Performance Analysis (C336) - Tutorial Sheet 2

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1 The Poisson Distribution

(a) Given a single trial, let p be the probability that an event A is the outcome of that trial. Now assume that the trials are repeated N times, such that each trial is independent. Denote by A_N the total number of occurrences of event A in the N trials. Show that the probability of A occurring k times, where $0 < k \leq N$, is given by:

$$P(A_N = k) \quad = \quad \binom{N}{k} p^k (1-p)^{N-k}$$

(b) Now suppose that the probability of the event A occuring in any given small time-step δt is $\lambda \delta t$, where λ is some constant. Consider N independent consecutive trials, each taking δt time, such that the time taken for the N trials is $t = N \delta t$. What is the probability that event A occurs k times during the N trials, where $0 < k \leq N$?

(c) Let A_t denote the number of times that event A occurs during time t. Using (b) or otherwise, show that the probability of the event A occuring k times during time t, for all $k > 0$, is:

$$P(A_t = k) \quad = \quad \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

(Hint: consider the expression from (b), letting $N \to \infty$ – then, since $\delta t = t/N$, $\delta t \to 0$ as $N \to \infty$)

2 Application to a simple server

The probability distribution in 1(c) is called the *Poisson Distribution*. In terms of applications to computer performance modelling, consider a server and requests being made to the server. Suppose that we think the probability of a request being made to the server in a given small time-step δt is proportional to the time step (with a given proportionality constant λ) and that each request to the server is independent of the other requests (this corresponds exactly to the "infinitesimal definition" given in the lectures). The Poisson distribution will then determine the probability of k requests to our server during a specified time t.

(a) What is the expected number of requests to the server during time t, under these assumptions?

(b) Calculate the variance in the number of requests to the server, under these assumptions.

(Note: The variance of a random variable X is defined to be $Var(X) = E(X^2) - E(X)^2$, where $E(X)$ is the expectation of the variable X)