

# Relational Model and Algebra

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# Topic 1: The Relational Model

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# Relations are sets of typed tuples

## Relations

Relations take the form  $R(A, B, \dots)$  where

- $R$  is the name of the relation
- $A, B, \dots$  is the set of attributes of the relation
  - Often write the set without commas:  $A, B, \dots \equiv AB \dots$ , and can refer to a set of attributes as  $\vec{A}$
  - The number of attributes  $n$  is the **arity** of the relation  
Can call  $R(A_1, \dots, A_n)$  an  $n$ -ary relation
  - $Domain(A)$  is the set of values (type) that the attribute can have
  - Will use  $Attrs(R)$  to find  $A, B, \dots$
- The **extent** of  $R(A, B, \dots)$  is the set of **tuples**
 $\{ \langle v_1^A, v_1^B, \dots \rangle, \langle v_2^A, v_2^B, \dots \rangle, \langle v_3^A, v_3^B, \dots \rangle, \dots \}$ 
  - $\forall x. v_x^A \in Domain(A)$
  - No duplicate tuples
  - Not ordered
  - All tuples have the same arity

# Relation=Table

$R$		
$A$	$B$	...
$v_1^A$	$v_1^B$	...
$v_2^A$	$v_2^B$	...
$v_3^A$	$v_3^B$	...
	$\vdots$	

## Set Semantics

- Order of columns not significant
- Order of rows not significant
- No duplicate rows

# Relation=Table

$R$		
$A$	$B$	...
$v_1^A$	$v_1^B$	...
$v_2^A$	$v_2^B$	...
$v_3^A$	$v_3^B$	...
	$\vdots$	

## Set Semantics

- Order of columns not significant
- Order of rows not significant
- No duplicate rows

- Attribute=Column

# Relation=Table

$R$		
$A$	$B$	...
$v_1^A$	$v_1^B$	...
$v_2^A$	$v_2^B$	...
$v_3^A$	$v_3^B$	...
⋮	⋮	⋮

## Set Semantics

- Order of columns not significant
- Order of rows not significant
- No duplicate rows

- Attribute=Column
- Tuple=Row

## Quiz 1.1: Equivalent Relations

Which is the odd one out?

A

branch		
sortcode	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

B

branch		
bname	sortcode	cash
'Wimbledon'	56	94340.45
'Goodge St'	34	8900.67
'Strand'	67	34005.00

C

branch		
sortcode	bname	cash
34	'Goodge St'	8900.67
56	'Wimbledon'	94340.45
67	'Strand'	34005.00

D

branch		
sortcode	bname	cash
56	'Wimbledon'	94340.45
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

## Handling 'missing' attribute values

Suppose we want to have a relation `account(no,type,cname,rate,sortcode)`, but not all accounts have a rate.

### Solution 1: Separate relations

account				
no	type	cname	sortcode	
100	'current'	'McBrien, P.'	67	
101	'deposit'	'McBrien, P.'	67	
103	'current'	'Boyd, M.'	34	
107	'current'	'Poulovassilis, A.'	56	
119	'deposit'	'Poulovassilis, A.'	56	
125	'current'	'Bailey, J.'	56	

account_rate	
no	rate
101	5.25
119	5.50

### Solution 2: NULL values

account				
no	type	cname	rate	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56



# Relational Keys

## Key

A **key** of a relation  $R(AB\dots)$  is a subset of the attributes for which the values in any extent are unique across all tuples

- Every relation has at least one key, which is the entire set of attributes
- A key is **violated** by there being two tuples in the extent which have the same values for the attributes of the key
- If  $A$  is a key, then so must  $AB$  be a key
- A **minimal key** is a set of attributes  $AB\dots$  for which no subset of the attributes is also a key
- The **primary key** is one of the keys of the relation: serves as the default key when no key explicitly stated

## Quiz 1.2: Violation of Relational Keys

movement			
mid	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	5/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

Which key is violated?

A

`movement(mid)`

B

`movement(no,amount)`

C

`movement(no,tdate)`

D

`movement(amount,tdate)`

## Quiz 1.3: Correct Keys for Relations

movement			
mid	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	5/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

Which key makes most sense in a bank UoD?

A

`movement(amount)`

B

`movement(no,amount)`

C

`movement(no,tdate)`

D

`movement(amount,tdate)`

# Relational Foreign Keys

## Foreign Key

A **foreign key**  $R(\vec{X}) \xrightarrow{fk} S(\vec{Y})$  of a relation  $R(AB\dots)$  is a subset  $\vec{X} \subseteq AB\dots$  of the attributes for which the values in the extent of  $R$  also appear as values of attributes  $\vec{Y}$  in the extent of  $S$ , and  $\vec{Y}$  is a key of  $S$ .

$\text{account}(\text{sortcode}) \xrightarrow{fk} \text{branch}(\text{sortcode})$

account				
<u>no</u>	type	cname	rate	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

key  $\text{branch}(\text{sortcode})$

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

## Quiz 1.4: Foreign Key Violation

$$\text{account}(\text{sortcode}) \xrightarrow{\text{fk}} \text{branch}(\text{sortcode})$$

account				
no	type	cname	rate	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

key branch(sortcode)

branch		
sortcode	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

Which update violates the foreign key?

A

INSERT INTO account  
(126,'business','McBrien, P.',1.00,67)

B

INSERT INTO branch  
(78,'Ealing',1000.00)

# Example Relational Schema

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

movement			
<u>mid</u>	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	8/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

key branch(sortcode)

key branch(bname)

key movement(mid)

key account(no)

movement(no)  $\xRightarrow{fk}$  account(no)

account(sortcode)  $\xRightarrow{fk}$  branch(sortcode)

## Topic 2: The Relational Algebra

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# Relational Algebra: A Query Language for the Relational Model

## Primitive operators of the Relational Algebra

Symbol	Name	Type
$\pi$	Project	Unary
$\sigma$	Select	Unary
$\times$	Cartesian Product	Binary
$\cup$	Union	Binary
$-$	Difference	Binary

- All operators take relations as input
- All operators produce one relation as their output
- Other (useful) operators may be defined in terms of the five primitive operators



Relational Algebra: Project  $\pi$ 

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

## Project Operator

$\pi_{no,type}$ account	
<u>no</u>	type
100	'current'
101	'deposit'
103	'current'
107	'current'
119	'deposit'
125	'current'

$\pi_{sortcode}$ account	
sortcode	
67	
34	
56	

Relational Algebra: Select  $\sigma$ 

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

## Select Operator

$\sigma_{\text{rate} > 0} \text{account}$				
<u>no</u>	type	cname	rate	sortcode
101	'deposit'	'McBrien, P.'	5.25	67
119	'deposit'	'Poulovassilis, A.'	5.50	56

Relational Algebra: Product  $\times$ 

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

$\sigma_{\text{rate}>0}\text{account}$				
<u>no</u>	type	cname	rate	sortcode
101	'deposit'	'McBrien, P.'	5.25	67
119	'deposit'	'Poulovassilis, A.'	5.50	56

## Product Operator

branch $\times$ $\sigma_{\text{rate}>0}\text{account}$							
<u>sortcode</u>	bname	cash	<u>no</u>	type	cname	rate	sortcode
56	'Wimbledon'	94340.45	101	'deposit'	'McBrien, P.'	5.25	67
56	'Wimbledon'	94340.45	119	'deposit'	'Poulovassilis, A.'	5.50	56
34	'Goodge St'	8900.67	101	'deposit'	'McBrien, P.'	5.25	67
34	'Goodge St'	8900.67	119	'deposit'	'Poulovassilis, A.'	5.50	56
67	'Strand'	34005.00	101	'deposit'	'McBrien, P.'	5.25	67
67	'Strand'	34005.00	119	'deposit'	'Poulovassilis, A.'	5.50	56

## Quiz 2.1: RA Queries

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

Which RA query lists the name of branches that have deposit accounts?

A

 $\pi_{\text{sortcode}} \sigma_{\text{type}='deposit'} \text{ACCOUNT}$ 

B

 $\pi_{\text{bname}}$ 
 $\sigma_{\text{account.sortcode}=\text{branch.sortcode} \wedge \text{type}='deposit'}$   
 (account  $\times$  branch)

C

D

# SPJ Queries

## Select Project Join (SPJ) queries

If a product of tables is formed, where a selection is then done that compares the attributes of those tables, we say that a **join** has been performed.

Normally not all columns of the product are returned, and therefore a project is also required.

## Branches with current accounts

$\pi_{\text{bname}, \text{no}} \sigma_{\text{branch.sortcode}=\text{account.sortcode} \wedge \text{account.type}=\text{'current'}} (\text{branch} \times \text{account})$

bname	no
'Goodge St'	103
'Wimbledon'	107
'Wimbledon'	125
'Strand'	100

Relational Algebra: Union  $\cup$ 

$\pi_{\text{sortcode as id}} \text{account}$	$\pi_{\text{no as id}} \text{account}$
id	id
67	100
34	101
56	103
	107
	119
	125

## Union Operator

$\pi_{\text{sortcode as id}} \text{account} \cup \pi_{\text{no as id}} \text{account}$
id
67
34
56
100
101
103
107
119
125

- relations must be **union compatible**

## Relational Algebra: Difference —

$\pi_{\text{no}}\text{account}$	$\pi_{\text{no}}\text{movement}$
no	no
100	100
101	101
103	103
107	107
119	119
125	

## Difference Operator

 $\pi_{\text{no}}\text{account} - \pi_{\text{no}}\text{movement}$ 

<u>no</u>
125

- relations must be **union compatible**

## Rules for Combining Operators

Since all operators produce a relation as output, *any* operator may produce one of the inputs to any other operator.

### well formed RA query

- the output of the nested operator must contain the attributes required by an outer  $\pi$  or  $\sigma$
- the two inputs to a  $\cup$  or  $-$  must contain the same number of attributes



## Quiz 2.2: Well formed queries

account				
no	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulouvassilis, A.'	NULL	56
119	'deposit'	'Poulouvassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

movement			
mid	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	8/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

Which RA query is well formed?

A

 $\sigma_{\text{type}='current'} \pi_{\text{no}} \text{account}$ 

B

 $\pi_{\text{no}} \text{account} - \pi_{\text{no}, \text{mid}} \text{movement}$ 

C

 $\pi_{\text{no}} \sigma_{\text{type}='current'} \text{account}$ 

D

 $\pi_{\text{no}} \pi_{\text{type}} \text{account}$

## Worksheet: Primitive Relational Algebra Operators

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

movement			
<u>mid</u>	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	8/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

key branch(sortcode)

key branch(bname)

key movement(mid)

key account(no)

$\text{movement}(\text{no}) \xRightarrow{fk} \text{account}(\text{no})$

$\text{account}(\text{sortcode}) \xRightarrow{fk} \text{branch}(\text{sortcode})$

## Topic 3: Relational Algebra Derived Operators

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Derived Relational Algebra: Natural Join  $\bowtie$ 

## Natural Join

$$R \bowtie S = \sigma_{R.A_1=S.A_1 \wedge \dots \wedge R.A_m=S.A_m} R \times S$$

## Natural Join

$$\text{branch} \bowtie \text{account} = \sigma_{\text{branch.sortcode}=\text{account.sortcode}} \text{branch} \times \text{account}$$

branch $\bowtie$ account						
sortcode	bname	cash	no	type	cname	rate
34	'Goodge St'	8900.67	103	'current'	'Boyd, M.'	NULL
56	'Wimbledon'	94340.45	107	'current'	'Poulovassilis, A.'	NULL
56	'Wimbledon'	94340.45	119	'deposit'	'Poulovassilis, A.'	5.50
56	'Wimbledon'	94340.45	125	'current'	'Bailey, J.'	NULL
67	'Strand'	34005.00	100	'current'	'McBrien, P.'	NULL
67	'Strand'	34005.00	101	'deposit'	'McBrien, P.'	5.25

## Quiz 3.1: Natural Join

What is the result of  $\pi_{no}(\text{account} \bowtie \text{movement})$ ?

A

 $\pi_{no}(\text{account} \bowtie \text{movement})$ 

no
100
101
103
107
119
125

B

 $\pi_{no}(\text{account} \bowtie \text{movement})$ 

no
100
101
103
107
119

C

 $\pi_{no}(\text{account} \bowtie \text{movement})$ 

no
125

D

 $\pi_{no}(\text{account} \bowtie \text{movement})$ 

no
----

Derived Relational Algebra: Semi Join  $\bowtie$ 

## Semi Join

$$R \bowtie S = R \bowtie \pi_{Attr(R) \cap Attr(S)}(S)$$

## Semi Join

$$\text{account} \bowtie \text{movement} = \text{account} \bowtie \pi_{no}(\text{movement})$$

account $\bowtie$ movement				
<u>no</u>	type	cname	rate	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56

# Derived Relational Algebra: Joins

## Natural Join

$$R \bowtie S = \sigma_{R.A_1=S.A_1 \wedge \dots \wedge R.A_m=S.A_m} R \times S$$

## Equi Join

$$R \stackrel{A=B}{\bowtie} S = \sigma_{R.A=S.B} R \times S$$

## Semi Join

$$R \ltimes S = R \bowtie \pi_{Attr(R) \cap Attr(S)}(S)$$

## Theta Join

$$R \stackrel{\theta}{\bowtie} S = \sigma_{\theta} R \times S$$

## Quiz 3.2: Understanding join operators

branch		
sortcode	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

account				
no	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

Which RA query produces the most tuples?

A

branch  $\bowtie$   $\text{branch.sortcode} < \text{account.sortcode}$  account

B

branch  $\bowtie$  account

C

branch  $\bowtie$  account

D

branch  $\bowtie$   $\text{branch.sortcode} = \text{account.sortcode}$  account



## Quiz 3.3: Foreign Keys and Natural Joins (1)

Suppose  $R$  and  $S$  only share attribute  $A$ , and there is a foreign key  $S(A) \xrightarrow{fk} R(A)$ .

If  $|R| = 100$  and  $|S| = 1,000$ , what is  $|R \bowtie S|$ ?

A

100

B

1,000

C

100,000

D

900

### Cardinality of a Relation

$|R|$  returns the number of tuples in the current extent of  $R$

## Quiz 3.4: Foreign Keys and Natural Joins (2)

Suppose  $R$  and  $S$  only share attribute  $A$ , and there is a foreign key  $R(A) \xrightarrow{fk} S(A)$ .

If  $|R| = 100$  and  $|S| = 1,000$ , what is  $|R \bowtie S|$ ?

A

100

B

1,000

C

100,000

D

900

Derived Relational Algebra: Intersection  $\cap$ 

## Intersection

$$R \cap S = R - (R - S)$$

 $\pi_{\text{no}}\text{account} \cap \pi_{\text{no}}\text{movement}$ 

$\pi_{\text{no}}\text{account}$
no
100
101
103
107
119
125

$\pi_{\text{no}}\text{account} - \pi_{\text{no}}\text{movement}$
<u>no</u>
125

$\pi_{\text{no}}\text{account} \cap \pi_{\text{no}}\text{movement}$
no
100
101
103
107
119

## Quiz 3.5: Intersection

	email
name	address
'McBrien, P.'	p.mcbrien@imperial.ac.uk
'Poulovassilis, A.'	ap@dcs.bbk.ac.uk
'Pietzuch, P.'	prp@doc.ic.ac.uk

What is the result of  $\pi_{\text{name}} \text{account} \cap \pi_{\text{name}} \text{email}$ ?

A

cname

'McBrien, P.'

'Boyd, M.'

'Poulovassilis, A.'

'Bailey, J.'

'Pietzuch, P.'

B

cname

'McBrien, P.'

'Boyd, M.'

'Poulovassilis, A.'

'Bailey, J.'

C

cname

'McBrien, P.'

'Poulovassilis, A.'

'Pietzuch, P.'

D

cname

'McBrien, P.'

'Poulovassilis, A.'

# Derived Relational Algebra: Division $\div$

## Division

$$R \div S = \pi_{Attrs(R)-Attrs(S)} R - \pi_{Attrs(R)-Attrs(S)}((\pi_{Attrs(R)-Attrs(S)} R \times S) - R)$$

## Division

$$\pi_{cname,type} \mathbf{account} \div \pi_{type} \mathbf{account} = \pi_{cname} \pi_{cname,type} \mathbf{account} - \pi_{cname}((\pi_{cname} \pi_{cname,type} \mathbf{account} \times \pi_{type} \mathbf{account}) - \pi_{cname,type} \mathbf{account})$$

$$\pi_{cname,type} \mathbf{account} \div \pi_{type} \mathbf{account} = \pi_{cname} \mathbf{account} - \pi_{cname}((\pi_{cname} \mathbf{account} \times \pi_{type} \mathbf{account}) - \pi_{cname,type} \mathbf{account})$$

$\pi_{cname,type} \mathbf{account}$	
cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Poulovassilis, A.'	'current'
'Poulovassilis, A.'	'deposit'
'Bailey, J.'	'current'

$\pi_{type} \mathbf{account}$
type
'current'
'deposit'

$\pi_{cname,type} \mathbf{account} \div \pi_{type} \mathbf{account}$
cname
'McBrien, P.'
'Poulovassilis, A.'

# Evaluation of Division

$\pi_{\text{cname}} \text{ account}$
cname
'McBrien, P.'
'Boyd, M.'
'Poulovassilis, A.'
'Bailey, J.'

$\pi_{\text{type}} \text{ account}$
type
'current'
'deposit'

# Evaluation of Division

$\pi_{\text{cname}} \text{ account}$
cname
'McBrien, P.'
'Boyd, M.'
'Poulovassilis, A.'
'Bailey, J.'

$\pi_{\text{type}} \text{ account}$
type
'current'
'deposit'



$\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}$	
cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Boyd, M.'	'deposit'
'Poulovassilis, A.'	'current'
'Poulovassilis, A.'	'deposit'
'Bailey, J.'	'current'
'Bailey, J.'	'deposit'

# Evaluation of Division

$\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}$	
cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Boyd, M.'	'deposit'
'Poulouvassilis, A.'	'current'
'Poulouvassilis, A.'	'deposit'
'Bailey, J.'	'current'
'Bailey, J.'	'deposit'

$\pi_{\text{cname,type}} \text{ account}$	
cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Poulouvassilis, A.'	'current'
'Poulouvassilis, A.'	'deposit'
'Bailey, J.'	'current'



# Evaluation of Division

$$\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}$$

cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Boyd, M.'	'deposit'
'Poulouvassilis, A.'	'current'
'Poulouvassilis, A.'	'deposit'
'Bailey, J.'	'current'
'Bailey, J.'	'deposit'

$$\pi_{\text{cname,type}} \text{ account}$$

cname	type
'McBrien, P.'	'current'
'McBrien, P.'	'deposit'
'Boyd, M.'	'current'
'Poulouvassilis, A.'	'current'
'Poulouvassilis, A.'	'deposit'
'Bailey, J.'	'current'



$$(\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}) \div \pi_{\text{cname,type}} \text{ account}$$

cname	type
'Boyd, M.'	'deposit'
'Bailey, J.'	'deposit'

## Evaluation of Division

$$(\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}) - \pi_{\text{cname,type}} \text{ account}$$

cname	type
'Boyd, M.'	'deposit'
'Bailey, J.'	'deposit'



$$\pi_{\text{cname}}((\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}) - \pi_{\text{cname,type}} \text{ account})$$

cname
'Boyd, M.'
'Bailey, J.'

## Evaluation of Division

 $\pi_{\text{cname}} \text{ account}$ 

cname

'McBrien, P.'

'Boyd, M.'

'Poulovassilis, A.'

'Bailey, J.'

 $\pi_{\text{cname}}((\pi_{\text{cname}} \text{ account} \times \pi_{\text{type}} \text{ account}) - \pi_{\text{cname,type}} \text{ account})$ 

cname

'Boyd, M.'

'Bailey, J.'

## Evaluation of Division

 $\pi_{\text{name}} \text{ account}$ 

cname

'McBrien, P.'

'Boyd, M.'

'Poulovassilis, A.'

'Bailey, J.'

 $\pi_{\text{name}}((\pi_{\text{name}} \text{ account} \times \pi_{\text{type}} \text{ account}) - \pi_{\text{name,type}} \text{ account})$ 

cname

'Boyd, M.'

'Bailey, J.'

 $\pi_{\text{name}} \text{ account} - \pi_{\text{name}}((\pi_{\text{name}} \text{ account} \times \pi_{\text{type}} \text{ account}) - \pi_{\text{name,type}} \text{ account})$ 

cname

'McBrien, P.'

'Poulovassilis, A.'

## Worksheet: Derived Relational Algebra Operators

branch		
<u>sortcode</u>	bname	cash
56	'Wimbledon'	94340.45
34	'Goodge St'	8900.67
67	'Strand'	34005.00

movement			
<u>mid</u>	no	amount	tdate
1000	100	2300.00	5/1/1999
1001	101	4000.00	5/1/1999
1002	100	-223.45	8/1/1999
1004	107	-100.00	11/1/1999
1005	103	145.50	12/1/1999
1006	100	10.23	15/1/1999
1007	107	345.56	15/1/1999
1008	101	1230.00	15/1/1999
1009	119	5600.00	18/1/1999

account				
<u>no</u>	type	cname	rate?	sortcode
100	'current'	'McBrien, P.'	NULL	67
101	'deposit'	'McBrien, P.'	5.25	67
103	'current'	'Boyd, M.'	NULL	34
107	'current'	'Poulovassilis, A.'	NULL	56
119	'deposit'	'Poulovassilis, A.'	5.50	56
125	'current'	'Bailey, J.'	NULL	56

key branch(sortcode)

key branch(bname)

key movement(mid)

key account(no)

$\text{movement}(\text{no}) \xRightarrow{fk} \text{account}(\text{no})$

$\text{account}(\text{sortcode}) \xRightarrow{fk} \text{branch}(\text{sortcode})$

## Topic 4: Relational Algebra Equivalences

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## Equivalences Involving Project

### Project and Project

$$\pi_{\vec{X}} \pi_{\vec{Y}} R \equiv \pi_{\vec{X}} R$$

You can eliminate any inner project (note that to be well formed  $\vec{X} \subseteq \vec{Y}$ )

### Project and Select

$$\pi_{\vec{X}} \sigma_{P(\vec{Y})} R \equiv \sigma_{P(\vec{Y})} \pi_{\vec{X}} R$$

If  $\vec{Y} \subseteq \vec{X}$  you can move a project of  $\vec{X}$  inside a select of  $\vec{Y}$

### Project and Product

$$\pi_{\vec{X}} (R \times S) \equiv \pi_{\vec{X} \cap \text{Attrs}(R)} R \times \pi_{\vec{X} \cap \text{Attrs}(S)} S$$

### Project and Union

$$\pi_{\vec{X}} (R \cup S) \equiv \pi_{\vec{X}} R \cup \pi_{\vec{X}} S$$

### Project and Difference

$$\pi_{\vec{X}} (R - S) \supseteq \pi_{\vec{X}} R - \pi_{\vec{X}} S$$

# Equivalences Involving Select

## Select and Project

$$\sigma_{P(\vec{X})} \pi_{\vec{X}} R \equiv \pi_{\vec{X}} \sigma_{P(\vec{X})} R$$

## Select and Select

$$\sigma_{P_x(\vec{X})} \sigma_{P_y(\vec{Y})} R \equiv \sigma_{P_x(\vec{X}) \wedge P_y(\vec{Y})} R$$

## Select and Product

$$\sigma_{P(\vec{X})} (R \times S) \equiv \sigma_{P(\vec{X})} R \times S \iff \vec{X} \subseteq \text{Attrs}(R)$$

If  $\vec{X} \subseteq \text{Attrs}(R)$  you can move a select predicate  $P(\vec{X})$  onto  $R$ .

## Select and Union

$$\sigma_{P(\vec{X})} (R \cup S) \equiv \sigma_{P(\vec{X})} R \cup \sigma_{P(\vec{X})} S$$

## Select and Difference

$$\sigma_{P(\vec{X})} (R - S) \equiv \sigma_{P(\vec{X})} R - S$$



## Quiz 4.1: Equivalent RA Expressions (Unary Operators)

Which RA expression is not equivalent to the other three?

A

$\pi_{no} \sigma_{type='current'} \mathbf{account}$

B

$\pi_{no} \sigma_{type='current'} \pi_{no,type,cname} \mathbf{account}$

C

$\pi_{no} \sigma_{type <> 'deposit'} \pi_{no,type,cname} \mathbf{account}$

D

$\pi_{no} \sigma_{type='current'} \sigma_{type <> 'deposit'} \mathbf{account}$

## Quiz 4.2: Query Evaluation

Which RA means that the  $\times$  operator handles fewer tuples?

A

$\sigma_{\text{account.no}=\text{movement.no}}$   
 $(\sigma_{\text{sortcode}=67} \text{ account} \times \sigma_{\text{amount}<0} \text{ movement})$

B

$\sigma_{\text{account.no}=\text{movement.no} \wedge \text{sortcode}=67}$   
 $(\text{account} \times \sigma_{\text{amount}<0} \text{ movement})$

C

$\sigma_{\text{account.no}=\text{movement.no} \wedge \text{amount}<0}$   
 $(\sigma_{\text{sortcode}=67} \text{ account} \times \text{movement})$

D

$\sigma_{\text{account.no}=\text{movement.no} \wedge \text{sortcode}=67 \wedge \text{amount}<0}$   
 $(\text{account} \times \text{movement})$

## Equivalences Involving Binary Operators

### Product and Union

$$R \times (S \cup T) \equiv (R \times S) \cup (R \times T)$$

### Product and Difference

$$R \times (S - T) \equiv (R \times S) - (R \times T)$$

### Union and Product

$R \cup (S \times T)$  unable to move  $\cup$  inside  $\times$

### Union and Difference

$R \cup (S - T)$  unable to move  $\cup$  inside  $-$

### Difference and Product

$R - (S \times T)$  unable to move  $-$  inside  $\times$

### Difference and Union

$$R - (S \cup T) \equiv (R - S) - T$$

## Quiz 4.3: Equivalent RA Expressions (Binary Operators)

Which equivalence does not hold?

A

$$(R \times S) \times T \equiv R \times (S \times T)$$

B

$$(R - S) - T \equiv R - (S - T)$$

C

$$(R \cup S) \cup T \equiv R \cup (S \cup T)$$

D

$$(R \cap S) \cap T \equiv R \cap (S \cap T)$$

# Worksheet: Equivalences Between RA Expressions

- 1  $\pi_{no,type} \sigma_{sortcode=56} \pi_{no,type,sortcode} \sigma_{type='deposit'} \mathbf{account}$
- 2  $\sigma_{account.no=movement.no} (\pi_{no,cname} \mathbf{account} \times \pi_{mid,no} \sigma_{amount>1000} \mathbf{movement})$
- 3  $\sigma_{account.no=movement.no} (\pi_{no,cname,rate} \mathbf{account} \times (\sigma_{amount>1000} \pi_{mid,no} \mathbf{movement} \cup \sigma_{amount<100} \pi_{mid,no} \mathbf{movement}))$
- 4  $\pi_{no,cname,tdate} \sigma_{amount<0 \wedge account.no=movement.no} \mathbf{account} \times \mathbf{movement}$