Weak Bisimulation

Observable transitions

$P \stackrel{\varepsilon}{\Longrightarrow} Q$ or $P \stackrel{\alpha}{\Longrightarrow} Q$ where α is a or $\overline{a}$ for action name a

$$\mathrm{R}(\stackrel{\varepsilon}{\Longrightarrow}) \qquad P \stackrel{\varepsilon}{\Longrightarrow} P \qquad \frac{P \stackrel{\tau}{\longrightarrow} P' \quad P' \stackrel{\varepsilon}{\Longrightarrow} Q}{P \stackrel{\varepsilon}{\Longrightarrow} Q}$$

$$\mathrm{R}(\stackrel{\alpha}{\Longrightarrow}) \qquad \frac{P \stackrel{\varepsilon}{\Longrightarrow} P' \quad P' \stackrel{\alpha}{\longrightarrow} Q' \quad Q' \stackrel{\varepsilon}{\Longrightarrow} Q}{P \stackrel{\alpha}{\Longrightarrow} Q}$$

We may construct observable transition graphs using observable transitions, just as transition graphs are constructed from standard transitions.

Example

$$
\begin{aligned}
P_0 &\stackrel{\text{def}}{=} a.P_0 + b.P_1 + \tau.P_1 \\
P_1 &\stackrel{\text{def}}{=} a.P_1 + \tau.P_2 \\
P_2 &\stackrel{\text{def}}{=} b.P_0
\end{aligned}
$$

$$
\begin{aligned}
Q_1 &\stackrel{\text{def}}{=} a.Q_1 + \tau.Q_2 \\
Q_2 &\stackrel{\text{def}}{=} b.Q_1
\end{aligned}
$$

Exercise

Give the observable transition graphs of P_0 and Q_1.

Weak (observable) Bisimulations

A binary relation B between processes is a weak (or observable) bisimulation provided that, whenever $(P, Q) \in B$ and β is a, $\overline{a}$ or ε for action name a, then

- if $P \stackrel{\beta}{\Longrightarrow} P'$ then $Q \stackrel{\beta}{\Longrightarrow} Q'$ for some Q' such that $(P', Q') \in B$, and
- if $Q \stackrel{\beta}{\Longrightarrow} Q'$ then $P \stackrel{\beta}{\Longrightarrow} P'$ for some P' such that $(P', Q') \in B$.

Two processes P and Q are weak bisimulation equivalent (or weakly bisimilar) if there is a weak bisimulation relation B such that $(P, Q) \in B$. We write $P \approx Q$ if P and Q are weakly bisimilar.

Example of Weak Bisimulation

To establish $P \approx Q$

1. present a candidate relation B with $(P, Q) \in B$;
2. prove that B is a weak bisimulation.

Example

$$\begin{aligned} P_0 &\stackrel{\text{def}}{=} a.P_0 + b.P_1 + \tau.P_1 \\ P_1 &\stackrel{\text{def}}{=} a.P_1 + \tau.P_2 \\ P_2 &\stackrel{\text{def}}{=} b.P_0 \end{aligned}$$

$$\begin{aligned} Q_1 &\stackrel{\text{def}}{=} a.Q_1 + \tau.Q_2 \\ Q_2 &\stackrel{\text{def}}{=} b.Q_1 \end{aligned}$$

Fact

The relation $B = \{(P_0, Q_1), (P_1, Q_1), (P_2, Q_2)\}$ is a weak bisimulation.

Properties

Weak bisimilarity $\approx$ satisfies analogous properties to strong bisimilarity:

- weak bisimilarity is an equivalence relation
- weak bisimilarity is a congruence with respect to all operators of CCS with the exception of $+$

  $$\tau.\mathrm{a}.0 \approx \mathrm{a}.0 \quad \text{but} \quad \tau.\mathrm{a}.0 + \mathrm{b}.0 \not\approx \mathrm{a}.0 + \mathrm{b}.0$$

- weak bisimilarity is the largest bisimulation
- strong bisimilarity $\sim$ implies weak bisimilarity $\approx$
- it is possible to adapt HML to get a correspondence between weak bisimulation and logical equivalence.

Exercise Which of the following are weakly bisimilar?

		Y/N
$a.\tau.b.0$	$a.b.0$	
$a.(b.0 + \tau.c.0)$	$a.(b.0 + c.0)$	
$a.(b.0 + \tau.c.0)$	$a.(b.0 + \tau.c.0) + a.c.0$	
$a.0 + b.0 + \tau.b.0$	$a.0 + \tau.b.0$	
$a.0 + b.0 + \tau.b.0$	$a.0 + b.0$	
$a.(b.0 + \tau.b.0)$	$a.b.0$	

Weak Bisimulation (Alternative)

A binary relation B between processes is an observation bisimulation just in case whenever $(P, Q) \in B$ and α is a, $\overline{a}$ or τ for action name a,

1. if $P \xrightarrow{\alpha} P'$ then $Q \overset{\hat{\alpha}}{\Longrightarrow} Q'$ for some Q' such that $(P', Q') \in B$, and

2. if $Q \xrightarrow{\alpha} Q'$ then $P \overset{\hat{\alpha}}{\Longrightarrow} P'$ for some P' such that $(P', Q') \in B$,

where $\hat{\alpha} = \alpha$ if α is a or $\overline{a}$, and $\hat{\alpha} = \varepsilon$ if α is τ.

Two processes are observation equivalent, denoted by $\approx'$, if they are related by an observation bisimulation relation.

Fact

1. B is a weak bisimulation if and only if B is an observation bisimimulation

2. $\approx = \approx'$

Example of Weak Bisimulation

To establish $P \approx Q$

1. present a candidate relation B with $(P, Q) \in B$;
2. prove that B is an observation bisimulation.

Establishing weak bisimilarity is so much easier using an observation bisimulation.

Example

$$\begin{aligned} P_0 &\stackrel{\text{def}}{=} a.P_0 + b.P_1 + \tau.P_1 \\ P_1 &\stackrel{\text{def}}{=} a.P_1 + \tau.P_2 \\ P_2 &\stackrel{\text{def}}{=} b.P_0 \end{aligned}$$

$$\begin{aligned} Q_1 &\stackrel{\text{def}}{=} a.Q_1 + \tau.Q_2 \\ Q_2 &\stackrel{\text{def}}{=} b.Q_1 \end{aligned}$$

Fact

The relation $B = \{(P_0, Q_1), (P_1, Q_1), (P_2, Q_2)\}$ is an observation bisimulation.

Protocol that may lose messages

$$
\begin{aligned}
\mathtt{Sender} &\stackrel{\text{def}}{=} \mathtt{in}(x).\overline{\mathtt{sm}}\langle x\rangle.\mathtt{Send1}\langle x\rangle \\
\mathtt{Send1}(x) &\stackrel{\text{def}}{=} \mathtt{ms}.\overline{\mathtt{sm}}\langle x\rangle.\mathtt{Send1}\langle x\rangle + \mathtt{ok}.\mathtt{Sender} \\
\mathtt{Medium} &\stackrel{\text{def}}{=} \mathtt{sm}(y).\mathtt{Med1}\langle y\rangle \\
\mathtt{Med1}(y) &\stackrel{\text{def}}{=} \overline{\mathtt{mr}}\langle y\rangle.\mathtt{Medium} + \tau.\overline{\mathtt{ms}}.\mathtt{Medium} \\
\mathtt{Receiver} &\stackrel{\text{def}}{=} \mathtt{mr}(x).\overline{\mathtt{out}}\langle x\rangle.\overline{\mathtt{ok}}.\mathtt{Receiver}
\end{aligned}
$$

$$
\begin{aligned}
\mathtt{Protocol} &\stackrel{\text{def}}{=} (\mathtt{Sender} \mid \mathtt{Medium} \mid \mathtt{Receiver})\backslash J \\
& \quad J = \{\mathtt{sm}, \mathtt{ms}, \mathtt{mr}, \mathtt{ok}\} \\
\mathtt{Cop} &\stackrel{\text{def}}{=} \mathtt{in}(x).\overline{\mathtt{out}}\langle x\rangle.\mathtt{Cop}
\end{aligned}
$$

Recall that $P\backslash J$ is alternative notation for new $J(P)$.

Example

$\texttt{Protocol} \approx \texttt{Cop}$

Let B be the following relation

$$
\begin{aligned}
&\{(\texttt{Protocol}, \texttt{Cop})\} \cup \\
&\{((\texttt{Send1}\langle m\rangle \mid \texttt{Medium} \mid \overline{\texttt{ok}}.\texttt{Receiver})\backslash J, \\
&\qquad \texttt{Cop}) : m \in D\} \cup \\
&\{((\overline{\texttt{sm}}\langle m\rangle.\texttt{Send1}\langle m\rangle \mid \texttt{Medium} \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}\langle m\rangle.\texttt{Cop}) : m \in D\} \cup \\
&\{((\texttt{Send1}\langle m\rangle \mid \texttt{Med1}\langle m\rangle \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}\langle m\rangle.\texttt{Cop}) : m \in D\} \cup \\
&\{((\texttt{Send1}\langle m\rangle \mid \texttt{Medium} \mid \overline{\texttt{out}}\langle m\rangle.\overline{\texttt{ok}}.\texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}\langle m\rangle.\texttt{Cop}) : m \in D\} \cup \\
&\{((\texttt{Send1}\langle m\rangle \mid \overline{\texttt{ms}}.\texttt{Medium} \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}\langle m\rangle.\texttt{Cop}) : m \in D\}
\end{aligned}
$$

B is an observation bisimulation