Modal transition systems: a foundation for three-valued program analysis

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Outline

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3. Property refutation and verification

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4. Semantic foundations

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6. Conclusions and future work

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Modal transition systems express such aspects and generalize Dams's work (Dams 1996) to *partial* systems.

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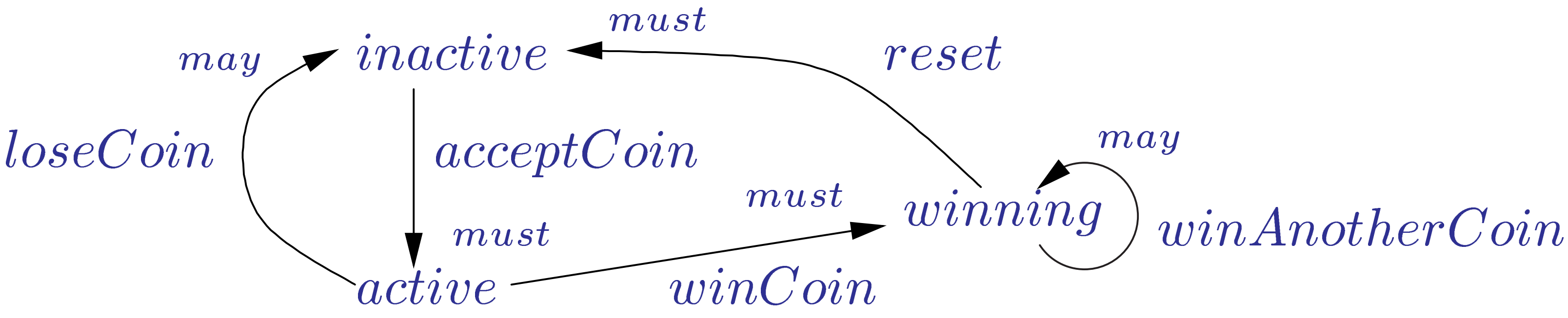
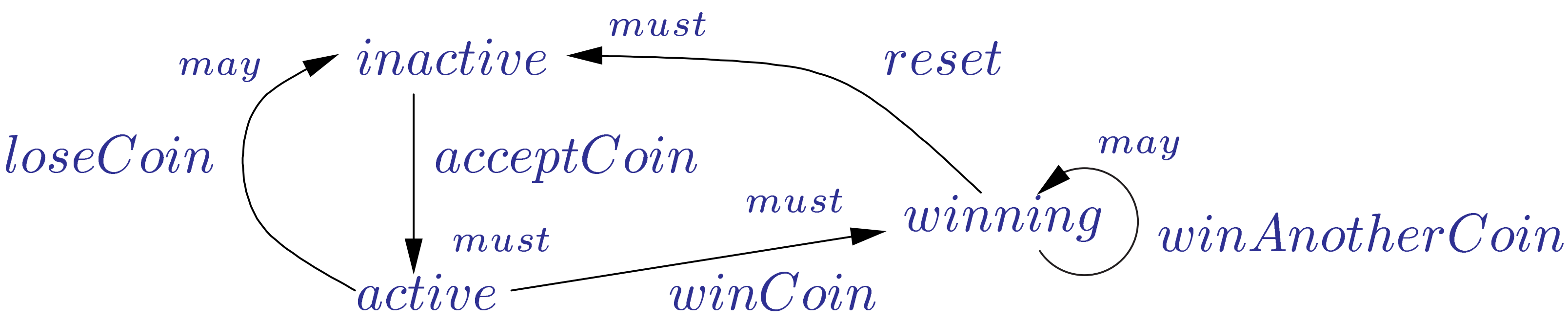
  - an a-transition from state s to s' must not occur (has to be absent from all refinements): not $s \to^{a}_{\Diamond} s'$;
- impose a consistency condition: $\to_{\Box} \subseteq \to_{\Diamond}$ (all required behavior is also possible).

A slot machine: specification



Consistency condition: *must*-transitions comprise $\rightarrow_\Box$, *may*- and *must*-transitions comprise $\rightarrow_\Diamond$.

A slot machine: specification and refinement



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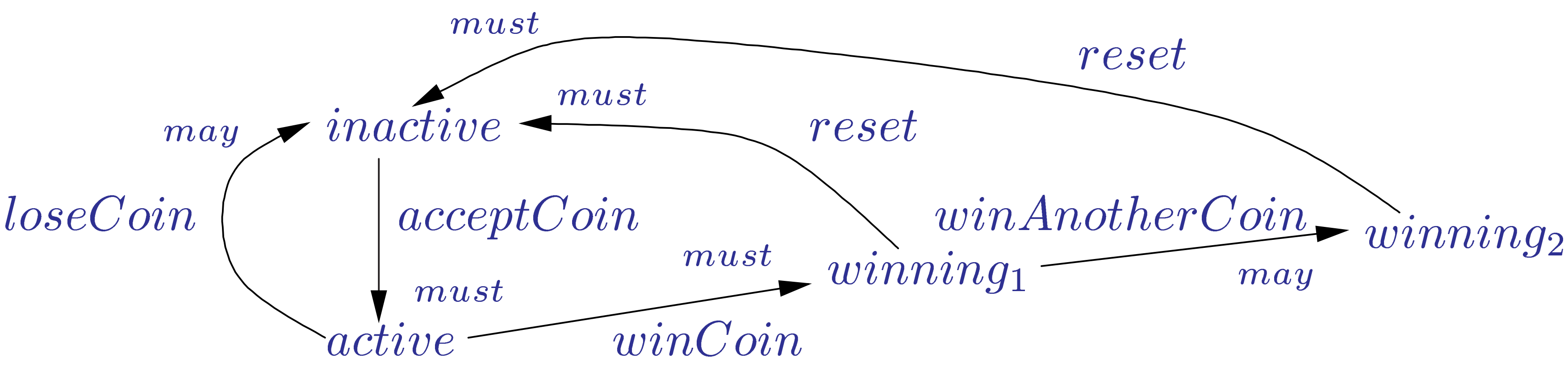

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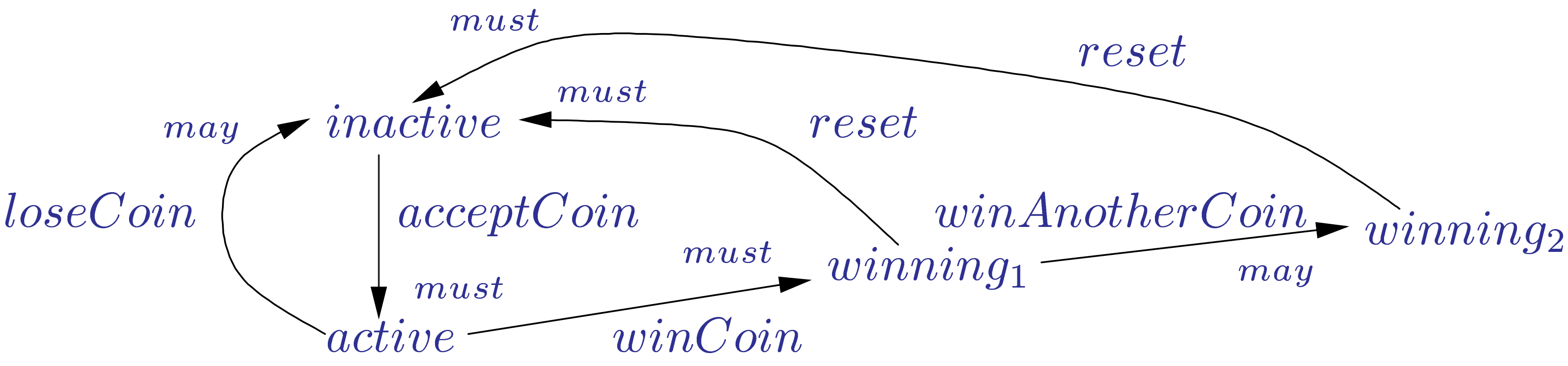
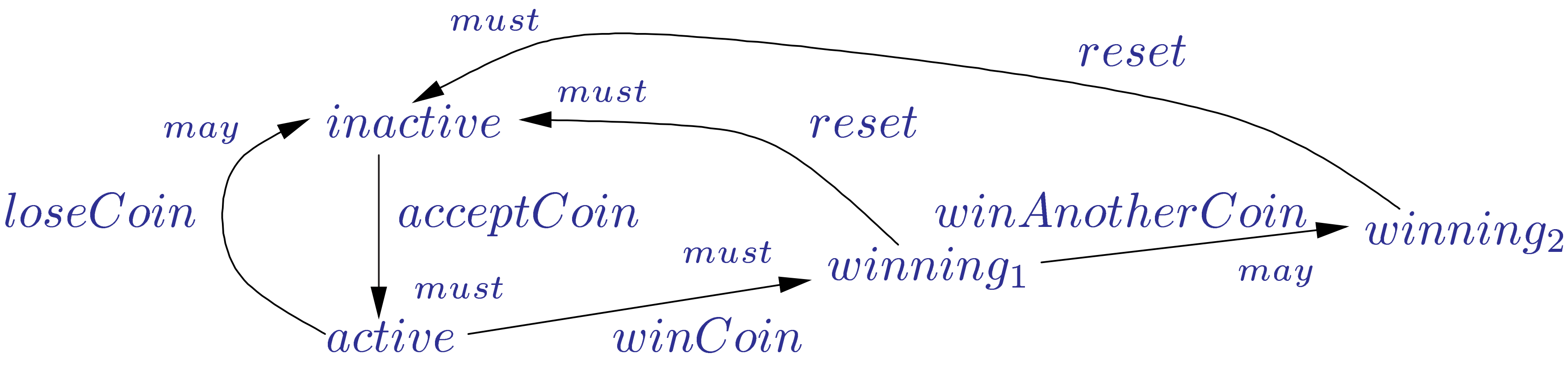
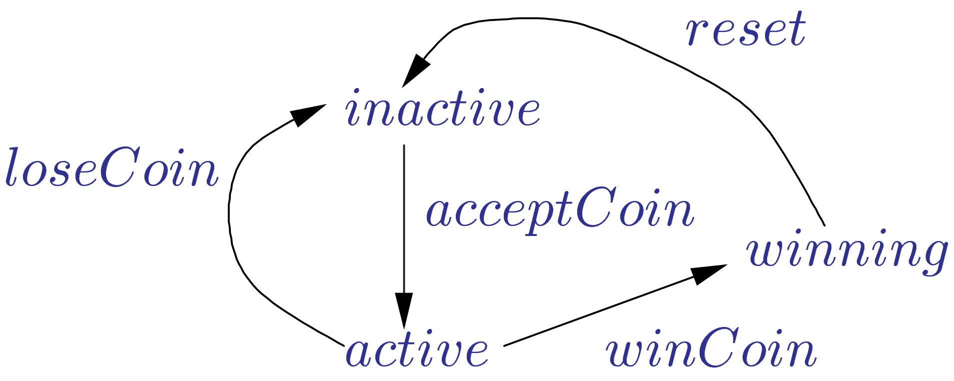


A slot machine: refinement and implementation



In implementations, all transitions are *must*-transitions:



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We write $\prec$ for the greatest such relation.

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Related work: Cousot & Cousot (2000) define a *universal* and an *existential* abstraction; the latter one, used *in isolation* restricts sound reasoning to *safety* properties.

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Kozen's modal mu-calculus (Kozen 1983) (subsumes tool logics):

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The definition of $\models^{may}$ is as for $\models^{must}$, but we swap $must \leftrightarrow may$ and $\Box \leftrightarrow \Diamond$.

E.g., $s \models^{may} \langle a \rangle\, \phi$ iff for some s', $s \rightarrow^{a}_{\Diamond} s'$ and $s' \models^{may} \phi$.

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means either refuting $t \models^{may} \phi$ or verifying $t \models^{must} \psi$.

Example refutation

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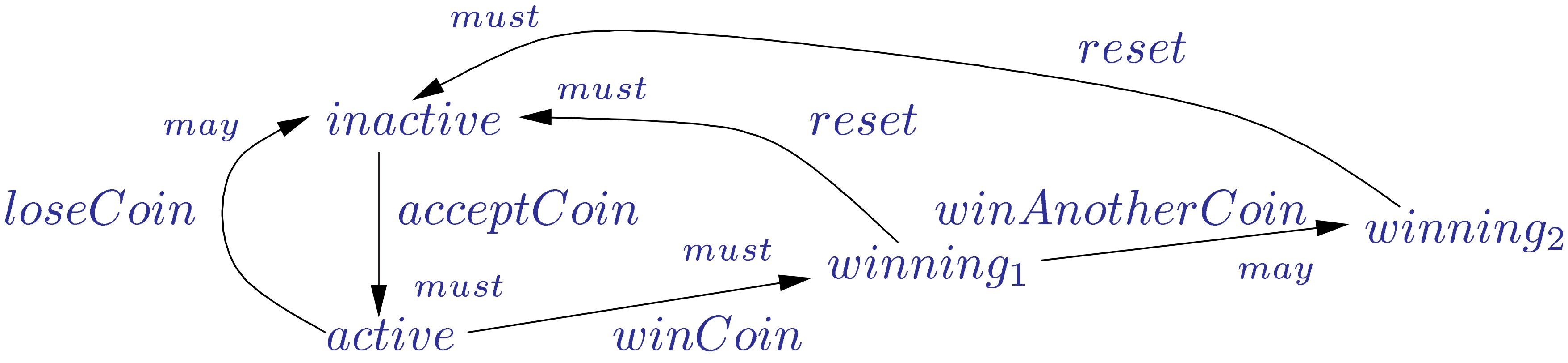

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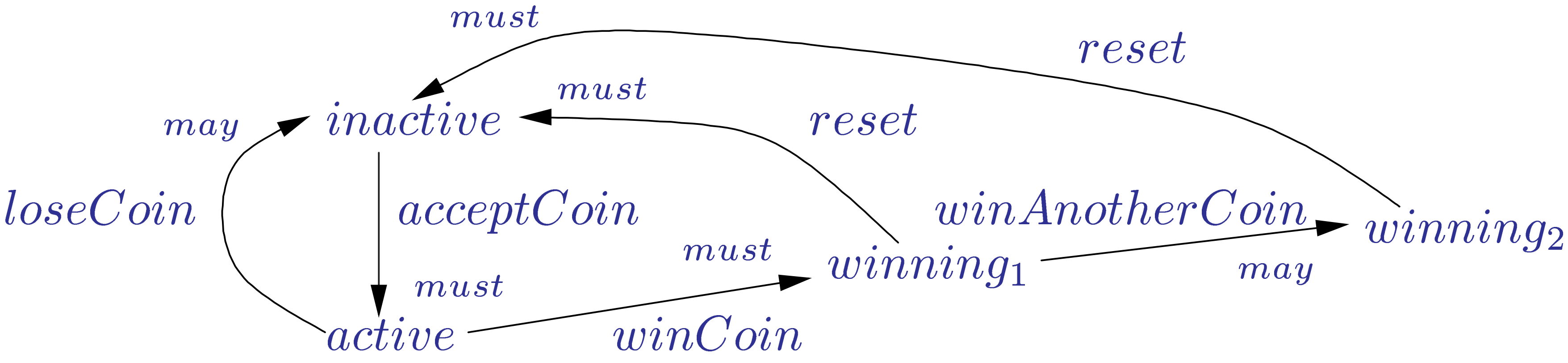

This makes sense as we have
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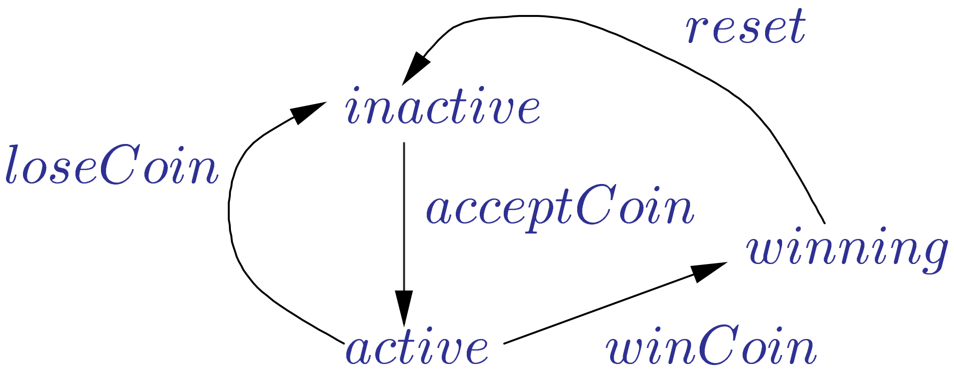

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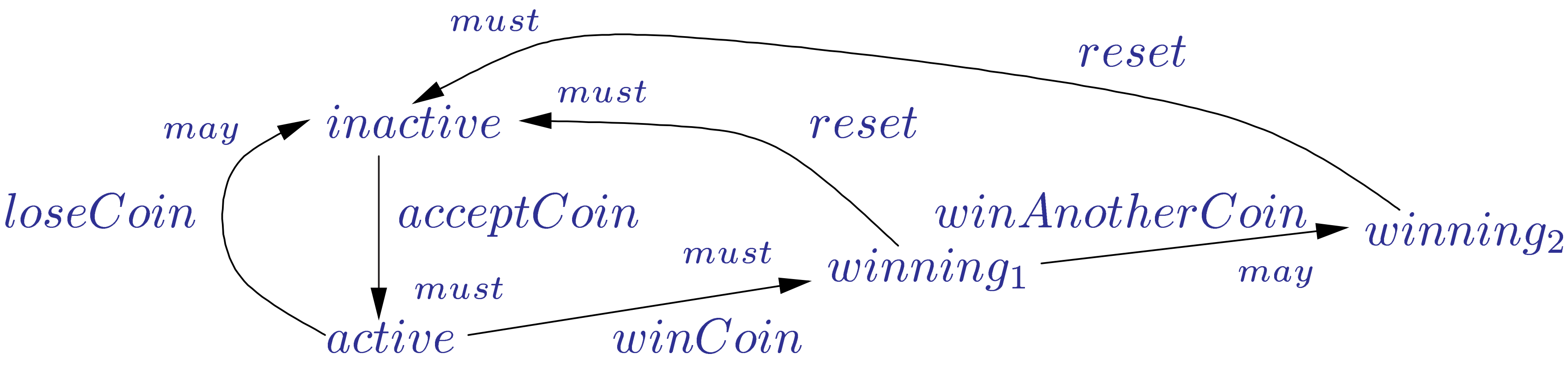

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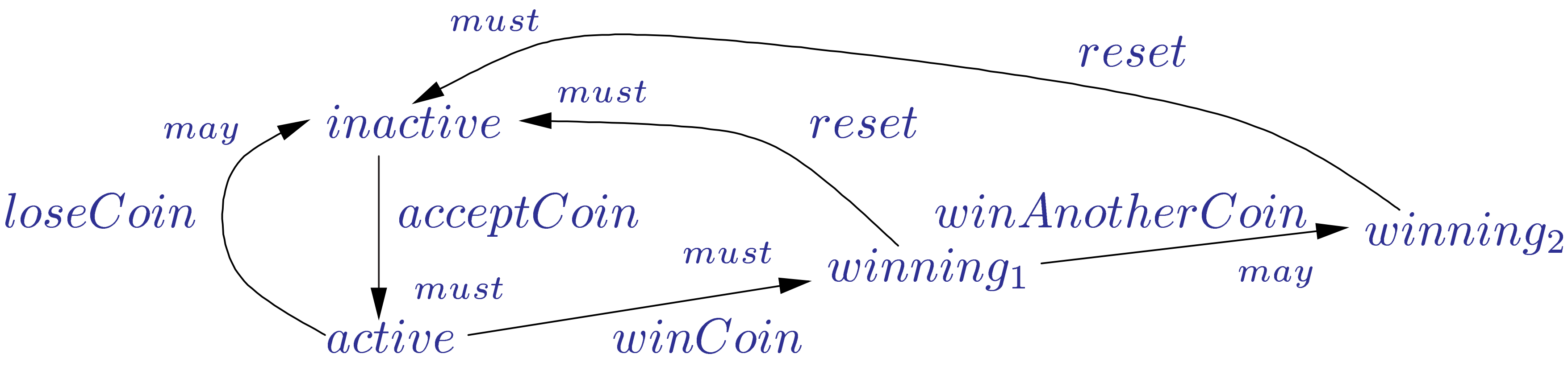
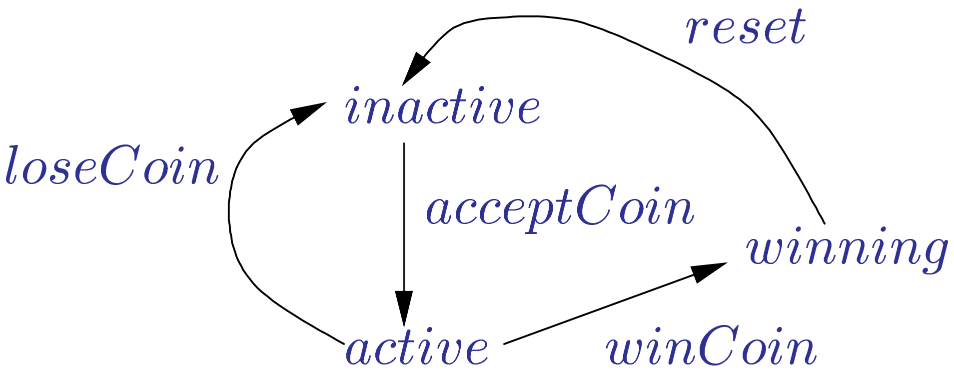


So this property holds for all implementations, in particular for



4 Semantic foundations

Mixed power domain (Gunter 1992 & Heckmann 1993):

$$\mathbf{M}[D] \stackrel{\text{def}}{=} \{(L, U) \in \mathcal{L}[D] \times \mathcal{U}[D] \mid L = {\downarrow}(L \cap U)\}.$$

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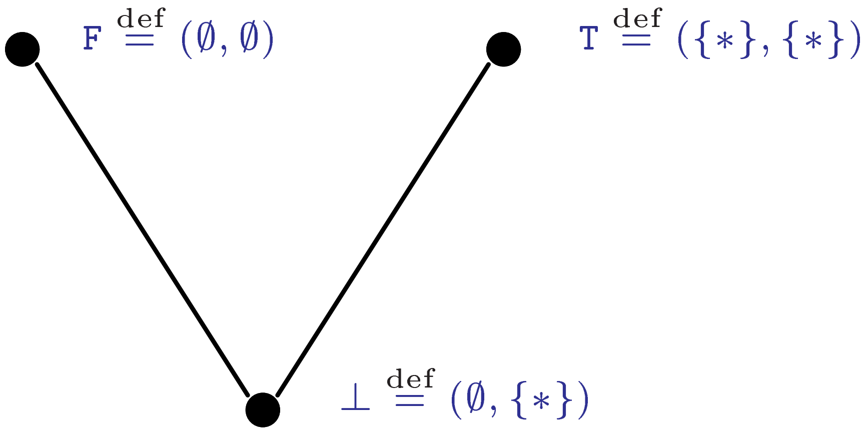

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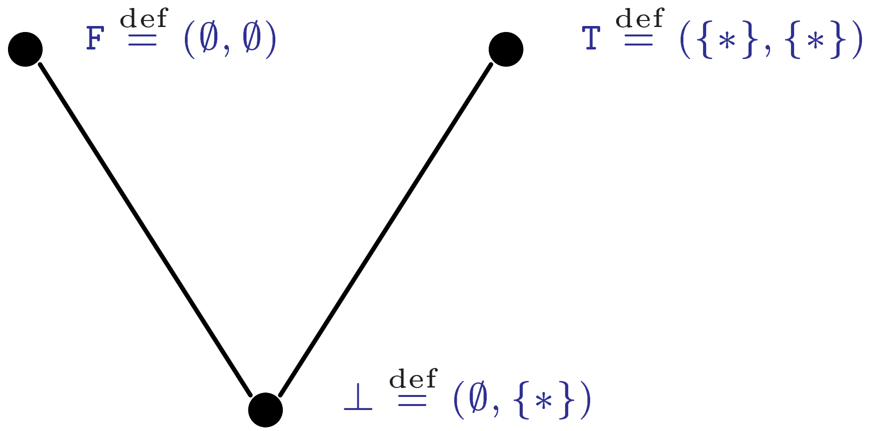

(The pair $(\{*\}, \emptyset)$ violates the consistency condition.)

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4. Domain-theoretic machinery a powerful tool for reasoning about higher-order and intuitionstic aspects.

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- are labeled transition systems $\mathcal{E} = (\Sigma_E, \mathtt{Act}, \rightarrow)$ together with a *divergence relation* $\uparrow\ \subseteq \Sigma_E \times \mathtt{Act}$;

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- $\mathcal{E}$ as a modal transition system $\mathcal{M}[\mathcal{E}]$:

$$
\begin{array}{lll}
s \rightarrow_a^{must} s' & \text{iff} & s \rightarrow_a s' \\
s \rightarrow_a^{may} s' & \text{iff} & s \rightarrow_a s' \text{ or } s \uparrow a.
\end{array}
$$

Partial bisimulation as abstraction

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Conclusion: The models, abstraction, and temporal logic semantics for partial bisimulation are a special instance of the corresponding notions for modal transition systems.

(b) Partial Kripke structures

- Bruns & Godefroid (1999) abstract propositional information at states ($L(s, p) \in \{\mathtt{F}, \mathtt{T}\}$) with a three-valued logic $\mathbf{3} \stackrel{\text{def}}{=} \{\perp < \mathtt{F}, \mathtt{T}\}$;

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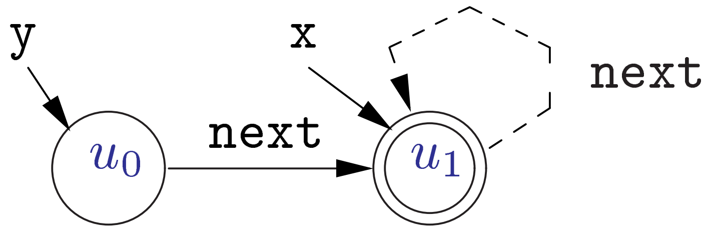
- model check CTL formulas over such partial state spaces.
- The order-isomorphism

$$
\begin{array}{rcll}
\mathbf{M}[\Sigma] & \cong & \Sigma \to \mathbf{3} & \qquad (\Sigma \text{ discrete}) \\
(f^{-1}\{\text{true}\}, f^{-1}\{\bot, \text{true}\}) & \cong & f &
\end{array}
$$

  recaptures their models, abstraction relation, and CTL semantics as an instance of the corresponding notions of modal transition systems.

(c) Modal shape graphs

Shape analysis: contents of heap storage is approximated by a graph. The graph



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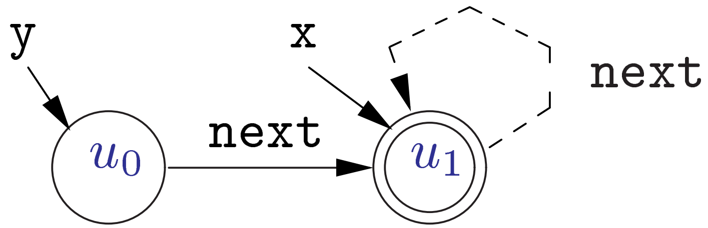

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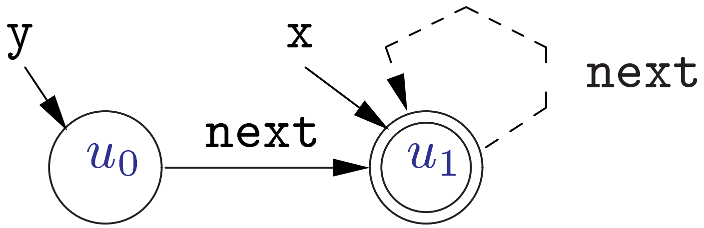

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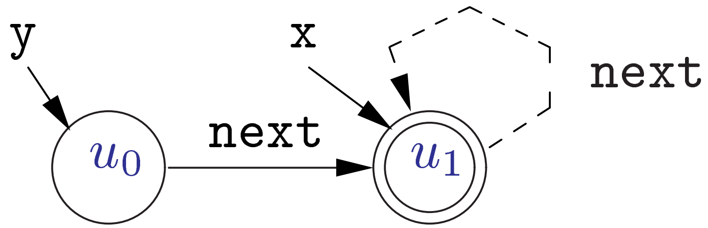

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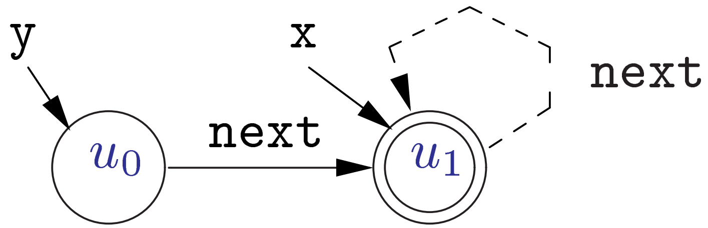

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- Local variables, `x` and `y`, point to objects.
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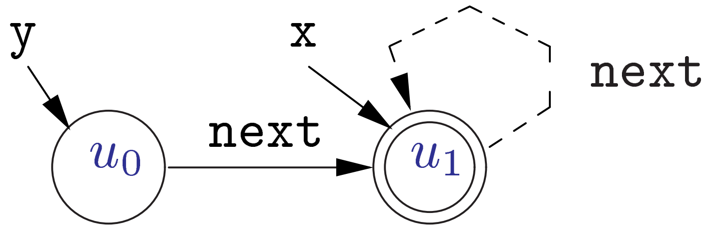
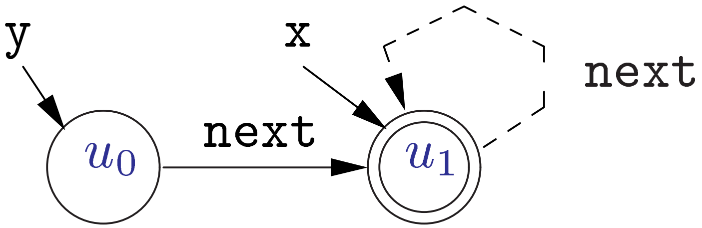
- Double-circled objects are "summary nodes", possibly representing more than one object.

The shape graph



The shape graph and its coding via predicates

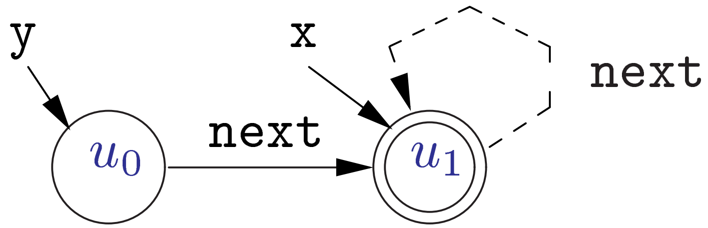


M. Sagiv, T. Reps, and R. Wilhelm (1999):

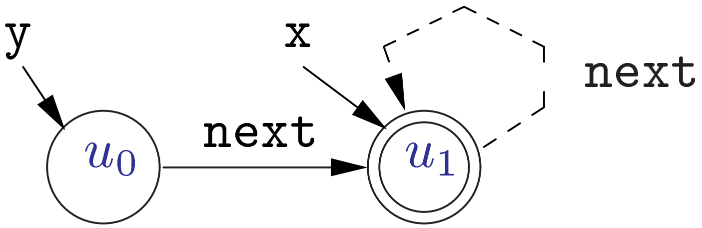
(Kleene ordering $0 < 1/2 < 1$)

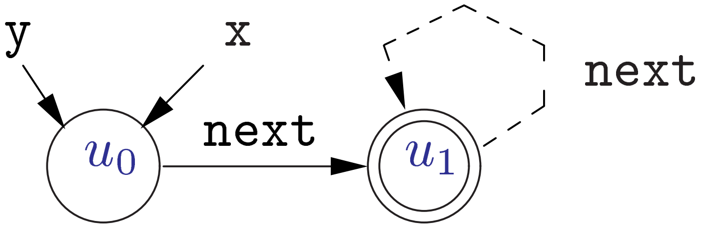
$$
\begin{array}{ll}
x(u_0) = 0 & next(u_0, u_0) = 0 \\
x(u_1) = 1 & next(u_0, u_1) = 1 \\
y(u_0) = 1 & next(u_1, u_0) = 0 \\
y(u_1) = 0 & next(u_1, u_1) = 1/2 \\
sm(u_0) = 0 & \\
sm(u_1) = 1 &
\end{array}
$$

The same shape graph



The same shape graph and the effect of `x = y`





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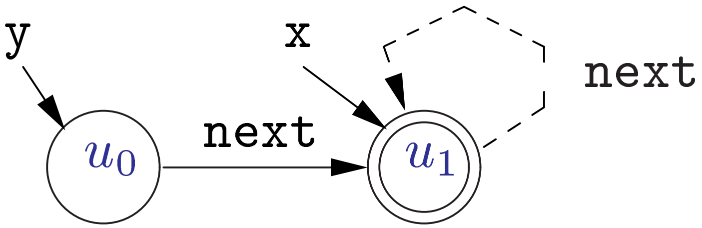
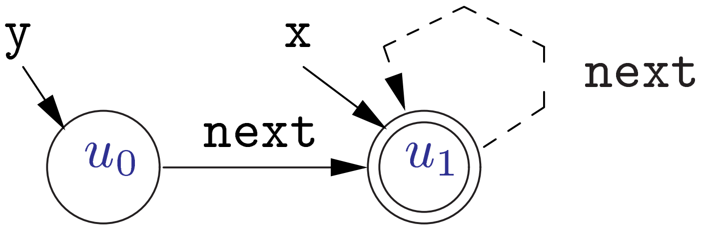
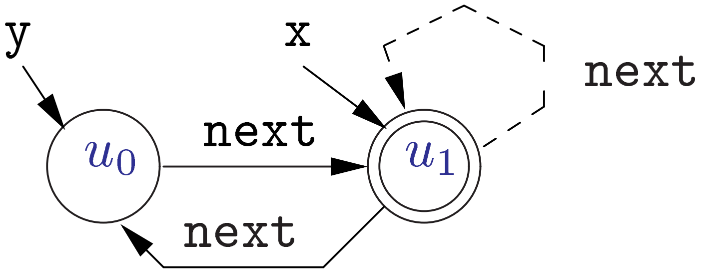


The shape graph and the effect of `x.next = y`





Notice: The *may*-transition out of the summary node persists.

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- their embedding theorem expresses soundness of verification;
- their `focus` operation, `instrumentation predicates`, and `coerce` operation have connections to assume-guarantee reasoning and generalized model checking (Bruns & Godefroid 2000);
- plenty of practically important properties — such as shape invariants, direction relationship, interference relationship, aliasing, cycles, etc — have simple encodings in CTL or LTL.

Conclusions and future work

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3. exploit all four possible interactions of path quantifiers and state-transition modalities.

4. await novel applications, e.g., compositional pointer analysis (Whaley & Rinard 1999) and computing abstract models (Godefroid, Huth, and Jagadeesan 2001).