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Access Control via Belnap Logic: Intuitive, Expressive, and Analyzable Policy Composition

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Outline of Access Control talk

- Motivation
- Belnap Logic
- Core Policy Language
- Expressiveness of Core Language
- Policy Analysis
- Conclusions

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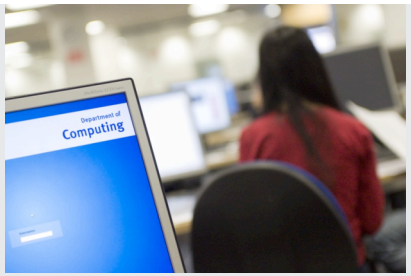
Motivation



Access control in IT systems increasingly relies on ability to compose policies

Composition framework should

- be intuitive, formal, expressive, and analyzable
- be independent of application domains but extendable to such domains
- support change management, separation of concerns, and reuse

We develop here such a framework based on Belnap Logic

Belnap Logic used in the past for reasoning in Artificial Intelligence

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$$p \vee (\neg(p \vee -p) \wedge q)$$

Belnap Logic

In the 1970ies, Belnap suggested the use of a four-valued logic

- Ordinary truth values for truth and falsity
- A third truth value that expresses lack of knowledge
- And a fourth truth value that expresses inconsistent knowledge

He developed a semantics and a sound and complete Hilbert style proof system for this logic

Belnap logic extends naturally to first- and higher-order logics

Key idea for us: Belnap's evidence-based notion of truth, e.g.

- conjunction of ''don't know'' and ''false'' is ''false''
- but conjunction of ''don't know'' and ''true'' is ''don't know''

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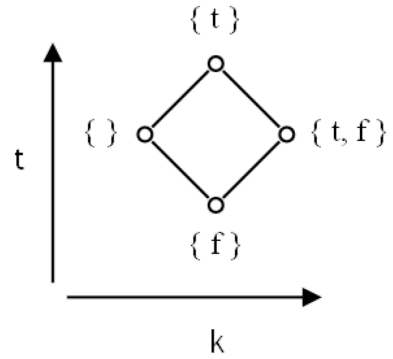
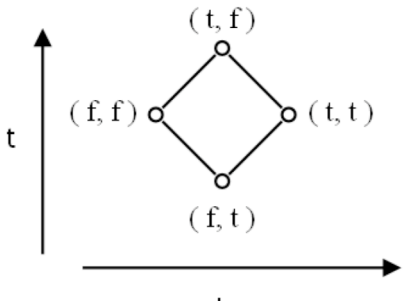
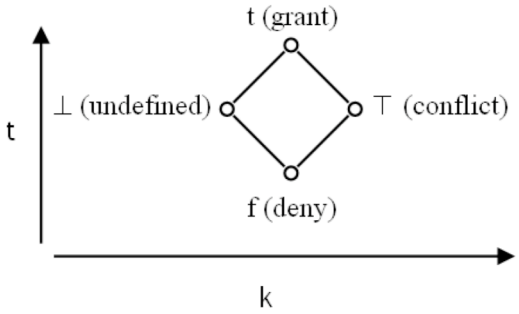
Belnap Logic: four values as composition of two



{..} collects results of policies p and q, e.g. {t,f} as conflict



(.,.) asks ``does single policy p (grant?,deny?)'', e.g. (t,t) means p grants and denies



Both interpretations captured abstractly

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Belnap space $(4, \leq_t, \leq_k, \neg)$

Belnap bilattice over $\mathbf{4} = \{\mathbf{t}, \mathbf{f}, \top, \bot\}$

Axioms:

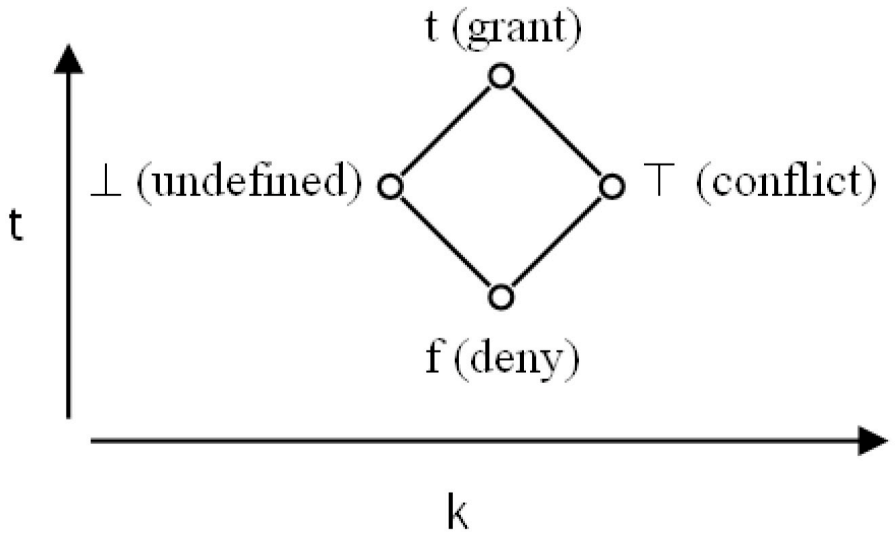
$$x \leq_t y \Rightarrow \neg y \leq_t \neg x,$$

$$x \leq_k y \Rightarrow \neg x \leq_k \neg y, \ and$$

$$\neg\neg x = x.$$

Truth negation $\neg$ swaps denials and grants, and leaves other two values fixed.

x-axis: knowledge ordering
y-axis: truth ordering



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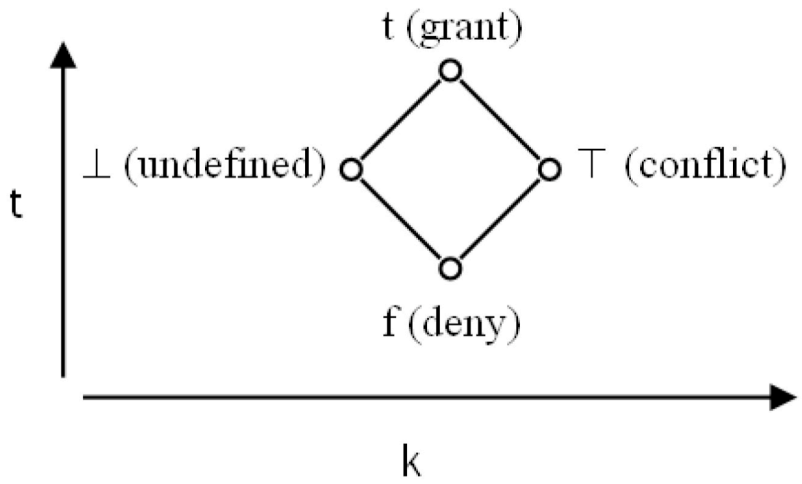
$\{\top, \neg, \wedge, \supset\}$

Belnap space functionally complete

Implication: $a \supset b = b$ if $a \in \{\mathbf{t}, \top\}$

$a \supset b = \mathbf{t}$ otherwise

Conjunction: $\wedge$ meet (aka infimum) in truth ordering



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Policy Language

Core language PBel, pronounced "pebble"

$$
\begin{array}{lll}
rp & ::= & \textit{Request Predicate} \\
\quad \mathsf{a} & & \text{Atomic} \\
\quad \mathsf{true} & & \text{Truth} \\
\quad \mathsf{false} & & \text{Falsity}
\end{array}
$$

$$
\begin{array}{lll}
p, p' & ::= & \textit{Policy} \\
\quad b \text{ if } rp & & \text{Basic policy} \\
\quad \top & & \text{Conflict} \\
\quad \neg p & & \text{Logical negation} \\
\quad p \wedge p' & & \text{Logical meet} \\
\quad p \supset p' & & \text{Implication}
\end{array}
$$

$\top$ *overloaded: denotes policy and denotes element of Belnap space*

Atomic request predicates a *denote sets of access requests.*

$b \in \{\mathbf{t}, \mathbf{f}\}$

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Syntactic sugar

PBel is a core language, similar to byte code for higher-level languages.

Convenient policy composition operators map (syntactically and semantically) into this core language, e.g.

$$
\begin{array}{lcl}
\mathbf{t} & = & \mathbf{t} \text{ if true} \\
\mathbf{f} & = & \neg\mathbf{t} \\
p \vee q & = & \neg(\neg p \wedge \neg q) \\
p \oplus q & = & (p \wedge \top) \vee (q \wedge \top) \vee (p \wedge q) \\
p[\mathbf{f} \mapsto q] & = & p \vee (\neg(p \vee -p) \wedge q) \\
p[\bot \mapsto q] & = & p \oplus (-(p \oplus \neg p) \otimes q) \\
p \text{ if } rp & = & p \otimes ((\mathbf{t} \text{ if } rp) \oplus (\mathbf{f} \text{ if } rp)) \\
p{\downarrow} & = & p[\top \mapsto \mathbf{f}][\bot \mapsto \mathbf{f}]
\end{array}
$$

$$
\begin{array}{lcl}
\bot & = & \mathbf{t} \text{ if false} \\
\top & & \text{already in PBel} \\
p \otimes q & = & (p \wedge \bot) \vee (q \wedge \bot) \vee (p \wedge q) \\
-p & = & (\neg p \supset \bot) \oplus (\neg(p \supset \bot)) \\
p[\mathbf{t} \mapsto q] & = & p \wedge (\neg(p \wedge -p) \vee q) \\
p[\top \mapsto q] & = & p \otimes (-(p \otimes \neg p) \oplus q) \\
p : q & = & (p \supset q) \otimes \neg(p \supset \neg q) \\
p{\uparrow} & = & p[\top \mapsto \mathbf{t}][\bot \mapsto \mathbf{t}]
\end{array}
$$

Abbreviation for priority composition: $p > q \;=\; p[\bot \mapsto q]$

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Models

Access-control model $\mathcal{M}$

consists of non-empty set of requests $\mathsf{R}_{\mathcal{M}}$

and interpretations of request predicates: $rp^{\mathcal{M}} \subseteq \mathsf{R}_{\mathcal{M}}$

$$\mathsf{true}^{\mathcal{M}} = \mathsf{R}_{\mathcal{M}} \ \ and \ \ \mathsf{false}^{\mathcal{M}} = \{\}$$

Example:

- set of requests as triples of form (subject,object, action, context)
- interpretation of atom *manager* is all triples whose subjects are managers
- interpretation of atom *lowThreat* is all triples whose context represents a low threat level

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Semantics

$$
\begin{aligned}
[\![b \text{ if } rp]\!]_{\mathcal{M}}(r) &= \begin{cases} b & \text{if } r \in rp^{\mathcal{M}} \\ \bot & \text{otherwise} \end{cases} \\
[\![\top]\!]_{\mathcal{M}}(r) &= \top \\
[\![\neg p]\!]_{\mathcal{M}}(r) &= \neg [\![p]\!]_{\mathcal{M}}(r) \\
[\![p \wedge q]\!]_{\mathcal{M}}(r) &= [\![p]\!]_{\mathcal{M}}(r) \wedge [\![q]\!]_{\mathcal{M}}(r) \\
[\![p \supset q]\!]_{\mathcal{M}}(r) &= [\![p]\!]_{\mathcal{M}}(r) \supset [\![q]\!]_{\mathcal{M}}(r)
\end{aligned}
$$

Meaning of policy p in model M maps requests r to element of Belnap space

Key point: policy composition is pointwise extension of Belnap operators

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Support for composite request predicates

Propositional logic structure on request predicates compiles into PBel, e.g.

$$(\mathbf{t} \text{ if } (\text{Manager} \wedge \text{OnDuty} \wedge \neg\text{Weekend} \wedge \text{ReadPDF})) > \mathbf{f}$$

is translated into PBel with function T below, for semantics on the left:

$$\begin{aligned}
[\![\mathsf{a}]\!]_{\mathcal{M}} &= \mathsf{a}^{\mathcal{M}} \\
[\![\text{true}]\!]_{\mathcal{M}} &= \mathsf{R}_{\mathcal{M}} \\
[\![\text{false}]\!]_{\mathcal{M}} &= \{\} \\
[\![\neg cp]\!]_{\mathcal{M}} &= \mathsf{R}_{\mathcal{M}} \setminus [\![cp]\!]_{\mathcal{M}} \\
[\![cp \wedge cp']\!]_{\mathcal{M}} &= [\![cp]\!]_{\mathcal{M}} \cap [\![cp']\!]_{\mathcal{M}} \\
[\![cp \vee cp']\!]_{\mathcal{M}} &= [\![cp]\!]_{\mathcal{M}} \cup [\![cp']\!]_{\mathcal{M}}
\end{aligned}$$

$$\begin{aligned}
T(\mathbf{t} \text{ if } \neg cp) &= T(\mathbf{t} \text{ if } cp) \supset \bot \\
T(\mathbf{t} \text{ if } cp \wedge cp') &= T(\mathbf{t} \text{ if } cp) \wedge T(\mathbf{t} \text{ if } cp') \\
T(\mathbf{t} \text{ if } cp \vee cp') &= \neg(T(\mathbf{f} \text{ if } cp) \wedge T(\mathbf{f} \text{ if } cp'))
\end{aligned}$$

$$\begin{aligned}
T(\mathbf{f} \text{ if } \neg cp) &= \neg(T(\mathbf{t} \text{ if } cp) \supset \bot) \\
T(\mathbf{f} \text{ if } cp \wedge cp') &= \neg(T(\mathbf{t} \text{ if } cp) \wedge T(\mathbf{t} \text{ if } cp')) \\
T(\mathbf{f} \text{ if } cp \vee cp') &= T(\mathbf{f} \text{ if } cp) \wedge T(\mathbf{f} \text{ if } cp')
\end{aligned}$$

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Safe sublanguages

Policy p is conflict-free if it never returns $\top$ for any request in any model.

Sublanguage in which all and only conflict-free policies can be written:

p, q ::=	*Conflict-free policy*
b if rp	Basic policy
$\bot$	Gap
$\neg p$	Logical negation
$p \wedge q$	Logical meet
$\mathbf{r} \supset q$	Implication
$p \vee q$	Logical Join
$p \otimes q$	Knowledge meet

$\mathbf{r}[\top \mapsto p]$	Conflict resolution
$p[v \mapsto q]$	Sequential composition
p if rp	Generalized basic policy
$p : q$	Guard connective
$\mathbf{r}\downarrow$	Pessimistic wrapper
$\mathbf{r}\uparrow$	Optimistic wrapper

Boldface r denotes any policy expression of PBel

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Safe sublanguage for gap-freedom

Policy p is gap-free if it never returns $\bot$ for any request in any model.

Sublanguage in which all and only gap-free policies can be written:

p, q ::=	*Gap-free policy*
b if true	Gap-free basic policy
$\top$	Conflict
$\neg p$	Logical negation
$p \wedge q$	Logical meet
$\mathbf{r} \supset q$	Implication
$p \vee q$	Logical join
$\mathbf{r} \oplus q$	Left knowledge join
$p \oplus \mathbf{r}$	Right knowledge join
$\mathbf{r}[\bot \mapsto p]$	Gap resolution
$p[v \mapsto q]$	Sequential composition
p if true	Generalized basic policy
$\mathbf{r}\downarrow$	Pessimistic wrapper
$\mathbf{r}\uparrow$	Optimistic wrapper

Boldface r denotes any policy expression of PBel

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Safe sublanguage for conclusive decisions

Sublanguage in which only (and all) policies can be written that are gap-free and conflict-free:

p, q ::=	*Conclusive policy*
b if true	Gap-free basic policy
$\neg p$	Logical negation
$p \wedge q$	Logical meet
$\mathbf{r} \supset q$	Implication
$p \vee q$	Logical join
$p[v \mapsto q]$	Sequential composition
p if rp	Generalized basic policy
$\mathbf{r}\downarrow$	Pessimistic wrapper
$\mathbf{r}\uparrow$	Optimistic wrapper

Boldface r denotes any policy expression of PBel

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Retrofitting policies

Recall abbreviation for priority composition: $p > q \;=\; p[\bot \mapsto q]$

Exceptional override: $(\mathbf{f} \text{ if } rp_{exc}) > p$

Behaves like policy p except at exceptional request set rp_{exc} *at which it denies*

Exclusive rights and exclusive prohibitions: $((\mathbf{t} \text{ if } rp_1) \oplus (\mathbf{f} \text{ if } rp_2)) > p$

Behaves like p except at two request sets that encode absolute rights and absolute prohibitions (respectively)

Retrofitted policy conflict-free iff two request sets are disjoint

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Composing request predicates as policies

$$(\mathbf{t} \text{ if } ChW) \wedge (\mathbf{t} \text{ if } RegisteredAnalyst) > \mathbf{f}$$

Might model that an access is granted if

- *the requester is a registered analyst*
- *and if the request is compliant with a Chinese Wall policy*

Otherwise, the request is denied.

Note: ChW is a request predicate that presumably stems from a policy.

PBel supports such demotions of policies to predicates (see analysis part below).

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Expressiveness of PBel: policy functions

Policy function for model $\mathcal{M}$ is total function $f : \mathsf{R}_{\mathcal{M}} \to \mathbf{4}$

Policy p expresses policy function f if $[\![p]\!]_{\mathcal{M}} = f$

Policy functions of form $[\![p]\!]_{\mathcal{M}} = f$ have the same output for requests that cannot be distinguished by any request predicate in p

All functions of that form are expressible as meanings of policies in PBel

These results customize to the safe sublanguages for gap-free, conflict-free, and conclusive policies (not shown in this talk)

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Expressing data-independent policy functions

Let R be a set of request predicates, e.g. those occurring in a policy p.

On each model $\mathcal{M}$ *we define equivalence relation* $\equiv^R_{\mathcal{M}}$ *to be* $\{(r, r') \in \mathsf{R}_{\mathcal{M}} \times \mathsf{R}_{\mathcal{M}} \mid \forall \mathsf{a} \in R\colon r \in \mathsf{a}^{\mathcal{M}} \text{ iff } r' \in \mathsf{a}^{\mathcal{M}}\}$.

A policy function $f\colon \mathsf{R}_{\mathcal{M}} \to \mathbf{4}$ *is* data-independent for R *in* $\mathcal{M}$ *iff* $r \equiv^R_{\mathcal{M}} r'$ *implies* $f(r) = f(r')$.

We write $PBel^R$ *for the set of PBel policies that contain only atoms from* R. *In particular,* $PBel^{\mathsf{AP}}$ *equals PBel.*

E.g. $(\mathbf{t} \text{ if } ChW) \wedge (\mathbf{t} \text{ if } RegisteredAnalyst) > \mathbf{f}$ *has four equivalence classes*

Result:

For each p *in* $PBel^R$, *policy function* $[\![p]\!]_{\mathcal{M}}$ *is data-independent for* R *in* $\mathcal{M}$.

Conversely, let f *be a policy function that is data-independent for* R *in* $\mathcal{M}$ *for finite set* $\mathsf{R}_{\mathcal{M}}$. *Then there is some* p *in* $PBel^R$ *with* $f = [\![p]\!]_{\mathcal{M}}$.

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Proof that data-independent function f is expressible

Policy p expressing policy function f is knowledge join of a grant and a denial part:

$$p = p_{\mathbf{t}} \oplus p_{\mathbf{f}}$$

Each part is the n-ary knowledge join of (negations of) policies p_r

$$p_{\mathbf{t}} = \sum_{r \in \mathsf{R}_{\mathcal{M}} \mid \mathbf{t} \leq_k f(r)} p_r \qquad p_{\mathbf{f}} = \sum_{r \in \mathsf{R}_{\mathcal{M}} \mid \mathbf{f} \leq_k f(r)} \neg p_r$$

Each building block p_r *is a characteristic function that grants on the equivalence class* $\equiv^R_{\mathcal{M}}$ *of request r and is undefined otherwise:*

$$p_r = \Big(\bigwedge_{\mathsf{a} \in R \mid r \in \mathsf{a}^{\mathcal{M}}} \mathbf{t} \text{ if a} \Big) \wedge \Big(\bigwedge_{\mathsf{a} \in R \mid r \notin \mathsf{a}^{\mathcal{M}}} (\mathbf{t} \text{ if a}) \supset \bot \Big)$$

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Expressiveness of PBel: policy composition

Example: majority vote of three policies should make as decision the majority of decisions

$$G(p_1, p_2, p_3) = (p_1 \wedge p_2) \vee (p_1 \wedge p_3) \vee (p_2 \wedge p_3)$$

Our pointwise composition means that any such function G is determined by a function

$$g\colon \mathbf{4}^n \to \mathbf{4}$$

G(p1,…,pn) at request r decides g(v1,…,vn) where vi is decision of pi on r

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All composition functions are expressible

Free algebra $\mathcal{A}$
generated from operators $\{\top, \neg, \wedge, \supset\}$
and variables $X_1, X_2, \ldots$

Result:

$$Let\ n \geq 0\ and\ g \in \mathbf{4}^n \rightarrow \mathbf{4}$$
$$Then\ there\ is\ a\ term\ t_g \in \mathcal{A}\ such\ that$$
$$[\![t_g(p_1, \ldots, p_n)]\!]\mathcal{M}(r) = g([\![p_1]\!]\mathcal{M}(r), \ldots, [\![p_n]\!]\mathcal{M}(r))$$
$$(\forall p_i \in PBel, \mathcal{M}, r \in \mathsf{R}_\mathcal{M})$$

Example: term for knowledge join $\oplus$ is

$$\neg(\neg(X_1 \wedge \top) \wedge \neg(\neg((\top \wedge X_2) \wedge \neg(X_1 \wedge X_2))))$$

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Policy Analysis

We develop a simple query language.

Many important policy analyses are expressible as queries in this language

Here, we ask whether queries hold in all models: validity checking

Validity checking is appropriate, e.g. for gap and conflict analysis

Queries can be assume-guarantee implications whose antecedents encode domain-specific or other assumptions

This policy analysis reduces to validity checking of propositional logic

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Query Language

cp, cp' ::= *Composite Request Predicate*

a	Atomic	$\neg cp$	Negation
true	Truth	$cp \wedge cp'$	Conjunction
false	Falsity	$cp \vee cp'$	Disjunction

ϕ, ϕ' ::= *Query*

$p \leq_t q$	Policy refinement in truth ordering
$p \leq_k q$	Policy refinement in information ordering
$\alpha \Rightarrow \phi$	Assume-guarantee query
$\phi \wedge \phi'$	Conjunction

where assumption α *ranges over composite request predicates*

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Query Examples

Policy q is more defined and more permissive than p

$$(p \leq_k q) \wedge (p \leq_t q)$$

Policy q is more defined but less permissive than p

$$(p \leq_k q) \wedge (q \leq_t p)$$

Policies p and q are equivalent

$$(p \leq_t q) \wedge (q \leq_t p)$$

Policy p has no gaps $p \leq_t p[\bot \mapsto \mathbf{f}]$

Policy p has no conflicts $p \leq_k p[\top \mapsto \mathbf{f}]$

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Query Semantics

$$\mathcal{M} \models p \leq_k q \text{ iff for all } r \in \mathsf{R}_{\mathcal{M}} \text{ we have } [\![p]\!]_{\mathcal{M}}(r) \leq_k [\![q]\!]_{\mathcal{M}}(r)$$
$$\mathcal{M} \models p \leq_t q \text{ iff for all } r \in \mathsf{R}_{\mathcal{M}} \text{ we have } [\![p]\!]_{\mathcal{M}}(r) \leq_t [\![q]\!]_{\mathcal{M}}(r)$$
$$\mathcal{M} \models \alpha \Rightarrow \phi \text{ iff } \| \alpha \|_{\mathcal{M}} \neq \mathsf{R}_{\mathcal{M}} \text{ or } \mathcal{M} \models \phi$$
$$\mathcal{M} \models \phi \wedge \psi \text{ iff } \mathcal{M} \models \phi \text{ and } \mathcal{M} \models \psi$$

Atomic queries interpreted universally: policy p is below policy q for all requests

Assume-Guarantee Implications modeled universally:

If all requests of the model satisfy assumption,
then the guarantee query has to be true in the model

Conjunction has standard interpretation

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Query Analysis

For each query ϕ we generate a composite request predicate $C(\phi)$ such that

Query ϕ is valid in all models

Iff

$C(\phi)$ is valid when interpreted as formula in propositional logic

Definition of $C(\phi)$ proceeds in two steps:

1. Capture constraints for grants and denials of policies contained in query
2. Capture the logical structure of the query in terms of these policy constraints

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Constraints for PBel policies

Each request r in model M determines model of propositional logic:

$$\rho_r^{\mathcal{M}}(\mathsf{a}) = \mathbf{t} \;\; if \; r \in \mathsf{a}^{\mathcal{M}} \qquad \rho_r^{\mathcal{M}}(\mathsf{a}) = \mathbf{f} \;\; if \; r \notin \mathsf{a}^{\mathcal{M}}$$

Semantics of that model matches that of request model M: $\rho_r^{\mathcal{M}}$
satisfies those composite request predicates that contain r in their interpretation

Define, below, propositional logic formula $p \Uparrow b$ *such that*

$$\rho_r^{\mathcal{M}} \models p \Uparrow b \;\; iff \; b \leq_k [\![p]\!]_{\mathcal{M}}(r)$$

$$(b' \text{ if } rp) \Uparrow b = \begin{cases} rp & \text{if } b = b' \\ \mathsf{false} & \text{otherwise} \end{cases}$$

$$\begin{aligned}
\top \Uparrow b &= \mathsf{true} \\
(p \wedge q) \Uparrow \mathbf{f} &= p \Uparrow \mathbf{f} \vee q \Uparrow \mathbf{f} \\
(p \supset q) \Uparrow \mathbf{f} &= p \Uparrow \mathbf{t} \wedge q \Uparrow \mathbf{f} \\
(\neg p) \Uparrow b &= p \Uparrow \neg b \\
(p \wedge q) \Uparrow \mathbf{t} &= p \Uparrow \mathbf{t} \wedge q \Uparrow \mathbf{t} \\
(p \supset q) \Uparrow \mathbf{t} &= \neg(p \Uparrow \mathbf{t}) \vee q \Uparrow \mathbf{t}
\end{aligned}$$

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Constraints for Queries

Query for knowledge ordering uses that truth and falsity are prime elements of the distributive lattice in the knowledge ordering

Query for assume-guarantee reasoning translates this into a propositional implication (as done in assume-guarantee reasoning for linear-time temporal logic)

$$C(p \leq_k q) = (p \Uparrow \mathbf{f} \rightarrow q \Uparrow \mathbf{f}) \wedge (p \Uparrow \mathbf{t} \rightarrow q \Uparrow \mathbf{t})$$
$$C(\alpha \Rightarrow \phi) = \alpha \rightarrow C(\phi)$$

Query for truth ordering uses characterization of truth ordering in terms of knowledge ordering

Query for conjunction is interpreted compositionally (sound for validity checking)

$$C(p \leq_t q) = (q \Uparrow \mathbf{f} \rightarrow p \Uparrow \mathbf{f}) \wedge (p \Uparrow \mathbf{t} \rightarrow q \Uparrow \mathbf{t})$$
$$C(\phi \wedge \psi) = C(\phi) \wedge C(\psi)$$

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Example Policy Analysis

$$p = (\mathbf{t} \text{ if } rd) \oplus (\mathbf{f} \text{ if } wr) \qquad q = p[\top \mapsto \mathbf{f}]$$

Valid query (assumes that no read action is also a write action):

$$\neg(rd \wedge wr) \;\Rightarrow\; (p \;\leq_t\; q)$$

Propositional constraints for policies:

$$\begin{aligned}
p \Uparrow \mathbf{t} &= rd \vee \mathsf{false} = rd \\
p \Uparrow \mathbf{f} &= \mathsf{false} \vee wr = wr \\
q \Uparrow \mathbf{t} &= p \Uparrow \mathbf{t} \wedge (\neg(p \Uparrow \mathbf{f}) \vee \mathbf{f} \Uparrow \mathbf{t}) = rd \wedge (\neg wr \vee \mathsf{false}) = rd \wedge \neg wr \\
q \Uparrow \mathbf{f} &= p \Uparrow \mathbf{f} \wedge (\neg(p \Uparrow \mathbf{t}) \vee \mathbf{t} \Uparrow \mathbf{t}) = wr \wedge (\neg rd \vee \mathsf{true}) = wr
\end{aligned}$$

Propositional constraint for above query is equivalent to valid formula

$$\begin{aligned}
C(\neg(rd \wedge wr) \Rightarrow p \leq_t q) &= \neg(rd \wedge wr) \rightarrow (q \Uparrow \mathbf{f} \rightarrow p \Uparrow \mathbf{f}) \wedge (p \Uparrow \mathbf{t} \rightarrow q \Uparrow \mathbf{t}) \\
&= \neg(rd \wedge wr) \rightarrow (wr \rightarrow wr) \wedge (rd \rightarrow (rd \wedge \neg wr))
\end{aligned}$$

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Some Related Work

- Halpern and Weissman 2003: policies specified in first-order logic, access granted if formula logically entails formal permission predicate
- XACML standard: has no formal semantics, its semantic values and policy combination algorithms can be interpreted within PBel for suitable interpretation
- Craven et al. 2009: policy analysis that takes account of obligations, authorizations, and system state
- Ni et al. 2009: D-algebras as functionally complete value spaces with algebraic operators, result spaces rather ad-hoc in nature
- Bauer et al. 2005: Polymer, access-control language for untrusted Java applications, policy is query method that returns one of six values for code execution request
- Moffett and Sloman 1994: early work on policy conflict analysis
- Ribeiro et al. 2001: access-control language SPL, three-valued, can be cleanly embedded into PBel

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Things we did but didn't mention here

- Support for attribute language for request predicates
- Clean encoding of policy language SPL in PBel
- Expressiveness results specialized to safe sub-languages, both for policy functions and for policy composition
- Symbolic query analysis, where policies are parameters
- Methods and interface specifications
- Request mappings, e.g. ``grant read access whenever write access is given''

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Conclusions

We developed an access-control policy language based on

- abstract request predicates that encapsulated domain-specific aspects of sets of access requests
- the 4-valued Belnap bilattice whose operators were extended pointwise to our language

This gave us a very expressive core language over which common policy combination idioms can be expressed.

The core language (and so any idioms compiling into it) has a simply query analysis that supports many of the desired policy analyses and reduces them to validity checking of propositional logic.

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Recommended Reading

Belnap, N. D.
A useful four-valued logic.
In “Modern Uses of Multiple-Valued Logic”, J. M. Dunn & G. Epstein (eds)
D. Reidel, Dordrecht, pages 8--37, 1977

Arieli, O. and Avron, A.
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