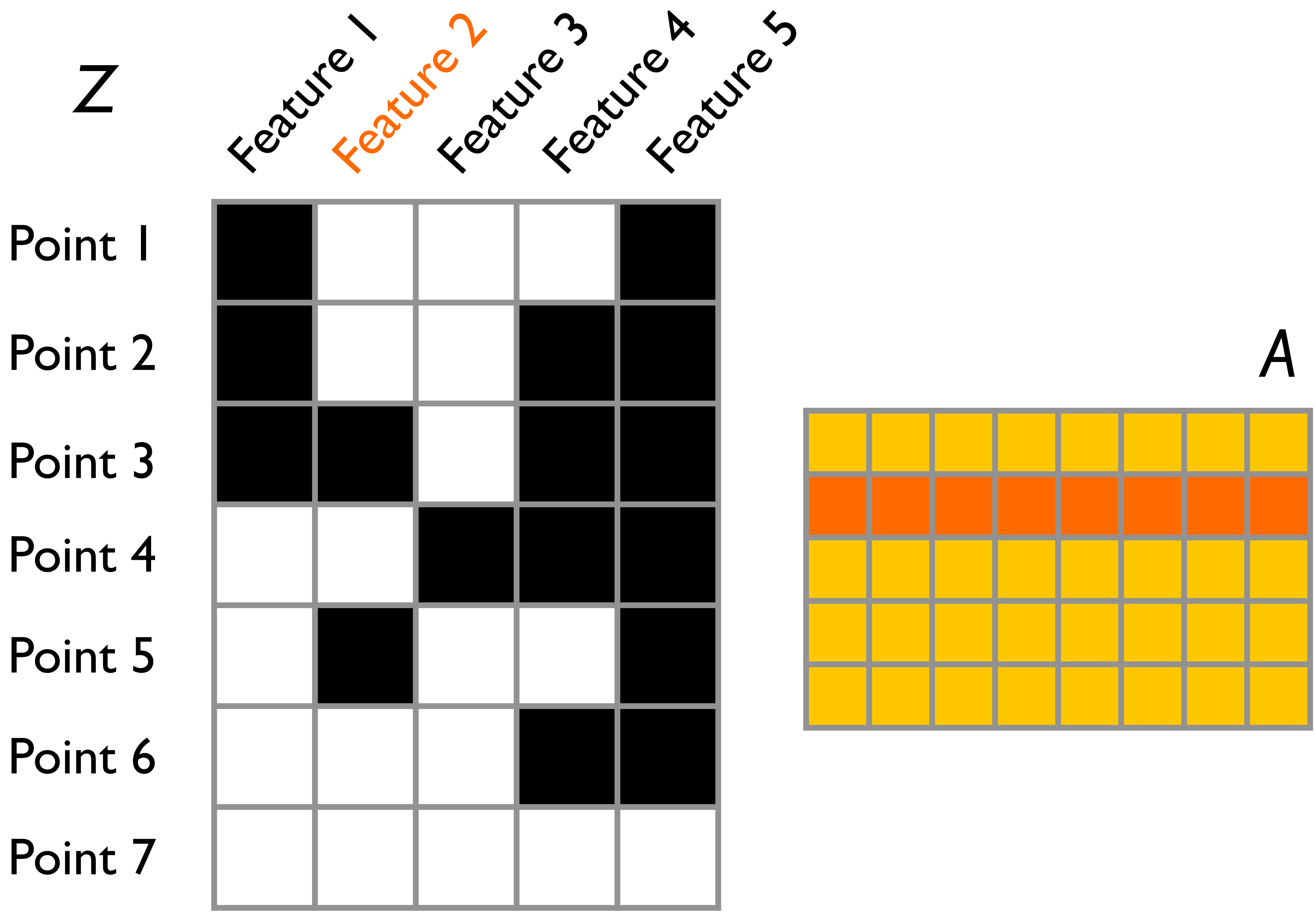
⋮		⋮
Beta process	→	Features

Features


Z
Feature 1
Feature 2
Feature 3
Feature 4
Feature 5
Point 1
Point 2
Point 3
Point 4
Point 5
Point 6
Point 7

Features



Z
Feature 1
Feature 2
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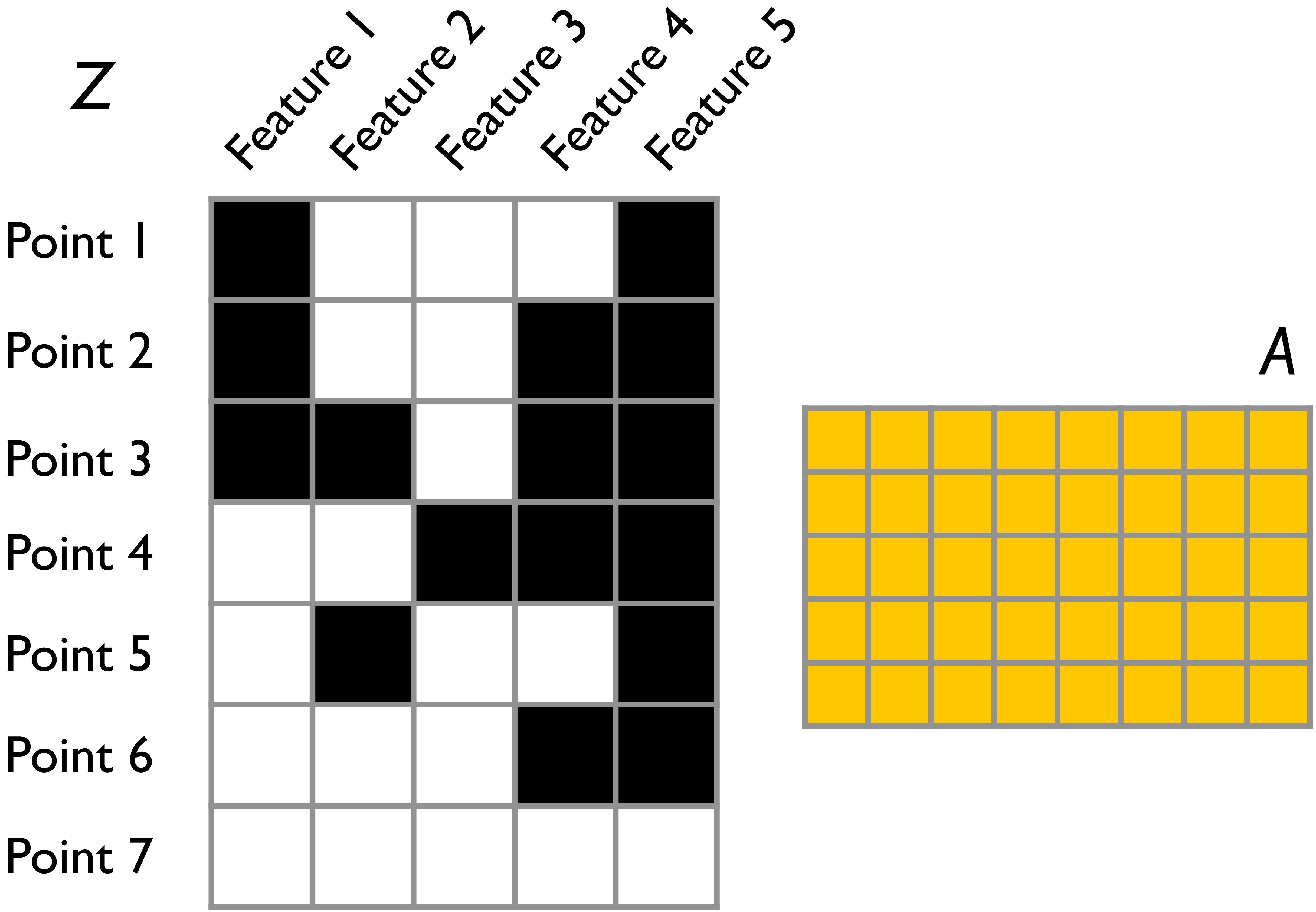
Feature 4
Feature 5
Point 1
Point 2
Point 3
Point 4
Point 5
Point 6
Point 7
A

Features


Z
Feature 1
Feature 2
Feature 3
Feature 4
Feature 5
Point 1
Point 2
Point 3
Point 4
Point 5
Point 6
Point 7
A

MAD-Bayes

Bayesian posterior

$$
\begin{aligned}
&\mathbb{P}(Z, A|X) \\
&\qquad \propto \frac{1}{(2\pi\sigma^2)^{ND/2}} \exp\left\{-\frac{1}{2\sigma^2}\mathbf{tr}((X - ZA)'(X - ZA))\right\} \\
&\qquad \cdot \frac{\gamma^{K^+} \exp\left\{-\sum_{n=1}^{N} \frac{\gamma}{n}\right\}}{\prod_{h=1}^{H} \tilde{K}_h!} \prod_{k=1}^{K^+} \frac{(S_{N,k} - 1)!(N - S_{N,k})!}{N!} \\
&\qquad \cdot \frac{1}{(2\pi\rho^2)^{K^+D/2}} \exp\left\{-\frac{1}{2\rho^2}\mathbf{tr}(A'A)\right\}.
\end{aligned}
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MAD-Bayes

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MAD-Bayes

BP-means objective

$$\mathrm{argmin}_{K^+,Z,A} \mathbf{tr}[(X - ZA)'(X - ZA)] + K^+\lambda^2.$$

MAD-Bayes

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BP-means algorithm

Iterate until no changes:

1. For n = 1, ..., N
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MAD-Bayes

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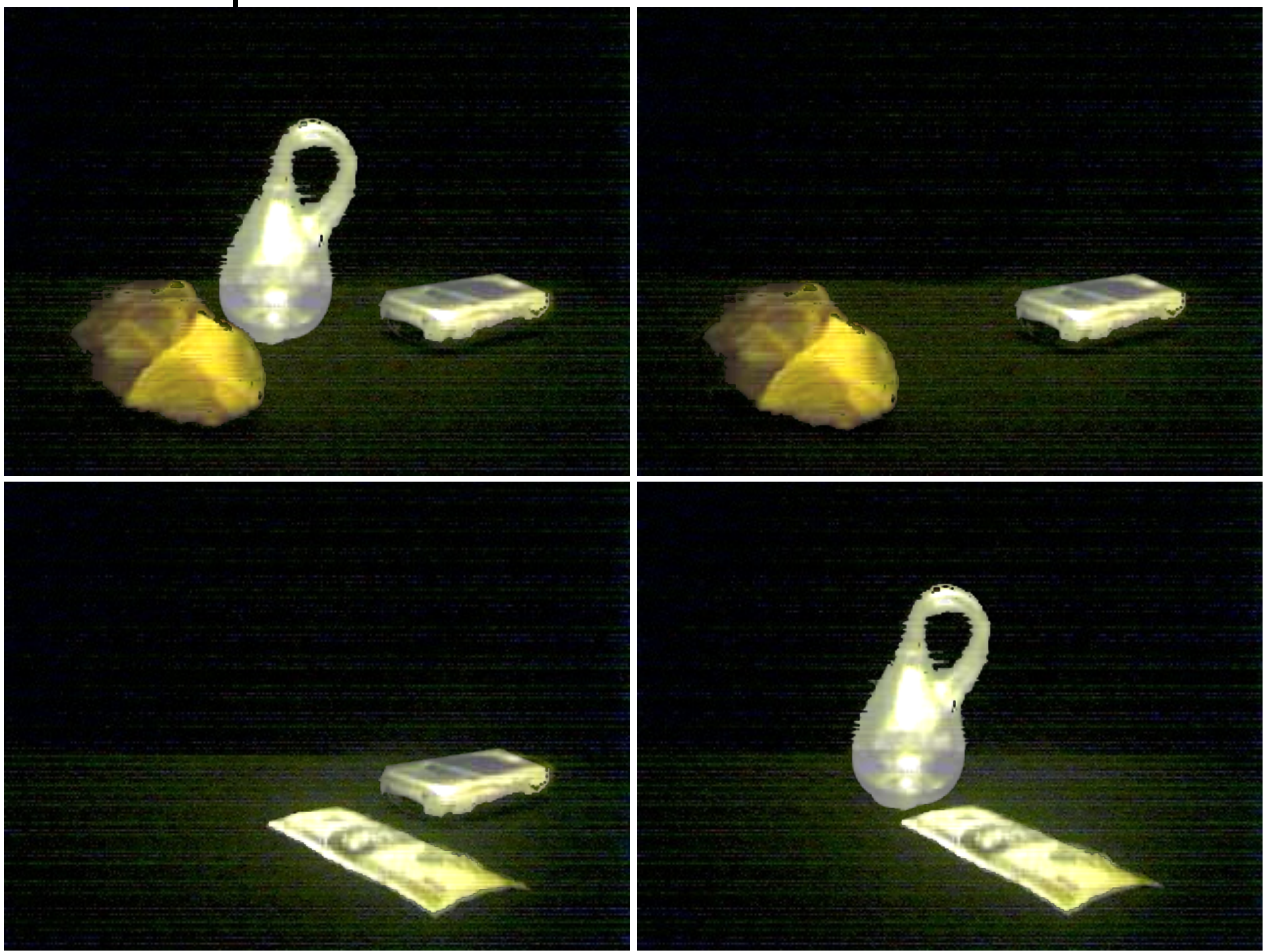
MAD-Bayes

Griffiths & Ghahramani (2006) computer vision problem “tabletop data”

[Griffiths, Ghahramani 2006]

MAD-Bayes

Griffiths & Ghahramani (2006) computer vision problem “tabletop data”

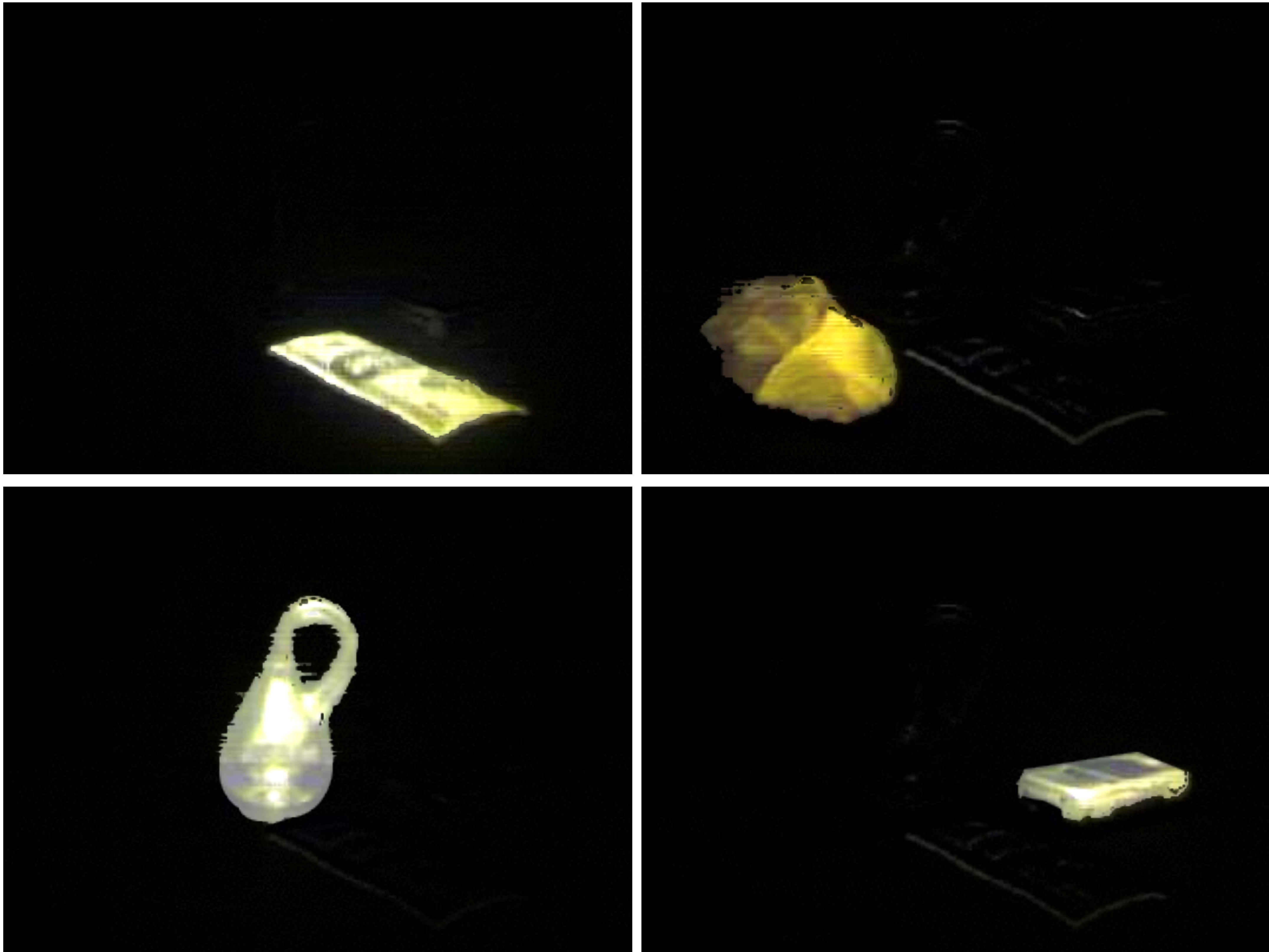


MAD-Bayes

BP-means features: table and four objects

MAD-Bayes

BP-means features: table and four objects



MAD-Bayes

Griffiths & Ghahramani (2006) computer vision problem "tabletop data"

Bayesian posterior
Gibbs sampler

BP-means algorithm

$8.5 * 10^3$ sec

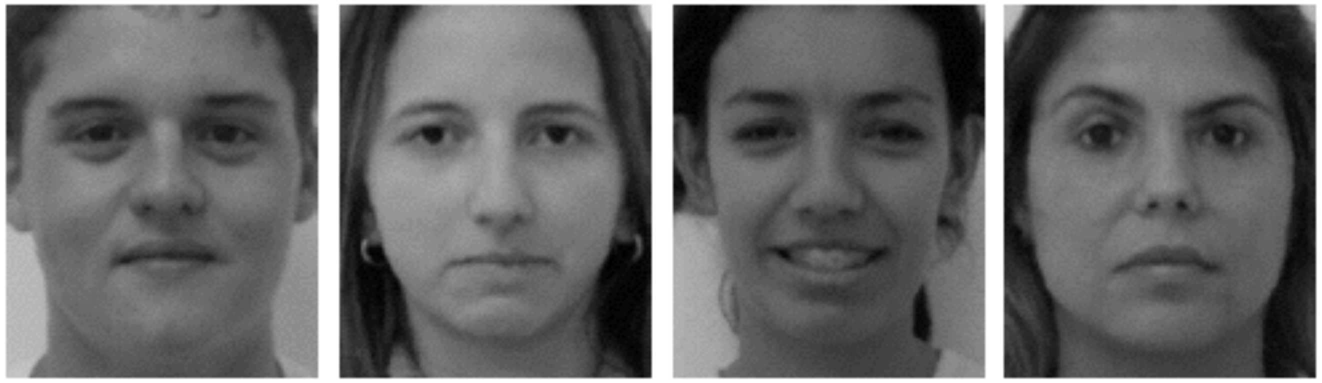
0.36 sec

Still faster by order of magnitude if restart 1000 times

Face data

Pre-aligned faces

Samples



[Thomaz, Giraldi 2010]

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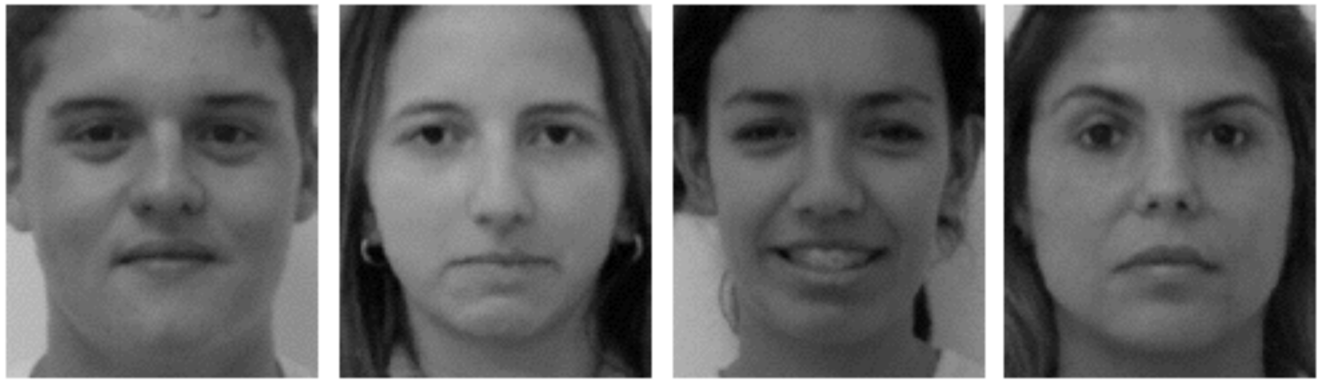
Pre-aligned faces

Samples



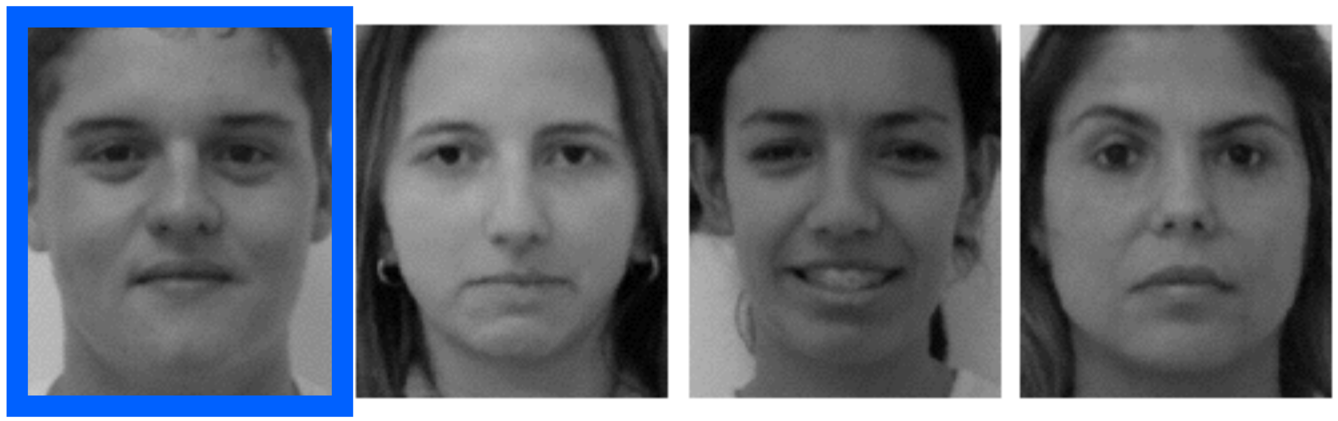
3 <u>**features**</u>

(BP-means)

Face data

Pre-aligned faces

Samples

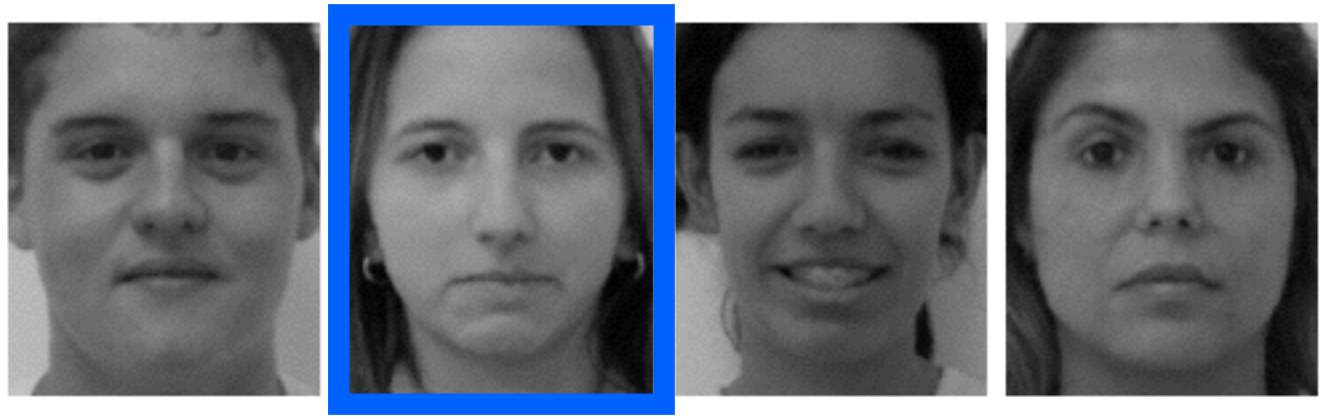


3 **<u>features</u>**
(BP-means)

Face data

Pre-aligned faces

Samples



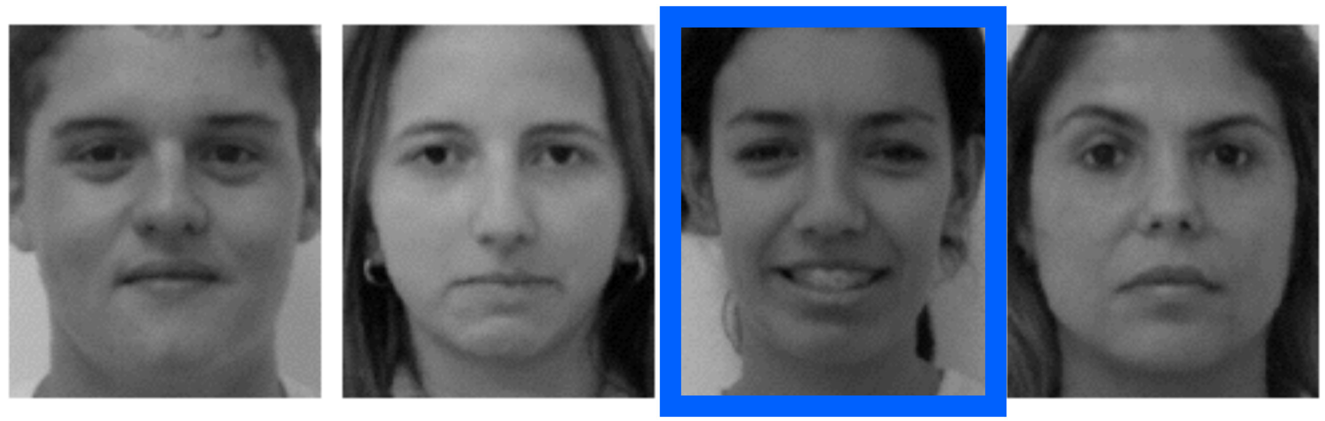
3 **features**

(BP-means)

Face data

Pre-aligned faces

Samples



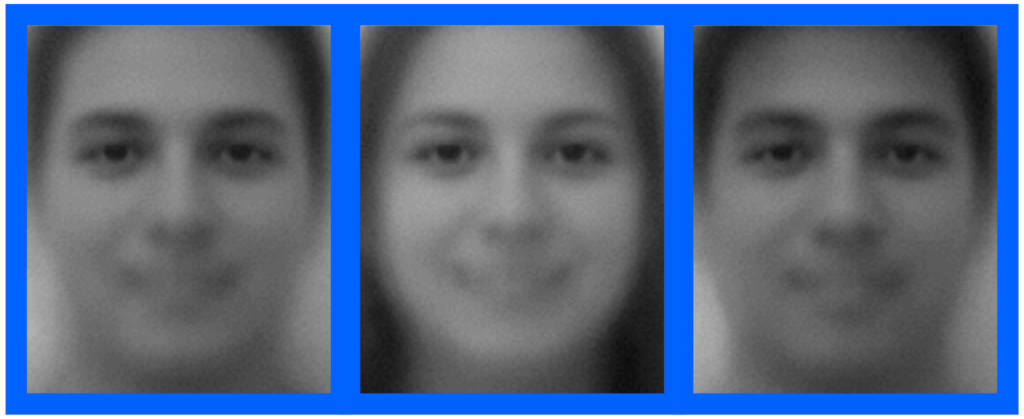
3 **features**
(BP-means)

Face data

Pre-aligned faces

Samples

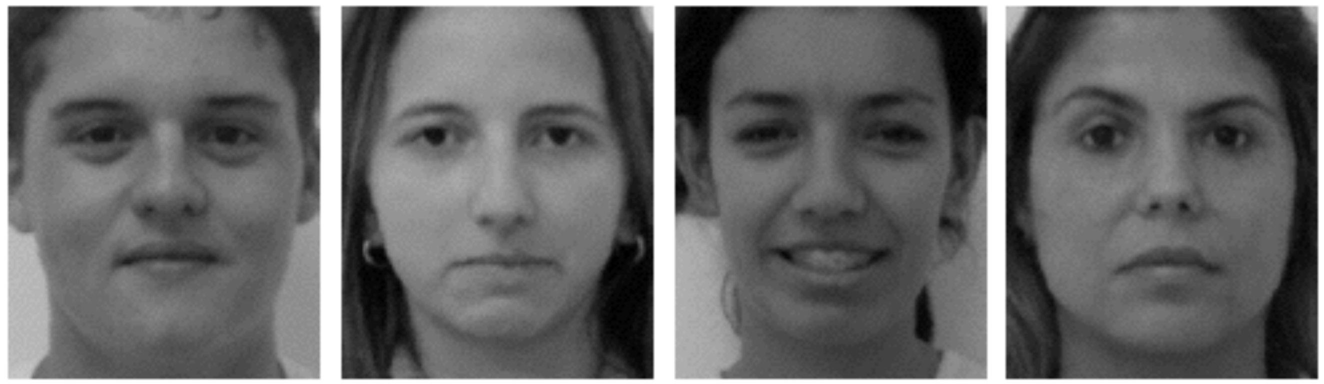
3 <u>**features**</u>
(BP-means)



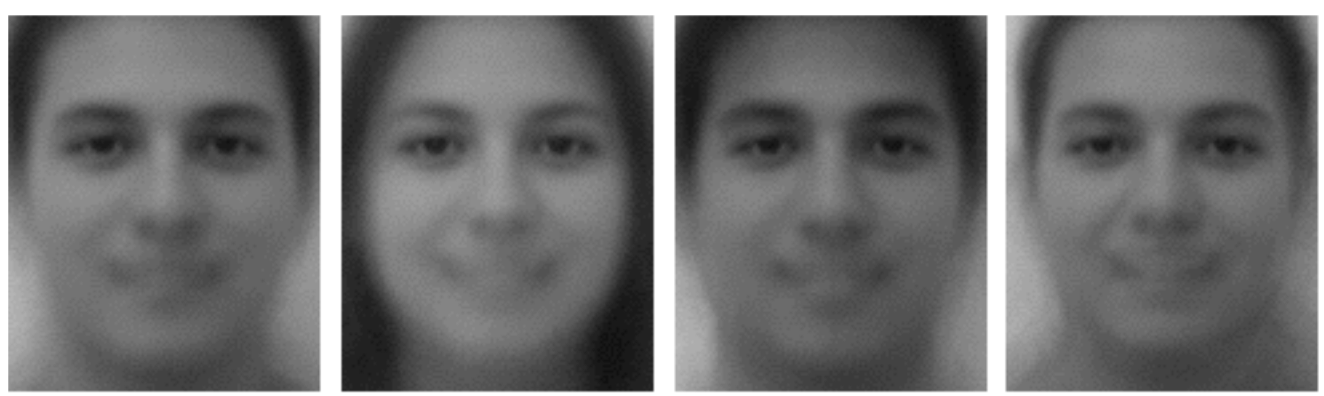
Face data

Pre-aligned faces

Samples



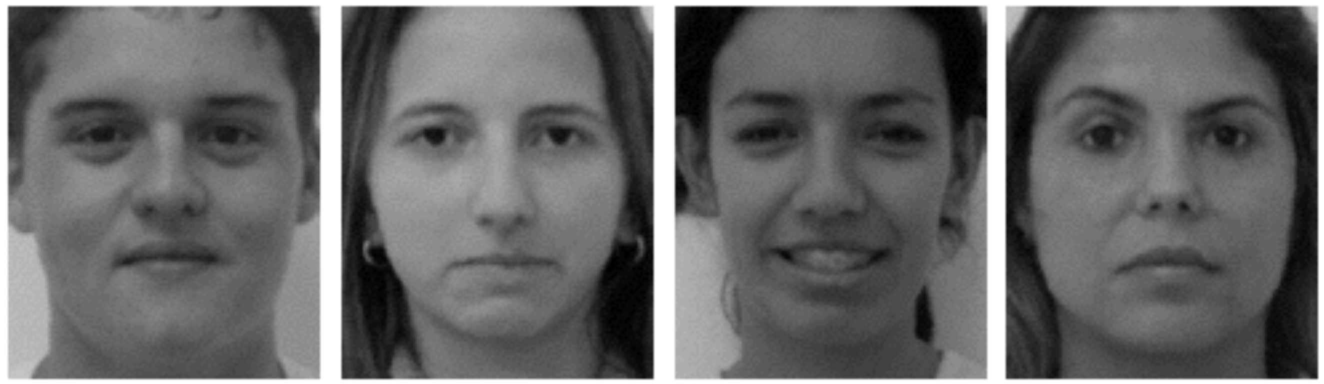
4 clusters
(K-means, K=4)



Face data

Pre-aligned faces

Samples



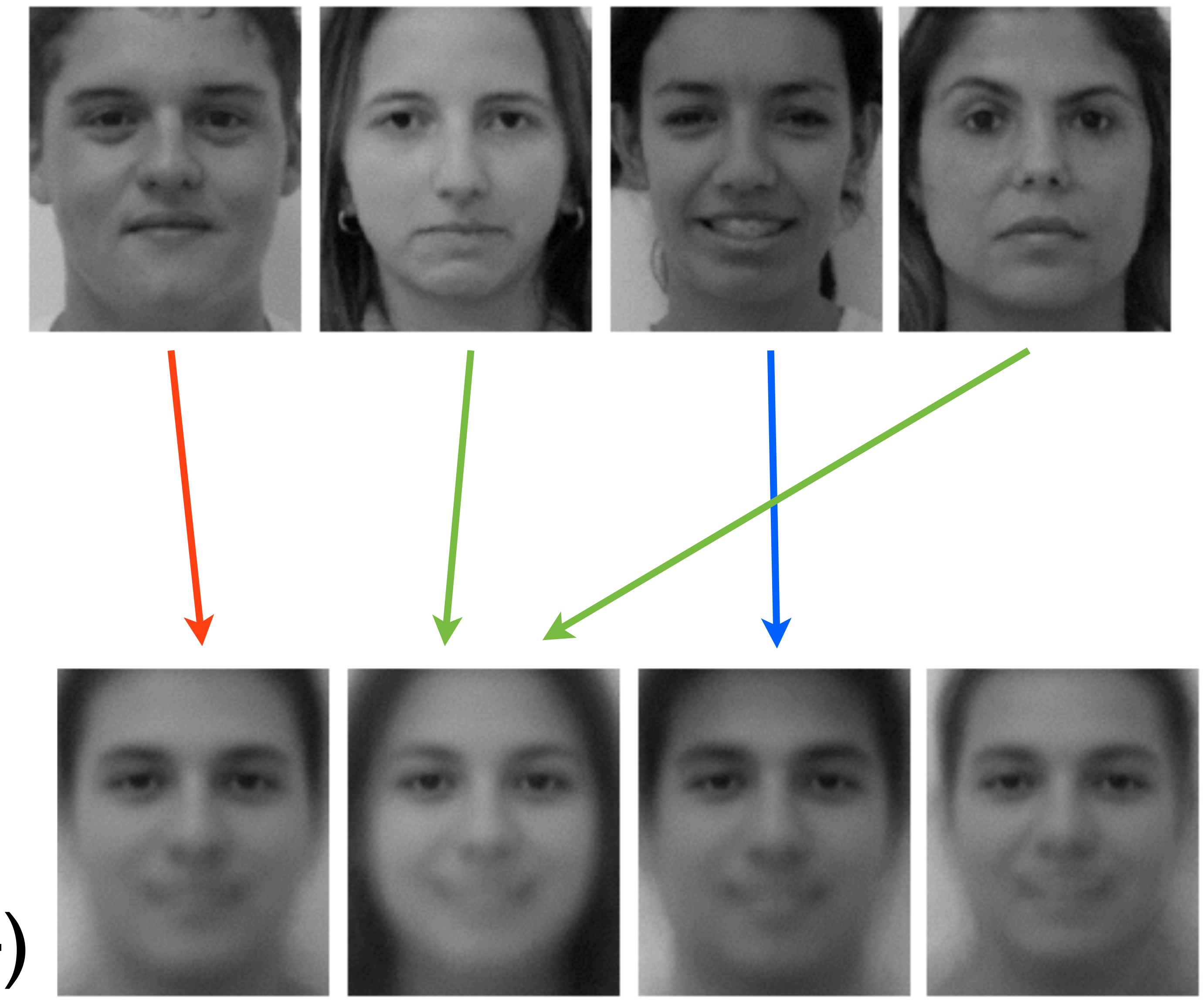
4 clusters
(K-means, K=4)

Face data

Pre-aligned faces

Samples

4 clusters
(K-means, K=4)



MAD-Bayes

Parallelism and optimistic concurrency control

	DP-means alg.	BP-means alg.
# data points	**134M**	**8M**
time per iteration	**5.5 min**	**4.3 min**

MAD-Bayes

Bayesian posterior		K-means-like objectives
Mixture of K Gaussians	→	K-means
Dirichlet process mixture	→	Unbounded number of clusters
Hierarchical Dirichlet process	→	Multiple data sets share cluster centers
⋮		⋮
Beta process	→	Features

MAD-Bayes conclusions

MAD-Bayes conclusions

- We provide new optimization objectives and regularizers

MAD-Bayes conclusions

- We provide new optimization objectives and regularizers
    - In fact, general means of obtaining more

MAD-Bayes conclusions

- We provide new optimization objectives and regularizers
  - In fact, general means of obtaining more
  - Straightforward, fast algorithms

References

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Further References

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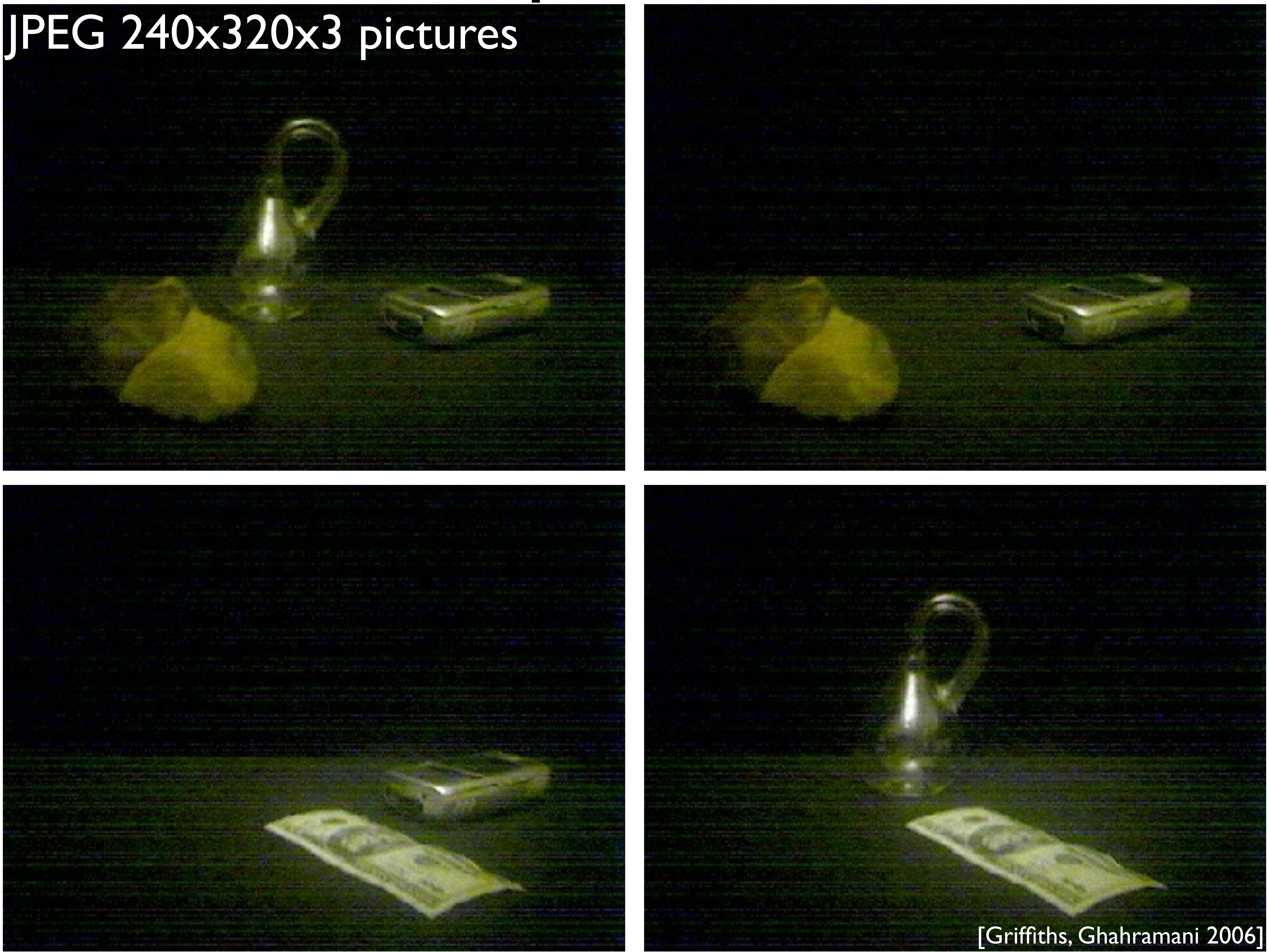
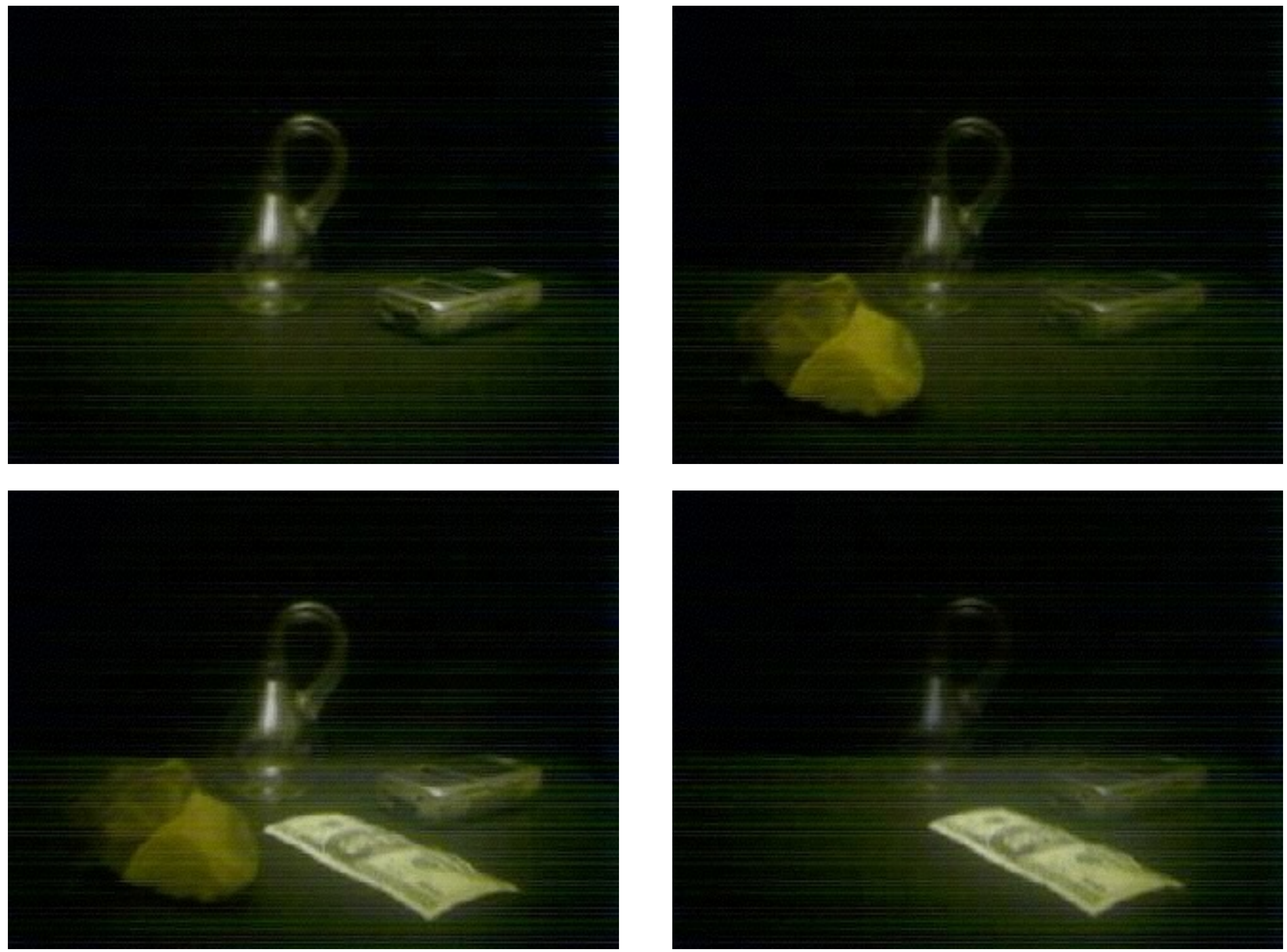
BP-means: Tabletop data



[Griffiths, Ghahramani 2006]

BP-means: Tabletop data results

K-means (K=4) cluster centers:



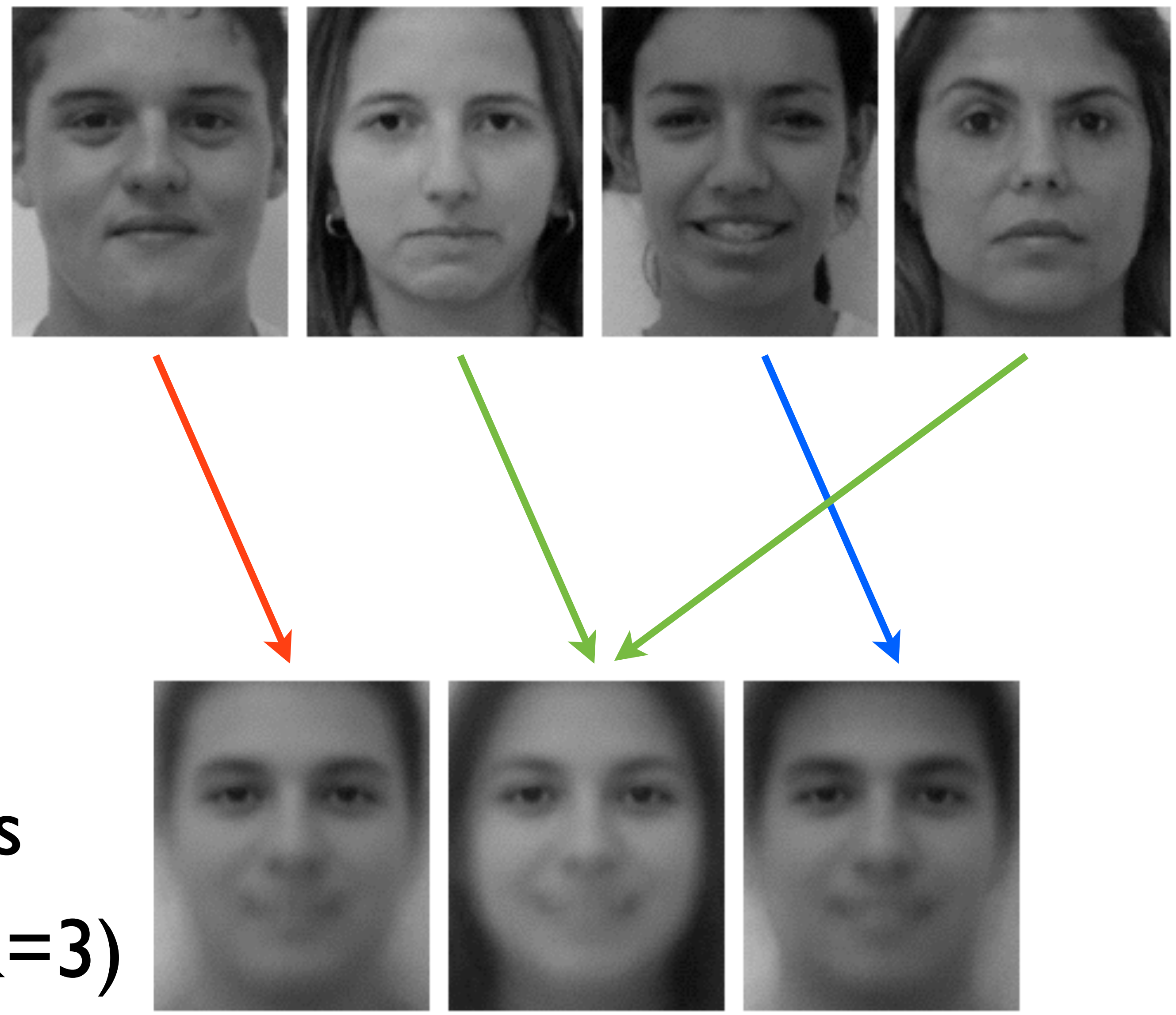
BP-means: Tabletop data results

BP-means features: table and four objects

Face data

Pre-aligned faces

Samples



3 clusters

(K-means, K=3)

Face data

Pre-aligned faces

Samples

4 clusters

(K-means, K=4)