p-Automata: New Foundations for Discrete-Time Probabilistic Verification

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Outline of talk

Motivation

Markov Chains and PCTL

Weak Stochastic Games

p-Automata

Acceptance Games

Expressiveness

Simulation

Conclusion

Motivation

Abstraction in Probabilistic Model Checking

- Probabilistic model checking increasingly important, widely used technique
- Advanced model-checking tools exist, e.g. PRISM (Oxford) and MRMC (Aachen)
- Scalability of analysis critical in many application domais
- Abstraction believed to be critical for scalability
- Effective abstraction techniques for probabilistic model checking: still an open research problem

Automata-Based Verification

Automaton A accepts as its language $\mathcal{L}(A)$ set of models M.

This approach supports important techniques:

- specifications and models have meaning-preserving representations as automata
- model checking reduces to acceptance of automata input
- satisfiability reduces to emptiness checks of automata
- automata closed under Boolean operations
- simulation under-approximates language containment
- uniform, strong framwork for sound abstraction of branching-time properties

Aim of this talk

Develop automata-based approach to probabilistic verification:

- supports all aforementioned techniques
- models: countable, discrete-time, labeled Markov chains
- specifications: subsume Probabilistic Computation Tree Logic (PCTL) [Hansson & Jonsson 1994]
- p-automata are themselves probabilistic specifications

Related work

- Probabilistic automata [Rabin 1963] define probabilistic languages of non-probabilistic models
- Automata for co-algebras [Venema 2006] have corresponding logic with finite-model property: hence they cannot express path modalities of PCTL
- Probabilistic processes [Larsen & Jonsson 1991] consider only refinement checking

These don't support all aforementioned techniques.

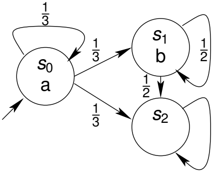
Markov Chains and PCTL

Models

Countable, discrete-time, labeled Markov chain M:

- set of atomic propositions $\mathbb{AP}$
- S countable set of locations
- $P\colon S \times S \to [0,1]$ stochastic matrix with $\sum_{s' \in S} P(s,s') = 1$ for all $s \in S$
- location $s^{\mathrm{in}} \in S$ designated initial one
- $L\colon S \to 2^{\mathbb{AP}}$ labeling function
  $L(s) =$ set of propositions true in location s
- $P(s,s') =$ probability that M, when in location s, transitions to location s' in one discrete time step

Example



- Three locations s_0 (initial), s_1, and s_2
- Two atomic propositions a and b; e.g. a true only at s_0
- Probability distribution $P(s_0, \cdot)$ uniform over all locations
- Sink state s_2 has implicit probability 1 self-loop
- $\{s_2\}$ terminal, maximal strongly connected component

PCTL Syntax

$\phi, \psi ::=$	*PCTL formulas*
$\mathsf{a}, \neg \mathsf{a}$	Atom
$\phi \wedge \psi$	Conjunction
$\phi \vee \psi$	Disjunction
$[\alpha]_{\bowtie p}$	Path Probability

$\alpha ::=$	*Path formulas*
$\mathsf{X}\,\phi$	Next
$\phi\,\mathsf{U}^{\leq k}\,\psi$	Until
$\phi\,\mathsf{W}^{\leq k}\,\psi$	Weak Until

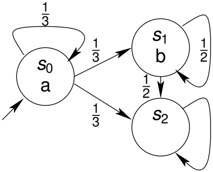
- $\mathsf{a} \in \mathbb{AP}$, $k \in \mathbb{N} \cup \{\infty\}$, $p \in [0, 1]$, $\bowtie\ \in \{>, \geq\}$
- full PCTL has this Greater Than Negation Normal Form

PCTL Semantics

$$\|a\| = \{s \in S \mid a \in L(s)\} \qquad \|\neg a\| = \{s \in S \mid a \notin L(s)\}$$
$$\|\phi \wedge \psi\| = \|\phi\| \cap \|\psi\| \qquad \|\phi \vee \psi\| = \|\phi\| \cup \|\psi\|$$
$$\|[\alpha]_{\bowtie p}\| = \{s \in S \mid \mathrm{Prob}_M(s, \alpha) \bowtie p\}$$

- paths: sequences $s_0 s_1 \ldots$ with $P(s_i, s_{i+1}) > 0$
- $s_0 s_1 \ldots \models \mathsf{X}\,\phi$ iff $s_1 \in \|\phi\|_M$
- $s_0 s_1 \ldots \models \phi\,\mathsf{U}^{\leq k}\psi$ iff there is $l \in \mathbb{N}$ such that $l \leq k$, $s_l \in \|\psi\|_M$ and for all $0 \leq j < l$ we have $s_j \in \|\phi\|_M$
- $s_0 s_1 \ldots \models \phi\,\mathsf{W}^{\leq k}\psi$ iff for all $l \in \mathbb{N}$ such that $0 \leq l \leq k$, either $s_l \in \|\phi\|_M$ or there is $0 \leq j \leq l$ with $s_j \in \|\psi\|_M$

Example



- Convention: write U for $U^{\leq\infty}$ and W for $W^{\leq\infty}$
- $s_0 \in \|[(\mathrm{a} \vee \mathrm{b})\,\mathrm{U}\,(\neg \mathrm{a} \wedge \neg \mathrm{b})]_{\geq 1}\|_M$ since measure of paths beginning at s_0 and satisfying $(\mathrm{a} \vee \mathrm{b})\,\mathrm{U}\,(\neg \mathrm{a} \wedge \neg \mathrm{b})$ is 1
- $s_0 \in \|[\mathrm{a}\,\mathrm{U}\,\mathrm{b}]_{\geq 0.5}\|_M$ as infinite path $s_0 s_0 \ldots$ has measure 0

Weak Stochastic Games

Stochastic game

Tuple $G = ((V,E),(V_0,V_1,V_p),\kappa,\alpha)$ where

- (V,E) directed graph
- (V_0,V_1,V_p) partitions V into Player 0, Player 1, and probabilistic configurations
- for each $v \in V_p$: $\kappa(v)$ probability distribution on $E(v) = \{v' \mid (v,v') \in E\}$ with $(v,v') \in E$ iff $\kappa(v)(v') \neq 0$
- $\alpha \subseteq V$ winning condition

Weakness

- Weak Stochastic game G: all its maximal, strongly connected components (SCC) V' in (V, E) satisfy

$$V' \subseteq \alpha \text{ or } V' \cap \alpha = \{\}$$

- Weak Game G: weak stochastic game without probabilistic configurations: $V_p = \{\}$.
- Markov chain: representable as weak stochastic game with $V_0 = V_1 = \{\}$ and $\alpha = V$.

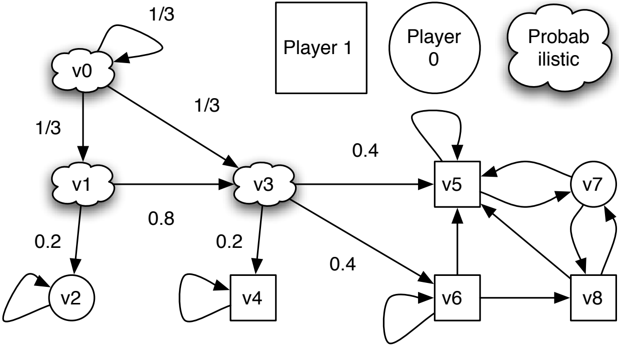
Plays and their wins

- Plays from v_0 are sequences $v_0 v_1 \ldots$ of configurations
  - $v_i \in V_0$: Player 0 chooses v_{i+1} with $(v_i, v_{i+1}) \in E$
  - $v_i \in V_1$: Player 1 chooses v_{i+1} with $(v_i, v_{i+1}) \in E$
  - $v_i \in V_p$: distribution $\kappa(v_i)$ chooses v_{i+1} at random
- WLOG: plays are infinite as Player 0 and Player 1 configurations don't deadlock.
- Play won by
  - player 0 if all configurations in some suffix of play are in α
  - Otherwise: player 1 wins play

Strategies and game values

- (pure memoryless) strategy $\sigma \in \Sigma$ for Player 0: function $\sigma\colon V_0 \to V$ with $(v, \sigma(v)) \in E$ for all $v \in V_0$
- strategy $\pi \in \Pi$ for Player 1: similar function $\pi\colon V_1 \to V$
- each pair $(\sigma, \pi) \in \Sigma \times \Pi$ determines Markov chain $M^{\sigma,\pi}$: all paths in G consistent with σ and π
- $\mathrm{val}_0^{\sigma,\pi}(v)$ measure of paths from v Player 0 wins in $M^{\sigma,\pi}$
- $\mathrm{val}_0(v) = \sup_{\sigma\in\Sigma} \inf_{\pi\in\Pi} \mathrm{val}_0^{\sigma,\pi}(v)$:
  game value for player 0 at v
- $\mathrm{val}_1(v) = \sup_{\pi\in\Pi} \inf_{\sigma\in\Sigma}(1 - \mathrm{val}_0^{\sigma,\pi}(v))$
  game value for player 1 at v
- strategies that achieve these values are optimal

A Weak Stochastic Game



- $\alpha = \{v_2, v_5, v_7, v_8\}$
- game values for player 0 are 1 at v_2, v_5, v_7, v_8; 0 at v_4 and v_6; 0.4 at v_3; 0.52 at v_1; and 0.46 at v_0

Determinacy and algorithms

- Stochastic games G are determined: for all $v \in V$

  $$\mathrm{val}_0(v) = 1 - \mathrm{val}_1(v)$$

- Let G be finite:
  - $\mathrm{val}_0(v)$ computable in NP & coNP
  - optimal strategies exist for both players
  - If G is weak, then $\mathrm{val}_0(v) \in \{0, 1\}$ and is linear-time computable

p-Automata

Structure

A p-automaton A is tuple

$$\langle \Sigma, Q, \delta, \varphi^{\text{in}}, \alpha \rangle$$

- Σ finite input alphabet
- Q set of states (not necessarily finite)
- $\delta\colon Q \times \Sigma \to B^{+}(Q \cup [\![Q]\!])$ transition function
- $\varphi^{\text{in}} \in B^{+}(Q \cup [\![Q]\!])$ initial condition
- $\alpha \subseteq Q$ acceptance condition

Guiding Intuition

- p-automata reuse ideas from alternating tree automata
- Need ability to quantify over probabilities of path sets
- Do this for regular path sets, not just for one time step.
- Need mechanism for decomposing probabilities and witnessing path sets.
- Value space $B^+(Q \cup [\![Q]\!])$ for transitions informed by that

Definition of $B^+(Q \cup [\![Q]\!])$

- $B^+(T)$ set of positive Boolean formulas generated from elements $t \in T$:

$$\varphi ::= t \mid \mathsf{ff} \mid \mathsf{tt} \mid \varphi \vee \varphi \mid \varphi \wedge \varphi$$

- Term set $[\![Q]\!]$ defined through:

$$\boxed{\begin{aligned}
[\![Q]\!]_{>} &= \{[\![q]\!]_{\bowtie p} \mid q \in Q, \bowtie \in \{\geq, >\}, p \in [0,1]\} \\
[\![Q]\!]^{*} &= \{*(t_1, \ldots, t_n) \mid n \in \mathbb{N}, \forall i\colon\ t_i \in [\![Q]\!]_{>}\} \\
[\![Q]\!]^{\overset{*}{\triangledown}} &= \{\overset{*}{\triangledown}(t_1, \ldots, t_n) \mid n \in \mathbb{N}, \forall i\colon\ t_i \in [\![Q]\!]_{>}\} \\
[\![Q]\!] &= [\![Q]\!]^{*} \cup [\![Q]\!]^{\overset{*}{\triangledown}}
\end{aligned}}$$

Intuition behind $B^+(Q \cup [\![Q]\!])$

- meaning of Boolean connectives as for alternating automata
- $[\![q]\!]_{\bowtie p}$ holds in location s if:
  measure of paths that begin in s and satisfy q is $\bowtie p$
- $\ast([\![q_1]\!]_{>p_1}, [\![q_2]\!]_{\geq p_2})$ means
  - q_1 and q_2 hold with probability greater than p_1 and greater than or equal to p_2, respectively
  - and sets supplying these probabilities are disjoint
- $\overset{\ast}{\vee}([\![q_1]\!]_{\geq p_1}, [\![q_2]\!]_{\geq p_2})$ has dual meaning

Example

$$A = \langle 2^{\{a,b\}}, \{q_1, q_2\}, \delta, [\![q_1]\!]_{\geq 0.5}, \{q_2\} \rangle$$

$$\begin{aligned}
&\delta(q_1, \{a, b\}) = \delta(q_1, \{a\}) = q_1 \vee [\![q_2]\!]_{\geq 0.5} \\
&\delta(q_2, \{b\}) = \delta(q_2, \{a, b\}) = [\![q_2]\!]_{\geq 0.5} \\
&\delta(q_1, \{\}) = \delta(q_1, \{b\}) = \delta(q_2, \{\}) = \delta(q_2, \{a\}) = \mathrm{ff}
\end{aligned}$$

- q_2 encodes recursive property ϕ = “b holds at location presently read by q_2, and ϕ holds with probability ≥ 0.5 in next locations”
- q_1 asserts it is possible to get to a location that satisfies q_2 along a path that satisfies a
- initial condition $[\![q_1]\!]_{\geq 0.5}$ encodes that set of paths satisfying “a U ϕ” has probability at least 0.5

Acceptance Games

Constraints for solvability of acceptance games

- p-automata can express recursive, probabilistic, regular path sets
- can do this also using $\ast$ and $\overset{\ast}{\vee}$ operator
- such properties may potentially be inconsistent, making the acceptance game insolvable
- current solution: constrain A, through its graph G_A
- partition graph G_A into maximal, strongly connected components (SCC)
- each SCC determines a weak stochastic or weak game
- solve these games bottom-up

Structure of acceptance game for $M \in \mathcal{L}(A)$

- Most configurations of these weak (stochastic) games in
$$S \times (Q \cup \{\mathsf{cl}_\mathsf{p}(\delta(q,\phi)) \mid q \in Q, \phi \in 2^{\mathbb{AP}}\})$$
where $\mathsf{cl}_\mathsf{p}(\eta)$ set of Boolean subformulas of η
- Initial configuration $(s^{\mathsf{in}}, \varphi^{\mathsf{in}})$ occurs as configuration in exactly one of these games
- A accepts M iff game value of $(s^{\mathsf{in}}, \varphi^{\mathsf{in}})$ in that game is 1

Graph $G_A = \langle Q', \rightarrow, \rightarrow_b, \rightarrow_u \rangle$ of p-Automaton A

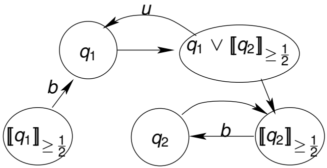
$$
\begin{aligned}
Q' &= Q \cup \mathrm{cl_p}(\delta(Q,\Sigma)) \\
\rightarrow &= \{(\varphi_1 \wedge \varphi_2, \varphi_i), (\varphi_1 \vee \varphi_2, \varphi_i) \mid \varphi_i \in Q' \setminus Q\} \cup \\
&\qquad \{(q, \delta(q,\sigma)) \mid q \in Q, \sigma \in \Sigma\} \\
\rightarrow_u &= \{(\varphi \wedge q, q), (q \wedge \varphi, q), (\varphi \vee q, q), (q \vee \varphi, q) \mid \varphi \in Q', q \in Q\} \\
\rightarrow_b &= \{(\varphi, q) \mid \varphi \in [\![Q]\!] \text{ and } q \in \mathrm{gs}(\varphi)\}
\end{aligned}
$$

- $\mathrm{gs}(\varphi)$ set of guarded states of φ: all $q' \in Q$ occurring in some term in φ
- $\rightarrow_b$ set of bounded transitions
- $\rightarrow_u$ set of unbounded transitions
- $\rightarrow$ set of simple transitions
- mark $(\varphi, q) \in \rightarrow_b$ with $*$ (resp. with $\overset{*}{\vee}$) if some $[\![q']\!]_{\bowtie p}$ occurs in φ within scope of a $*$ (resp. $\overset{*}{\vee}$)

Example G_A

$$A = \langle 2^{\{\mathrm{a},\mathrm{b}\}}, \{q_1, q_2\}, \delta, [\![q_1]\!]_{\geq 0.5}, \{q_2\} \rangle$$

$$\begin{aligned}
&\delta(q_1, \{\mathrm{a}, \mathrm{b}\}) = \delta(q_1, \{\mathrm{a}\}) = q_1 \vee [\![q_2]\!]_{\geq 0.5} \\
&\delta(q_2, \{\mathrm{b}\}) = \delta(q_2, \{\mathrm{a}, \mathrm{b}\}) = [\![q_2]\!]_{\geq 0.5} \\
&\delta(q_1, \{\}) = \delta(q_1, \{\mathrm{b}\}) = \delta(q_2, \{\}) = \delta(q_2, \{\mathrm{a}\}) = \mathrm{ff}
\end{aligned}$$

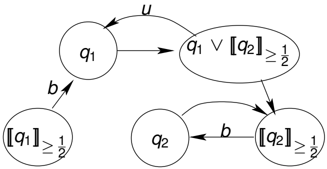


Uniform Weak p-Automata

- p-automaton A uniform if:
  - cycles in G_A have transitions only in $\rightarrow \cup \rightarrow_b$ or only in $\rightarrow \cup \rightarrow_u$
  - cycles in $\langle Q, \rightarrow \cup \rightarrow_b \rangle$ have markings $\{\}$, $\{*\}$ or $\{\overset{*}{\vee}\}$, not $\{*, \overset{*}{\vee}\}$.
  - preorder that encodes reachability in $\rightarrow \cup \rightarrow_b \cup \rightarrow_u$, induces finitely many equivalence classes $(\!(q)\!)$.
- (not necessarily uniform) p-automaton A weak if for all $q \in Q$, either $(\!(q)\!) \cap Q \subseteq \alpha$ or $(\!(q)\!) \cap \alpha = \{\}$.
- acceptance game for $M \in \mathcal{L}(A)$ well-defined for uniform weak p-automata
- acceptance game exponential in size of input M and size of automaton A

Uniform Weak p-automaton A

$$\delta(q_1, \{\mathrm{a}, \mathrm{b}\}) = \delta(q_1, \{\mathrm{a}\}) = q_1 \vee [\![q_2]\!]_{\geq 0.5}$$
$$\delta(q_2, \{\mathrm{b}\}) = \delta(q_2, \{\mathrm{a}, \mathrm{b}\}) = [\![q_2]\!]_{\geq 0.5}$$
$$\delta(q_1, \{\}) = \delta(q_1, \{\mathrm{b}\}) = \delta(q_2, \{\}) = \delta(q_2, \{\mathrm{a}\}) = \mathrm{ff}$$



- A uniform: SCC $(\!(q_1)\!) = \{q_1, q_1 \vee [\![q_2]\!]_{\geq 0.5}\}$ no bounded transitions, SCC $(\!(q_2)\!) = \{q_2, [\![q_2]\!]_{\geq 0.5}\}$ no unbounded transitions, SCC $(\!([\![q_1]\!]_{\geq 0.5})\!) = \{[\![q_1]\!]_{\geq 0.5}\}$ trivial
- A weak: $\alpha = \{q_2\}$.

Expressiveness

Dual of $A = \langle \Sigma, Q, \delta, \varphi^{\mathrm{in}}, \alpha \rangle$: $M \in \mathcal{L}(\mathrm{dual}(A))$ iff $M \notin \mathcal{L}(A)$

$$\mathrm{dual}(A) = \langle \Sigma, \overline{Q}, \overline{\delta}, \mathrm{dual}(\varphi^{\mathrm{in}}), Q \setminus \alpha \rangle$$

- $\overline{Q} = \{\overline{q} \mid q \in Q\}$ and $\overline{\delta}(\overline{q}, \sigma) = \mathrm{dual}(\delta(q, \sigma))$

$$\boxed{\begin{array}{rcl}
\mathrm{dual}(\varphi_1 \overset{*}{\vee} \varphi_2) & = & \mathrm{dual}(\varphi_1) * \mathrm{dual}(\varphi_2) \\
\mathrm{dual}(\varphi_1 * \varphi_2) & = & \mathrm{dual}(\varphi_1) \overset{*}{\vee} \mathrm{dual}(\varphi_2) \\
\mathrm{dual}(\varphi_1 \wedge \varphi_2) & = & \mathrm{dual}(\varphi_1) \vee \mathrm{dual}(\varphi_2) \\
\mathrm{dual}(\varphi_1 \vee \varphi_2) & = & \mathrm{dual}(\varphi_1) \wedge \mathrm{dual}(\varphi_2) \\
\mathrm{dual}(q) & = & \overline{q} \\
\mathrm{dual}(\overline{q}) & = & q \\
\mathrm{dual}([\![q]\!]_{\bowtie p}) & = & [\![\overline{q}]\!]_{\mathrm{dual}(\bowtie p)} \\
\mathrm{dual}(\geq p) & = & > 1 - p \\
\mathrm{dual}(> p) & = & \geq 1 - p
\end{array}}$$

Boolean Connectives, Bisimulation

Let input alphabet Σ be $2^{\mathbb{AP}}$.

- Set of languages accepted by p-automata with Σ is closed under Boolean operations
- Language containment of p-automata with Σ reduces to language emptiness of such p-automata, and vice versa
- For p-automaton $A = \langle 2^{\mathbb{AP}}, Q, \delta, \varphi^{\mathsf{in}}, \alpha \rangle$ and probabilistically bisimilar Markov chains M_1, M_2 over $\mathbb{AP}$:

$$M_1 \in \mathcal{L}(A) \text{ iff } M_2 \in \mathcal{L}(A)$$

Representing Markov chains

Convert Markov chain $M = (S, P, L, s^{\text{in}})$ into p-automaton

$$A_M = \langle 2^{\mathbb{AP}}, Q, \delta, \varphi^{\text{in}}, \alpha \rangle$$

- $\mathcal{L}(A_M)$ set of Markov chains bisimilar to M
- conversion uses linear order on each successor set:

$$\begin{array}{rcl}
Q & = & \{(s, s') \in S \times S \mid P(s, s') > 0\} \\
\delta((s, s'), L(s)) & = & *(\llbracket (s', s'') \rrbracket_{\geq P(s', s'')} \mid P(s', s'') > 0) \\
\delta((s, s'), \sigma) & = & \text{ff} \qquad\qquad \text{if } \sigma \neq L(s) \\
\varphi^{\text{in}} & = & *(\llbracket (s^{\text{in}}, s') \rrbracket_{\geq P(s^{\text{in}}, s')} \mid P(s^{\text{in}}, s) > 0) \\
\alpha & = & Q
\end{array}$$

- Only bounded transitions and $*$ operator, so uniform weak

Representing PCTL formulas

Convert PCTL formula ϕ over $\mathbb{AP}$ into p-automaton

$$A_\phi = \langle 2^{\mathbb{AP}}, \mathrm{cl_t}(\phi) \cup \mathbb{AP}, \rho_X, \rho_\epsilon(\phi), F \rangle$$

- $\mathcal{L}(A_\phi)$ exactly Markov chains satisfying ϕ
- resembles translation from CTL to alternating tree automata:
  - $\mathrm{cl_t}(\phi)$ set of temporal subformulas of ϕ
  - F consists of $\mathbb{AP}$ and all ψ of $\mathrm{cl_t}(\phi)$ not of form $\psi_1 \mathrm{U} \psi_2$
- function ρ_X: unfolds fixed points, replaces PCTL [] with $[\![]\!]$
- function ρ_ϵ: for initial state, replaces [] with $[\![]\!]$

Example for $\varphi = [\mathrm{a}\,\mathrm{U}\,[\,\mathrm{X}\,\mathrm{b}]_{>0.5}]_{\geq 0.3}$

$$A_\varphi = \langle 2^{\{\mathrm{a},\mathrm{b}\}}, \mathrm{cl}_{\mathrm{t}}(\varphi) \cup \{\mathrm{a}, \mathrm{b}\}, \rho_X, \rho_\epsilon(\varphi), F\rangle$$

- $\mathrm{cl}_{\mathrm{t}}(\varphi) = \{\mathrm{a}\,\mathrm{U}\,[\,\mathrm{X}\,\mathrm{b}]_{>0.5},\ \mathrm{X}\,\mathrm{b}\}$
- $\rho_\epsilon(\varphi) = (\mathrm{a} \wedge [\![\mathrm{a}\,\mathrm{U}\,[\,\mathrm{X}\,\mathrm{b}]_{>0.5}]\!]_{\geq 0.3}) \vee [\![\,\mathrm{X}\,\mathrm{b}]\!]_{>0.5}$
- $F = \{\,\mathrm{X}\,\mathrm{b}, \mathrm{a}, \mathrm{b}\}$
- $\rho_X(\,\mathrm{X}\,\mathrm{b}) = \mathrm{b}$
- $\rho_X(\mathrm{a}\,\mathrm{U}\,[\,\mathrm{X}\,\mathrm{b}]_{>0.5}) = (\mathrm{a} \wedge \mathrm{a}\,\mathrm{U}\,[\,\mathrm{X}\,\mathrm{b}]_{>0.5}) \vee [\![\,\mathrm{X}\,\mathrm{b}]\!]_{>0.5}$

p-Automata Are More Expressive

- p-automata more expressive than Markov chains (trivial)
- Routine (counting argument) to show that p-automata are more expressive than PCTL formulas
- Would like to capture a fixed-point logic that corresponds to p-automata (not yet done, don’t yet know how)

Simulation

Under-approximating language containment

- decidability status of $\mathcal{L}(A) \subseteq \mathcal{L}(B)$ not known at present
- seek “efficient” simulation $A \leq B$ between p-automata such that $A \leq B$ implies $\mathcal{L}(A) \subseteq \mathcal{L}(B)$
- we developed such a simulation notion that borrows from
    - fair simulation
    - simulation for alternating word automata
    - probabilistic bisimulation
    - and from our acceptance games $M \in \mathcal{L}(A)$
- For A and B finite, or for A representing some Markov chain, the above under-approximation holds

Conclusion

What We Did

- presented notion of p-automaton A which accepts or rejects an entire Markov chain M as input
- reduced acceptance games for $M \in \mathcal{L}(A)$ to solving a weak stochastic game, at most exponential in size of automaton and Markov chain
- showed p-automata to be closed under Boolean operations, their languages to be closed under bisimulation
- represented both Markov chains and PCTL formulas as p-automata
- developed notion of simulation that “efficiently” under-approximates language inclusion

What We Want To Do

- Decidability of non-emptiness for qualitative p-automata? (Only thresholds > 0 and ≥ 1.)
- Decidability of non-emptiness for full p-automata?
- Determinism and non-determinism for p-automata?
- How to define and solve acceptance game for non-uniform p-automata?
- p-automata as acceptors of Markov decision processes?
- Retrofit existing tools with support for p-automata?
- How to use p-automata for CEGAR?

Thank You for Your Kind Attention

Questions?