An abstraction framework for mixed non-deterministic and probabilistic systems

Michael Huth, Computing, Imperial College London.

Acknowledgments: Christel Baier, Marta Kwiatkowska, Roberto Segala, and many more.

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- lists caveats for effective use of abstract MCPs; and
- present research challenges for abstract MCPs of mixed models.

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- $\models: \mathcal{M} \times \Phi \to \texttt{Ans}$ is a specification of how to compute answers to queries over models.

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We denote this MCP by MDP subsequently.

An abstraction in MDP

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int aRandomUpdate(x : int) { // D. Monniaux
// pre-cond.: 0 <= x <= 2
 for (i = 0; i < 5; i++) {
  // coin_flip() returns 0 or 1 with prob. 0.5
  x = x + coin_flip(); }
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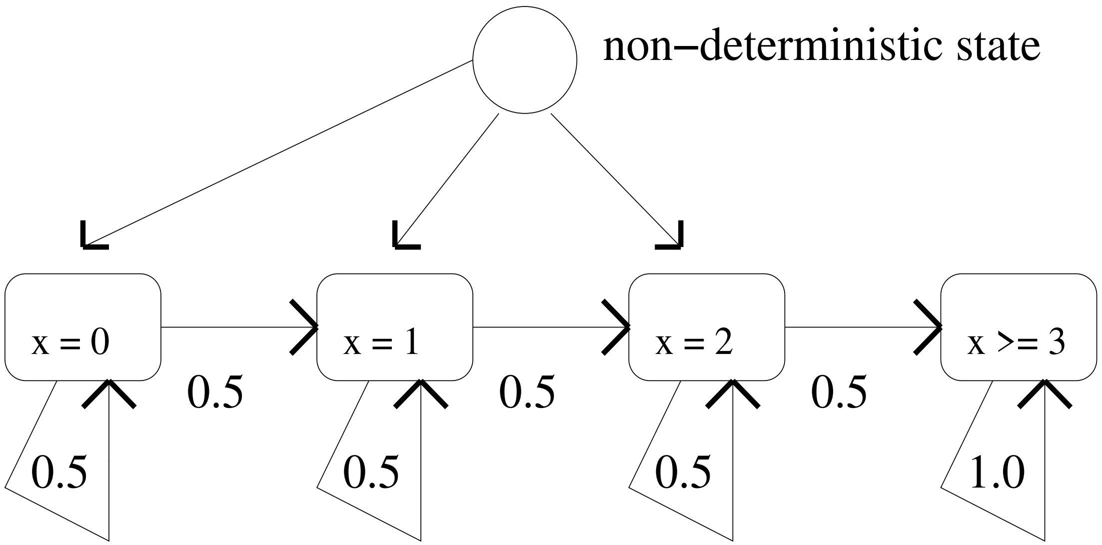

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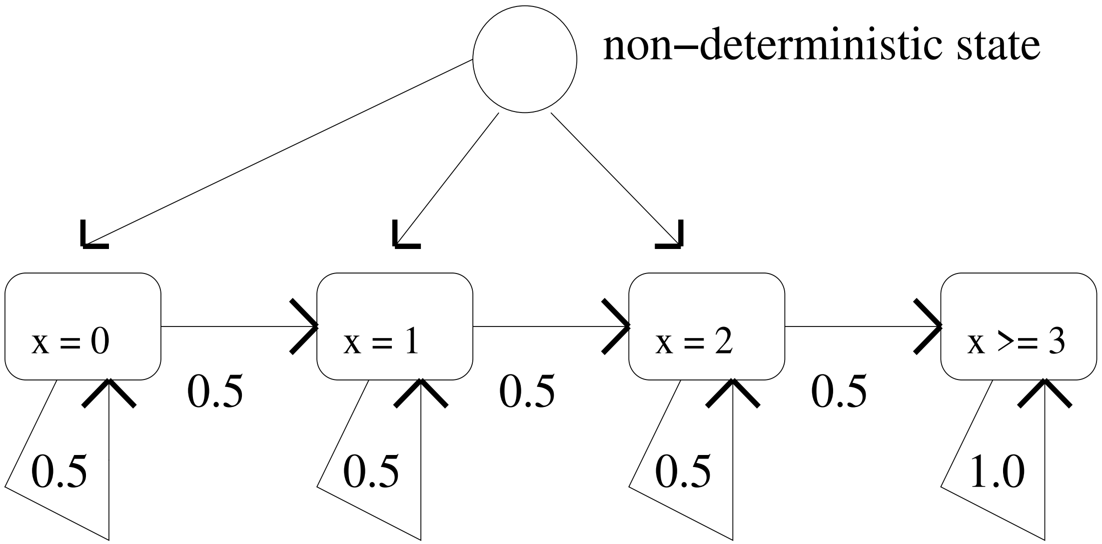

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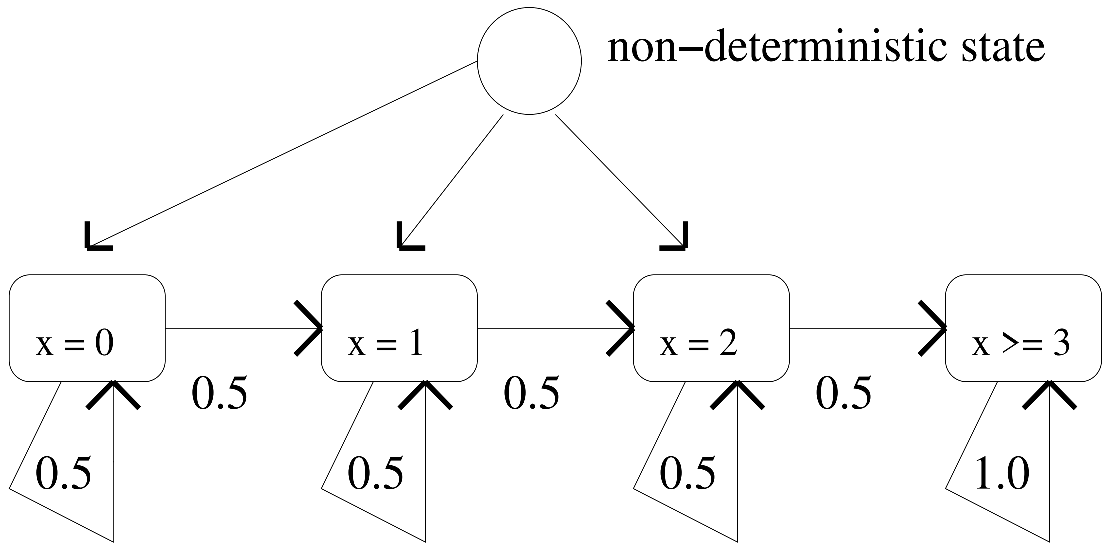

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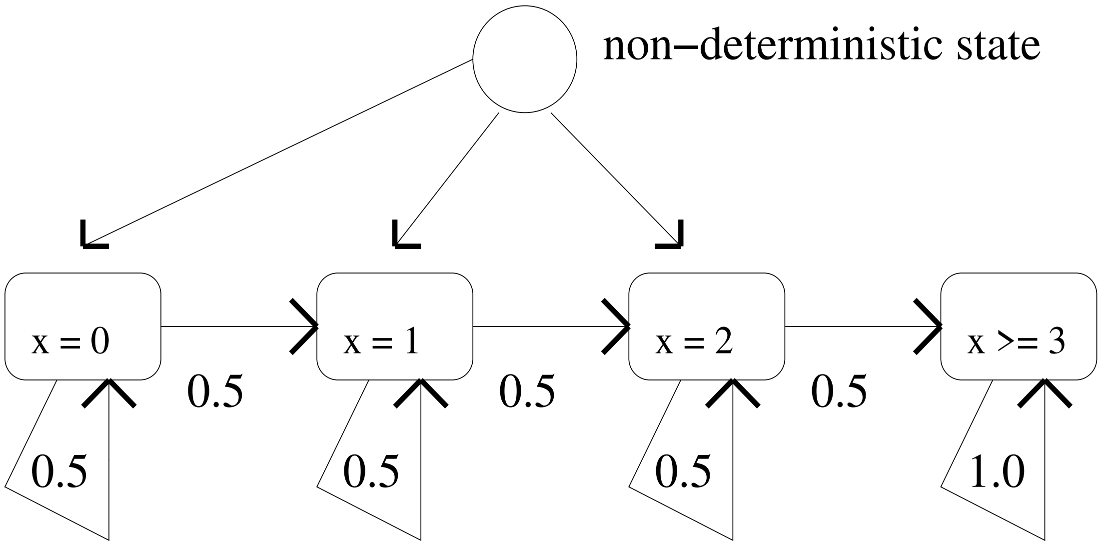

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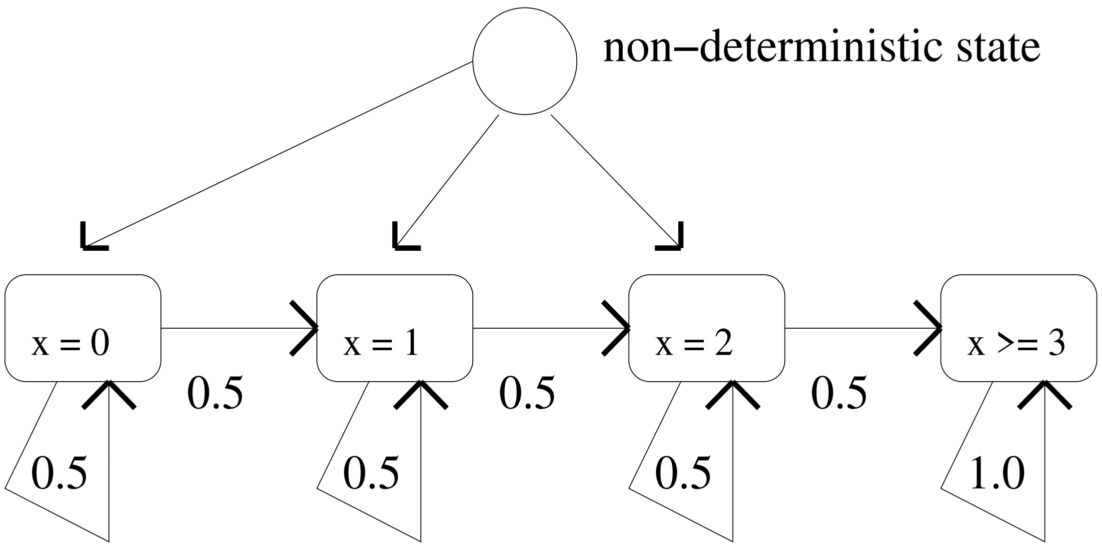

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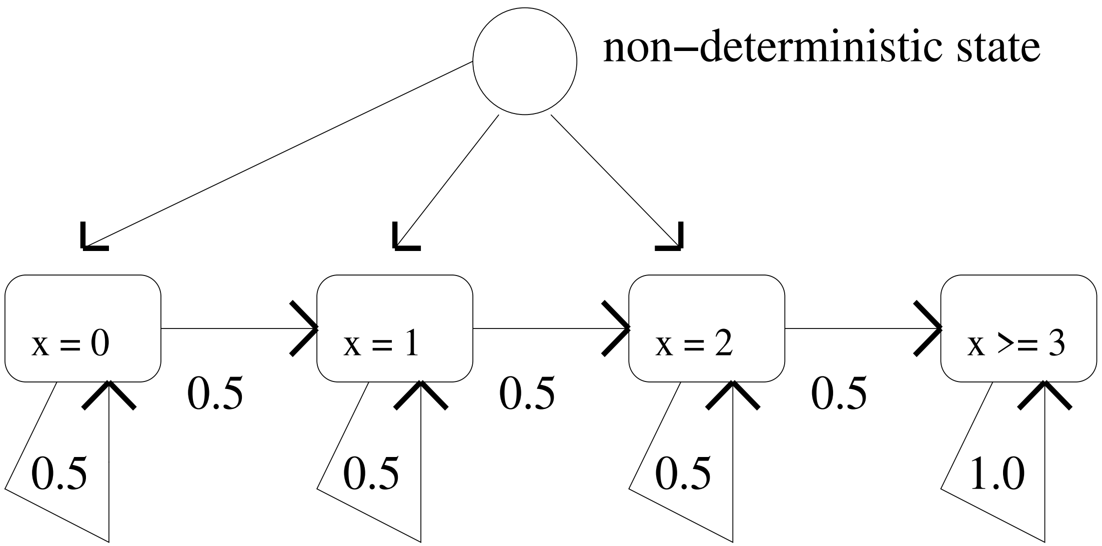

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- probabilistic behavior: random coin flip;
- abstraction: driven by pre- and post-conditions;
- $(M \models (0 \leq x \leq 2) \rightarrow \mathsf{G}(x < 3)) = [0, 0]$; computed through bounded reachability: $\mathsf{true}\,\mathsf{U}^{\leq 5}\,(x >= 3)$.

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Recommended reading: D. Monniaux's doctoral thesis: abstract interpretation for Markov decision processes; and Monto Carlo methods.

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- checks in MDP undecidable in general.

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- conjunction $\phi_1 \wedge \phi_2$ complexity bottleneck in MDP: conditional probabilities, non-compositional.

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- makes time complexity single exponential in size of formula, has linear space complexity.

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- is a form of probabilistic testing;
- randomization can improve complexity: trace equivalence in labeled transition systems is PSPACE complete, but trace equivalence in fully probabilistic systems is time polynomial.

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- **caveat abstractor**: e.g. semantics of safety properties in MDP does not change if strategies are history-*dependent*; *not* true for liveness properties.

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- sound abstraction for first-order logic with transitive closure possible for Markov decision processes: three-valued Kleene semantics on top of probabilistic one [Huth'02].

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- existing work for mixed models?

Specifying abstract MCPs

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Soundness:

$$(M, M') \in \alpha \;\&\; (\phi, \phi') \in \beta \;\Rightarrow\; \gamma(M' \models' \phi') \leq (M \models \phi).$$

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- as in first or previous item, but F, U, and G turn into bounded instances: potentially unsound;

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- as in first or previous item, but F, U, and G turn into bounded instances: potentially unsound;
- possibilistic abstractions: M' qualitative model, $\mathtt{Ans}'$ discrete domain; often sound for "probability $= 1$" type questions.

MDP as sound abstraction

$$\phi ::= \text{true} \mid o \mid \neg\phi \mid \phi \wedge \phi \mid [\mathsf{X}\,\phi]_{\geq p} \mid [\phi\,\mathsf{U}\,\phi]_{\geq p} \qquad (\text{LTL}_{\geq})$$

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- $\models$ equals $\models_{\geq} \times \models_{\geq}$,
- Φ equals $\mathrm{LTL}_{\geq} \times \mathrm{LTL}_{\geq}$,
- $\mathtt{Ans}$ equals $\{\mathrm{false} < \mathrm{true}\} \times \{\mathrm{true} < \mathrm{false}\}$,
- α is the identity,
- β contains only pairs $((\phi^\flat, \phi^\sharp), \phi)$ — $[r, s]$-semantics provides thresholds,
- and $\gamma\colon \mathtt{Ans}' \to \mathtt{Ans}$ maps intervals $[r, s]$ to $(\alpha^\forall(r), \alpha^\exists(s)) \in \mathtt{Ans}$; $\alpha^\forall, \alpha^\exists\colon [0,1] \to \{\mathrm{false} < \mathrm{true}\}$, $(\alpha^\forall)^{-1}(\mathrm{false}) = \{0\}$ and $(\alpha^\exists)^{-1}(\mathrm{true}) = \{1\}$.

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- alas, some continuous-space labeled Markov chains aren't equivalent to any finite-state model for a simple probabilistic modal logic Φ;
- probabilistic bisimulations, establish instances of query equivalences for suitable Φ;
- weak probabilistic bisimulations may/may not be consistent with query semantics.

Precise abstract MCPs

$$\mathrm{A}_{\phi}^{M} = \{\gamma(M' \models \phi') \mid (M, M') \in \alpha,\ (\phi, \phi') \in \beta\}$$

collects all information about $(M \models \phi)$ we could ever infer from abstract and sound checks.

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- Example: **finite-model property** for Kripke structures; $(M, M') \in \alpha$ implies M' finite; $\mathtt{Ans} = \{\text{false} < \text{true}\}$; if ϕ holds in M, does it hold in a finite abstraction M'?

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- finite acyclic labeled Markov processes are precise for continuous-state labeled Markov processes [Deharnais et al.'00];

- metric $d([r, s], [r', s']) \stackrel{\text{def}}{=} |r - r'| + |s - s'|$ measures accuracy of $\gamma(M' \models' \phi')$ with respect to $(M \models \phi)$;
- begs the question of what $(M \models \phi)$ is, blurs under-specification of adversary and approximation.

Approximation Problem via Measurements

- answer domains `Ans` have a measurement [Keye'00]
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- allow measuring precision of abstraction without knowing un-abstracted value;
- reformulated approximation problem:

  "Given $\epsilon > 0$, $M \in \mathcal{M}$, and $\phi \in \Phi$, are there $M' \in \mathcal{M}'$ and $\phi' \in \Phi'$ such that $(M, M') \in \alpha$, $(\phi, \phi') \in \beta$, and

$$\mu(\gamma(M' \models' \phi')) < \epsilon?''$$

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- **diagnostics:** need ability to attach information to extensional replies (e.g. $[r, s]$ for MDP);
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  - specifying/computing executable abstractions cost more than executing un-abstracted models?
  - heuristics or other support to guide choice of abstraction?
- **abandoning soundness:** limited bounded model checking, probabilistic testing, Monte Carlo methods, lack of finite-model property, approximative bisimilarity and information flow [Di Pierro et el.'02], etc.

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- develop diagnostics for the evidence of answers; develop probabilistic games as such diagnostics;
- push established methods and results from qualitative model checking into the model checking of mixed models.

Conclusions

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