Hybrid logics, abstraction, and probabilities

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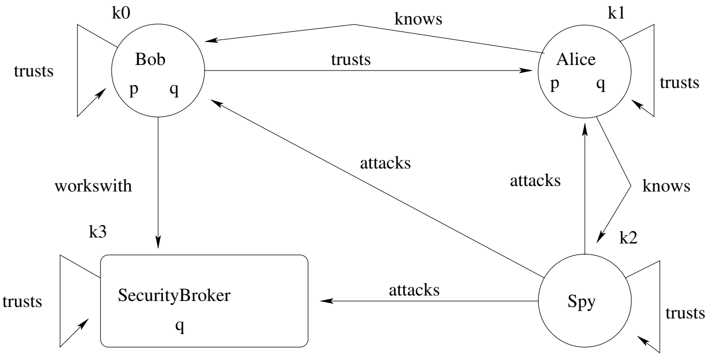
Hybrid models and logics



- nominals, e.g. Bob, true at exactly one world
- formulas may refer to nominals n, e.g. "at n, ϕ" or "there is a cycle: $\exists n$, $EF\ n$" (not expressible in modal logic)
- an abstraction of the model above

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- **behavior:** "an audio plug-in may be present in a browser"
- **interface:** "`requires balance >= 0`"
- **topology:** "a node may have no neighbor in its broadcast range"
- **space-time:** "packets will get through in an ad-hoc network if no node is ever hostile."

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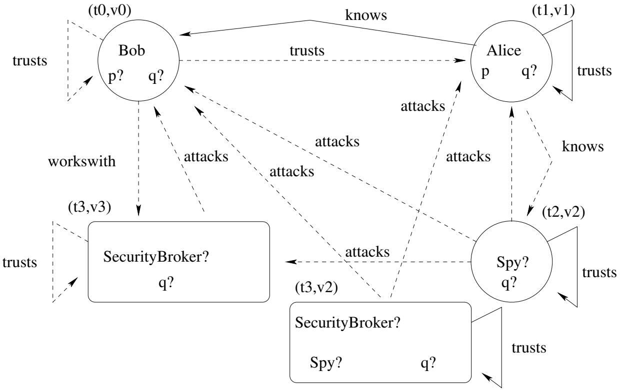
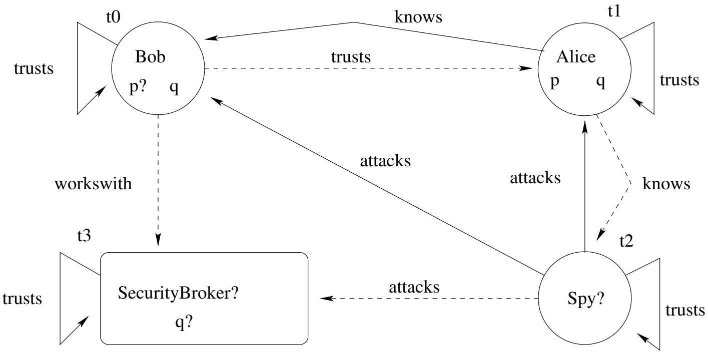
- loss of precision: $[p \mapsto 1/2] \models^{th} p \vee \neg p$ but $[p \mapsto 1/2] \not\models^{val} p \vee \neg p$

Under-specifying temporal models



- abstracts ▸ initial model, abstracts ▸ another model
- ? and dashed lines $\equiv 1/2$, solid lines and atoms with no ? $\equiv 1$
- SecurityBroker possibly true at two worlds, refinement can only realize one choice

Refinement



a refinement of the model on previous slide, refinement preserves guarantees and introduces no “new” possibilities

Compositional and thorough semantics

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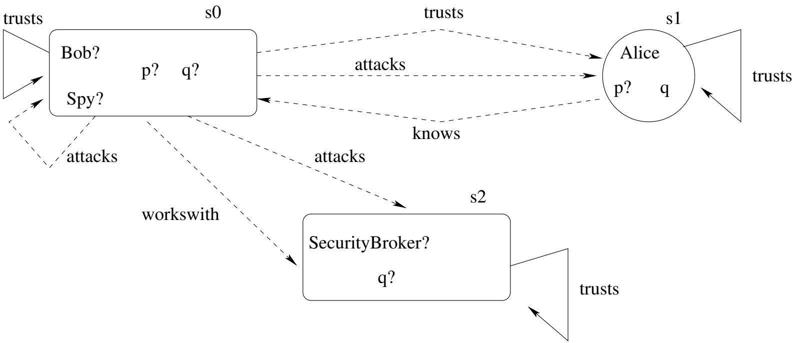
- compositional semantics sound: $M \models^{val} \phi \Rightarrow M \models^{th} \phi$ for all M, ϕ
- compositional semantics incomplete: inherited from propositional logic, e.g. $\langle\alpha\rangle\phi \vee \neg\langle\alpha\rangle\phi$

Example re-visited



- $s_0 \not\models^{val} \langle \mathrm{attacks} \rangle Alice \vee \neg \langle \mathrm{attacks} \rangle Alice$ as $(s_0, \mathrm{attacks}, s_1)$ possible but $(s_0, \mathrm{attacks}, \cdot)$ not guaranteed

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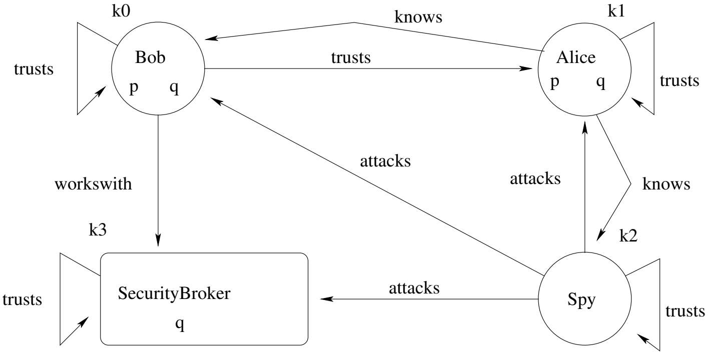
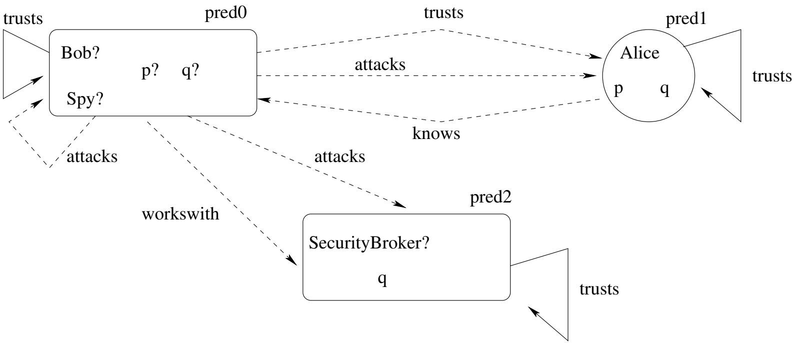
- obtain: $\forall n \in Nom\colon L^{\forall}(n)$ empty or singleton; if singleton, then $L^{\exists}(n) = L^{\forall}(n)$;
  morale: turn these constraints into model axioms, even for relational abstractions

Example of predicate abstraction



- predicate abstraction of the model above with $\phi_1 = Alice$ and $\phi_2 = SecurityBroker$ results in ▸ this abstraction

Example of predicate abstraction con't



- the predicate abstraction of the ▸ concrete model with $\phi_1 = \mathit{Alice}$ and $\phi_2 = \mathit{SecurityBroker}$

Interlude: multiple-model checking

(Ongoing work with Altaf Hussain.)

- $M \models^{val} \phi$ and $M \models^{th} \phi$ reason about set $\mathcal{C}(M) = \{N \text{ 2-valued} \mid N \text{ refines } M\}$, link to "abstract interpretation."

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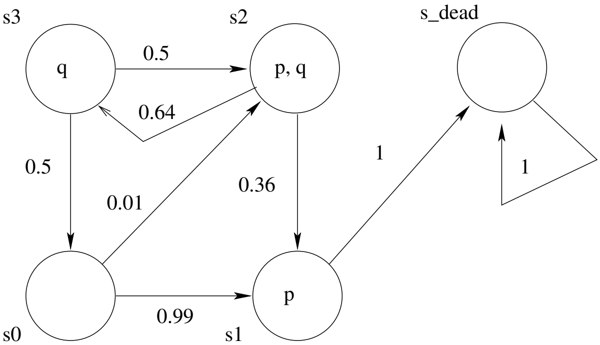
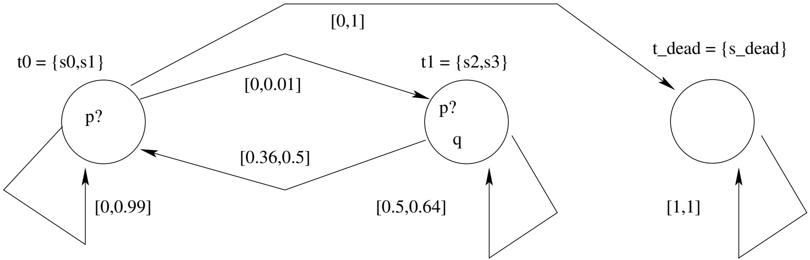
- Seek good analogue of efficient $M \models^{val} \phi$ in this setting.

A probabilistic system



- discrete-time labeled Markov chain
- transition = probability measure over state space

An abstraction of that probabilistic system



- predicate abstraction of model on previous slide
- intervals approximate non-additive Choquet capacities
- nominals (if present) are treated as before

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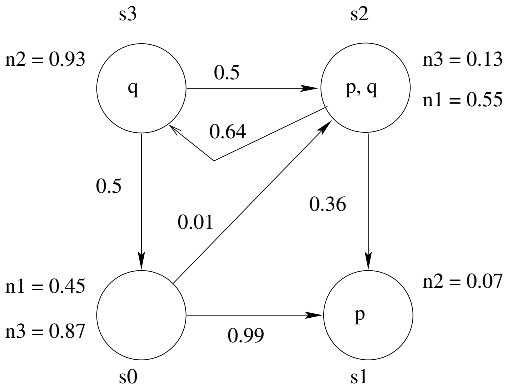
- Optimal finite state abstractions for finite set of properties of some probabilistic logic?

Probabilistic nominals



- probabilistic system as before
- but now nominals governed by probability distribution
- atomic events for nominals of the form “*n* is at state *s*”

Hybrid probabilistic computation tree logic

$$\phi ::= \ldots\ PCTL\ \ldots \mid @_n^{\sqsupseteq p}.\phi \mid \downarrow(n,\delta).\phi \mid \exists(n,\Delta').\phi$$

$\Delta' \subseteq \Delta$ set of probability measures

- "at n, ϕ holds with probability $\sqsupseteq p$:" $s \models_L @_n^{\sqsupseteq p}.\phi$ iff $\sum\{L(n,s') \mid s' \models_{L[n\mapsto\delta_{s'}]} \phi\} \sqsupseteq p$; reflects conditional probabilities of n's being at s'; where $L[n \mapsto \delta](n) = \delta$ and $L[n \mapsto \delta](m) = L(m)$ if $m \neq n$

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- "if n is rebound to δ, ϕ holds:"
  $s \models_L \downarrow(n,\delta).\phi$ iff $s \models_{L[n \mapsto \delta]} \phi$
- "it is possible to rebind n in Δ' such that ϕ holds:"
  $s \models_L \exists(n,\Delta').\phi$ iff for some $\delta \in \Delta'$: $s \models_{L[n \mapsto \delta]} \phi$

expressiveness of hybrid PCTL

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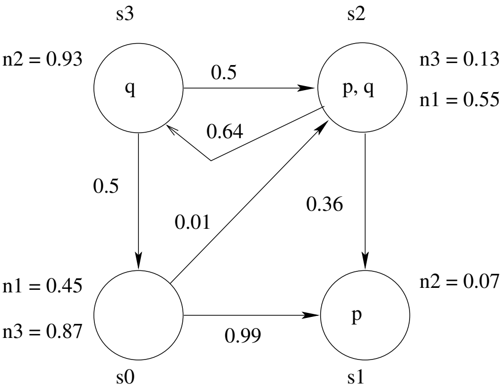
- subsumes $(M, s) \models_L \exists n.\phi$ through $(M, s) \models_L \exists(n, \{\delta_t \mid t \in \Sigma\}).\phi$
- can express probabilistic recurrence, e.g. that state s is on a cycle with probability at least .9999, as

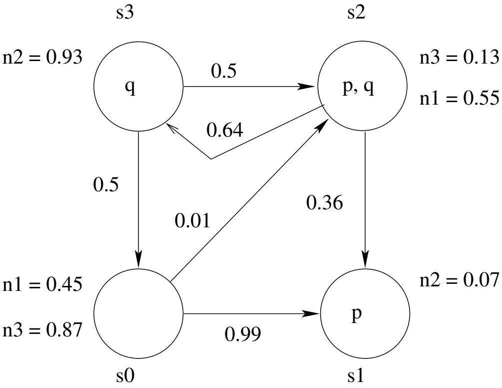
$$s \models_L \downarrow(n, \delta_s).[\mathit{true}\,\mathrm{U}\,n]_{\geq .9999}$$

Example probabilistic check



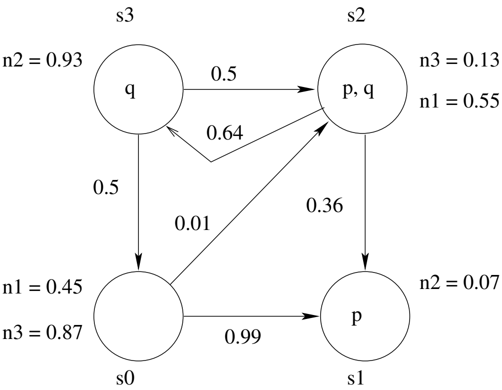
check $s_3 \models_L @^{>0.1}_{n_3}[true \, \mathrm{U} \, n_3]_{\geq 0.01}$ i.e.
is sum of all $L(n_3, s)$, with $s \models_{L[s \mapsto \delta_s]} [true \, \mathrm{U} \, n_3]_{\geq 0.01}$, > 0.1?
only s_0 and s_2 are relevant states s here

Example probabilistic check (2)



at s_0 for $L[n_3 \mapsto \delta_{s_0}]$, the probability that s_0 is on a cycle is $0.01 \cdot 0.64 \cdot 0.5 \cdot (\sum_{i=0}^{\infty}(0.64 \cdot 0.5)^i) = 0.00948529 \cdots \not\geq 0.01$ so $L(s_0, n_3) = 0.87$ does not contribute to that sum

Example probabilistic check (3)



at s_2 for $L[n_3 \mapsto \delta_{s_2}]$, the probability that s_2 is on a cycle is $0.64 \cdot 0.5 + 0.64 \cdot 0.5 \cdot 0.01 = 0.3232 \geq 0.01$ so $L(s_2, n_3) = 0.13$ is only contributor to that sum $\Rightarrow s_3 \models_L @_{n_3}^{>0.1}[\mathit{true}\ \mathrm{U}\ n_3]_{\geq 0.01}$ holds as $0.13 > 0.1$

Some references

- Huth, M. Abstraction and Probabilities for Hybrid Logics. *Electronic Notes in Theoretical Computer Science* 112 (2005) 61–76.
- Huth, M. On Finite-State Approximants for Probabilistic Computation Tree Logic. To appear in *Theoretical Computer Science*, 2005.
- Hussain, A. & Huth, M. On model checking multiple hybrid views. Extended version of paper given at the 1th International Symposium on Leveraging Applications of Formal Methods, Paphos, Cyprus, October 2004. Invited journal submission.

- Detailed references to related and originating work (e.g. by Kim Larsen) can be found in these papers.