C240 Computability and Complexity : Tutorial 1

We discuss the 'paradox' of the P_ns on pages 8-9 of the notes. Recall that P_0, P_1,… are all the programs of our programming language, in alphabetical order. The program P was:

```
1 repeat forever
  2  generate the next program P_n in the list
  3      run P_n as far as the nth bit of the output
  4      if P_n terminated or prompted for input
     before
         the nth bit was output then
  5           output 1
  6      else if the nth bit of P_n's output is 0 then
  7           output 1
  8      else if the nth bit of P_n's output is 1 then
  9           output 0
  10      end if
  11      end repeat
```

1. P must be some P_n, but for any n its output differs from P_n's at the nth bit. This is impossible. What is wrong with our reasoning?
2. Suppose $P=P_{19}$, (say). What would happen on the 19^{th} loop of P?
3. Suppose that H(x) is a procedure in our language, having the following property: for any program Q (supplied as a text string), H(Q) = 1 if Q halts when run, H(Q) = 0 otherwise
   (i) modify P using H to obtain a genuinely paradoxical program [Hint: use H(run P_n as far as the nth bit of the output)]
   (ii) Deduce that H does not exist.
4. (i) what is the least number that is not the answer to an English question having fewer than 200 letters?
   (ii)C.C.Chang and H.J.Keisler kindly dedicated their book 'Model Theory' to all those people who haven't got a book dedicated to them. Is it dedicated to you or not?
   (iii) What are the implications for your reasoning powers if the following sentence is (a) true, or (b) false? "The reader has no way of convincing him/herself that this sentence is true".