COURSE 436
PERFORMANCE ANALYSIS
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Coursework 1
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Unassessed

1. The bottleneck in a network of service centres is the one with the maximum demand, i.e. average amount of service required by all tasks in unit time. It can be shown that demand is proportional to server utilisation.

   (a) In a closed network, i.e. one with a constant population of tasks, K, what happens to the location of the bottleneck as K increases?

   (b) What happens to the bottlenecks utilisation as $K \to \infty$?

   (c) What happens to the population at non-bottleneck servers as $K \to \infty$?

   (d) Which servers should you speed up to best improve performance?

   (e) What should the server utilisations be for optimal performance?

2. Consider a simple queue with arrival rate λ, mean service time m and standard deviation of service time σ.

   (a) If σ is always proportional to m, what would you expect to happen to response time if the arrival rate and service rate both double?

   (b) If σ doubles whilst λ and m remain fixed, what would you expect to happen to average response time?

3. Consider a queue in steady state with arrival rate λ, service rate μ and utilisation U. What is the throughput? How is this related to the arrival rate? Hence prove that $U = \lambda/\mu$.

4. Suppose that the amount of time that a light-bulb works before burning itself out is exponentially distributed with mean ten hours. Suppose that a person enters a room in which such a light-bulb is burning. If this person desires to work five hours, then what is the probability that he will complete his work without the bulb burning out? What can be said if the distribution is not exponential?

5. Given two exponentially distributed random variables X_1 and X_2, determine the probability that one is smaller than the other?

6. The times taken for transactions from sites A and B are exponentially distributed with means $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ minutes respectively on the host computer system. If two transactions T_A and T_B arrive from A and B respectively and their service is started immediately in both cases, what is the probability that T_B finishes first? What if they are not both started at the same time?

7. Customers arrive at a supermarket according to a Poisson process with rate λ per hour. Suppose that two customers arrive during the first hour. Find the probability that

- both arrived in the first 20 minutes;
- at least one of them arrived in the last 30 minutes.

8. Let T_n be the instant of the nth arrival in a Poisson process with rate λ. Show that the distribution function of T_n, $F_n(x)$ is given by:

$$F_n(x) = 1 - \sum_{k=0}^{n-1} \frac{(\lambda x)^k}{k!} e^{-\lambda x}$$