Semantic Minimizations for Temporal Logics

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Joint work with Patrice Godefroid

1. Propositional logic
2. Temporal logics
3. References & Acknowledgments

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- **space-time:** "packets will get through in ad-hoc network"

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- $M \models^{th} \phi$ and $M \models^{th} \neg\phi$ record consensus of $\gamma(M)$ on ϕ

Thorough semantics as validity problem

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- Under-approximation as soundness: for all M and ϕ, $M \models \phi$ implies $M \models^{th} \phi$

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- Proof uses mutually recursive CNFs. Establishes that $\models$ is functionally complete for functions $\{0, 1/2, 1\}^n \to \{0, 1/2, 1\}$ that are monotone in information ordering $1/2 < 0, 1$

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  - non-monotone $(p \to q) \wedge (q \to p)$
- $p \vee \neg p$ not pessimistically self-minimizing

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- This talk sheds some light on only some of these questions.

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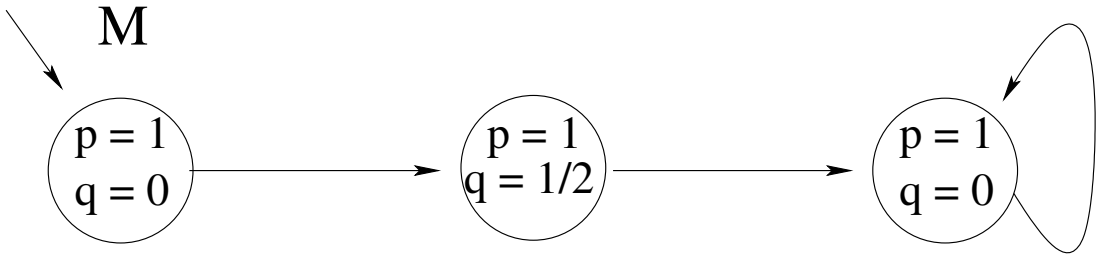
- refinement, thorough & compositional semantics also extended over time

Example of partial Kripke structure

```
void someArithmetic() {
  x,y = 1,0;
  x,y = 2*f(x),f(y);
  x,y = x+1,0; }
```



- `f: int -> int` unknown function
- model M above obtained by predicate abstraction with respect to p = "x is odd" and q = "y is odd"
- $M \not\models \mathsf{EG}\neg q \vee \mathsf{EF}(\neg p \wedge q)$ but $M \models^{th} \mathsf{EG}\neg q \vee \mathsf{EF}(\neg p \wedge q)$, note the mixed polarity for q

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  1. $\forall\, q \in \mathrm{AtomProp}\colon L_1(s_1, q) \leq_I L_2(s_2, q)$,

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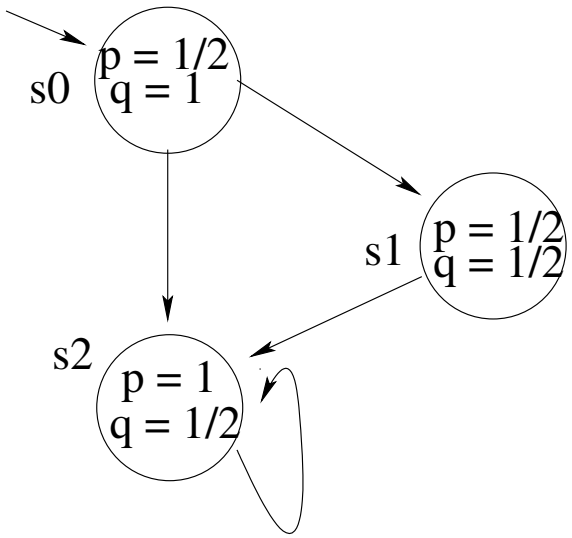
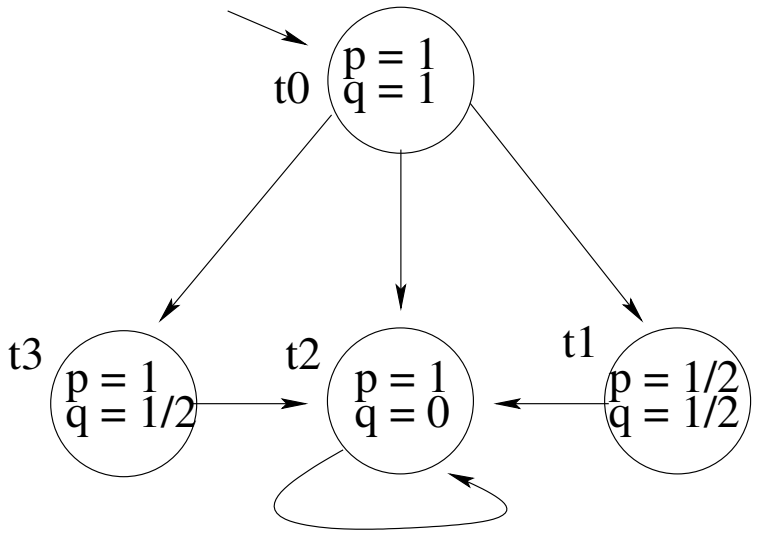
Example of refinement





- t0 refines s0, t1 refines s1, and t2 and t3 refine s2

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- write $\phi_\exists$ ($\phi_\forall$) if the negation normal form of ϕ is known to be an existential (resp., universal) one

Patterns of temporal self-minimization

$$
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\mathrm{ps} & ::= & \mathcal{M} \mid \mathcal{R} \mid \neg\mathrm{os} \mid \mathrm{ps} \wedge \mathrm{ps} \mid \mathrm{ps}_{\forall\#} \vee \mathrm{ps}_{\forall\#} \\
 & & \mathsf{EX}\mathrm{ps} \mid \mathsf{AX}\mathrm{ps} \mid \mathsf{EG}\mathrm{ps} \mid \mathsf{AG}\mathrm{ps} \\
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\mathrm{os} & ::= & \mathcal{M} \mid \mathcal{R} \mid \neg\mathrm{ps} \mid \mathrm{os} \vee \mathrm{os} \mid \mathrm{os}_{\exists\#} \wedge \mathrm{os}_{\exists\#} \\
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 & & \mathsf{EG}\mathrm{os}_{\exists} \mid \mathsf{E}[\mathrm{os}_{\exists}\mathsf{U}\mathrm{os}] \mid \mathit{ref}(\mathit{OS})
\end{array}
$$

- $\mathcal{M}$ ranges over "monotone" ϕ
- $\mathcal{R}$ ranges over all ϕ_M
- $\mathit{ref}(\mathcal{O})$ is defined as $(\bigwedge_{O\in\mathcal{O}} \mathsf{EX}O) \wedge (\mathsf{AX}\bigvee\mathcal{O})$
- various proof techniques,
  e.g. multiple-model checking for $\mathcal{R}$ in os

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- consequence: $M \models^{th} \phi$ for PML in ALOGTIME in size of model M

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- this existence result does not seem to have consequences on complexity of $M \models^{th} \phi$ for modal mu-calculus

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- so not all formulas of CTL or CTL* have semantic minimizations in CTL*

Some references

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- Detailed references to related and originating work (e.g. by *Kim Larsen*) can be found in these papers.

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