C240 Computability and Complexity
Tutorial 3

If $n \geq 0$ is a number, let 'n' $\in \{0,1\}^*$ be the binary expansion of n, without leading zeros, written for your convenience with the least significant digits on the left.

So, for example, '8' = 0001 $\in \{0,1\}^*$, and '0' = 0,

1. Design a 2-tape Turing machine A with f_A('n','m')= 'n+m' for all $n,m \geq 0$ (so A adds two numbers in binary).
   You can cheat and assume that initially 'n' is on tape 1, and 'm' on tape 2,
   State the input and full alphabet of A.

2. Explain briefly how to design a multi-tape Turing machine M such that f_M('n'.'m') = 'n.m'. That is, M multiplies two binary numbers.
   Hint: remember that multiplication is repeated addition; you can use A as a subroutine

3. A palindrome (of {0,1}) is a word $w = s_1s_2s_3s_4,,s_n$ in $\{0,1\}^*$ such that $w = s_ns_{n-1}\ldots s_3s_2s_1$. for example, 0110,010,0 and . are palindromes; 011 is not.
   Design a flowchart or give pseudocode for a Turing machine M with input alphabet $I = \{0,1\}$ such that for all $w \in I^*$, M halts and succeeds on w iff w is a palindrome.
   Write 2 versions of this TM: with one tape and with 2 tapes.
   What is the full alphabet of your TM?