Making Complex Decisions

Paolo Turrini

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Introduction to Artificial Intelligence
2nd Part

AlphaGo beats World Go Champion

AlphaGo beats World Go Champion

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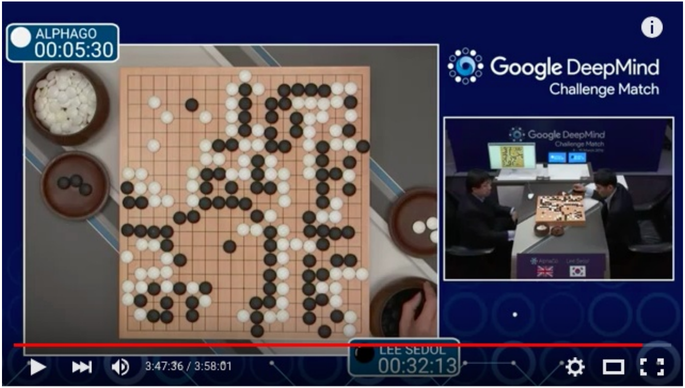

AlphaGo beats World Go Champion

Algorithm vs intuition

The five-day battle is being seen as a major test of what scientists and engineers have achieved in the sphere of artificial intelligence.

Go is a 3,000-year old Chinese board game and is considered to be a lot more complex than chess where artificial intelligence scored its most famous victory to date when IBM's Deep Blue beat grandmaster Gary Kasparov in 1997.

But experts say Go presents an entirely different challenge because of the game's incomputable number of move options which means that the computer must be capable of human-like "intuition" to prevail.

AlphaGo beats World Go Champion

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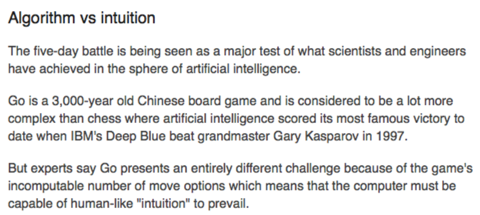
Welcome to scientific journalism!

AlphaGo beats World Go Champion



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Welcome to scientific journalism!

It's the number of possible positions the fundamental difference, together with the branching factor.

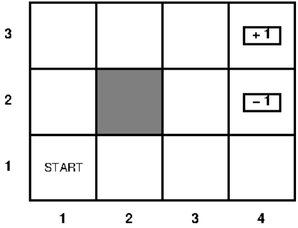
Outline

- Time
- Patience
- Risk

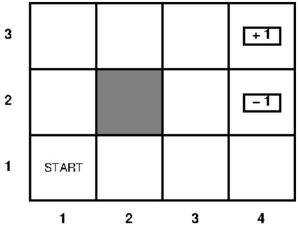
The main reference

Stuart Russell and Peter Norvig
Artificial Intelligence: a modern approach
Chapters 17

The World

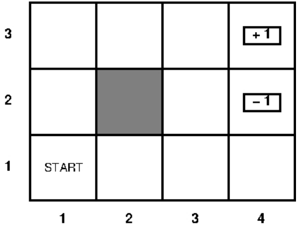


The World



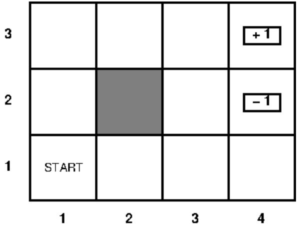
- Begin at the start state

The World



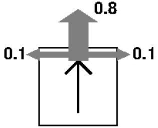
- Begin at the start state
- The game ends when we reach either goal state $+1$ or -1

The World

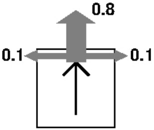


- Begin at the start state
- The game ends when we reach either goal state $+1$ or -1
- Collision results in no movement

The Agent

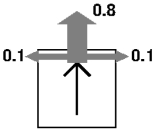


The Agent



The agent goes:

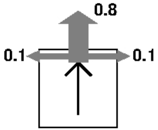
The Agent



The agent goes:

- towards the intendend direction with probability 0.8

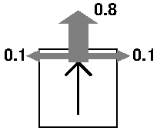
The Agent



The agent goes:

- towards the intendend direction with probability 0.8
- to the left of the intended direction with probability 0.1

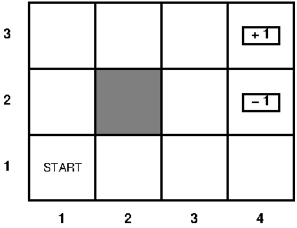
The Agent

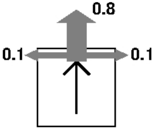


The agent goes:

- towards the intendend direction with probability 0.8
- to the left of the intended direction with probability 0.1
- to the right of the intended direction with probability 0.1

The Agent and the World





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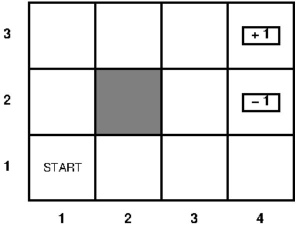
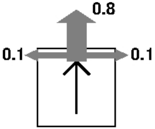
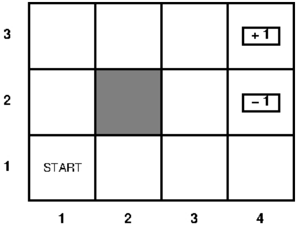
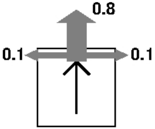




The environment is **fully observable**:

The Agent and the World

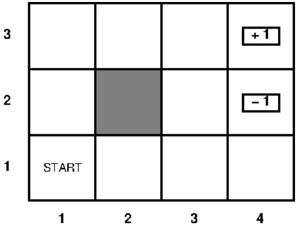


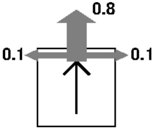


The environment is **fully observable**:

- the agent always knows what the world looks like: e.g., there is a wall, where the wall is, how to get to the wall . . .

The Agent and the World

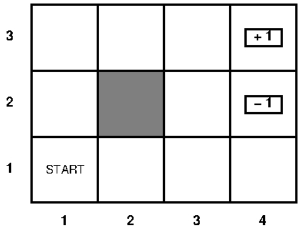


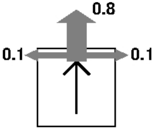


The environment is **fully observable**:

- the agent always knows what the world looks like: e.g., there is a wall, where the wall is, how to get to the wall . . .
- the agent always knows his or her position during the game, even though some trajectories might not be reached with certainty.

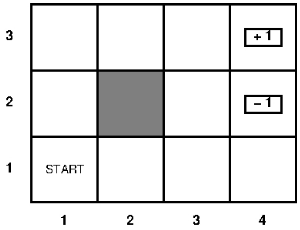
The Agent and the World

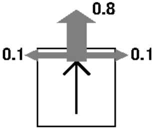




The environment is **Markovian**:

The Agent and the World

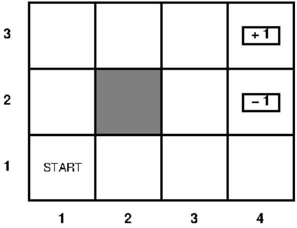


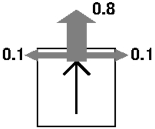


The environment is **Markovian**:

- the probability of reaching a state, only depends on the state the agent is in and the action she performs.

The Agent and the World





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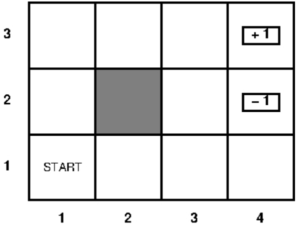
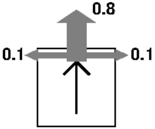
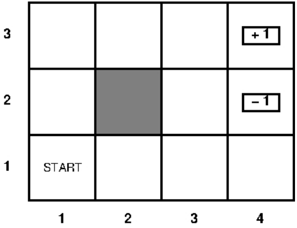
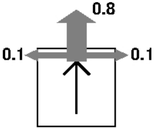




- $[x, y]_t$ is the fact that the agent is at square $[x, y]$ at time t

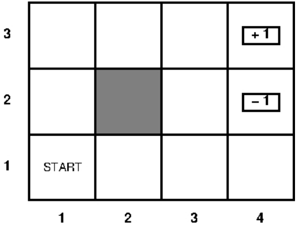
The Agent and the World

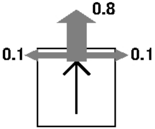




- $[x, y]_t$ is the fact that the agent is at square $[x, y]$ at time t
- $(x, y)_t$ is the fact that the agent *intends* to go to $[x, y]$ at time t

The Agent and the World





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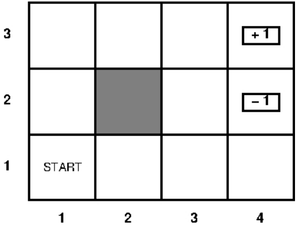
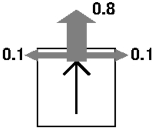
$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}) =$$

The Agent and the World





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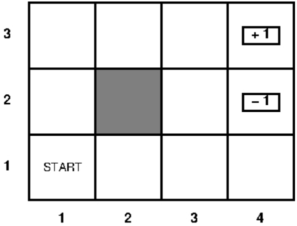
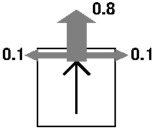
$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}) =$$
$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}, [x-5, y-6]_{t-20}) =$$

The Agent and the World

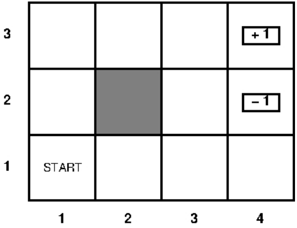


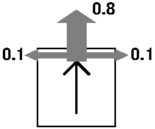


- $[x, y]_t$ is the fact that the agent is at square $[x, y]$ at time t
- $(x, y)_t$ is the fact that the agent *intends* to go to $[x, y]$ at time t

$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}) =$$
$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}, [x-5, y-6]_{t-20}) =$$
$$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}, (x-4, y-6)_{t-20})$$

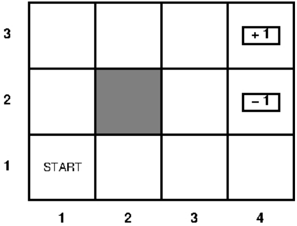
The Agent and the World

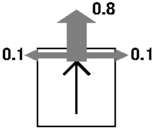




These properties allow us to make plans.

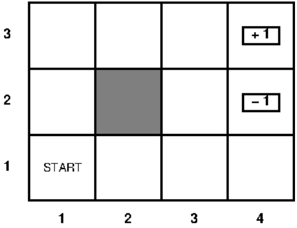
The Agent and the World

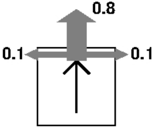




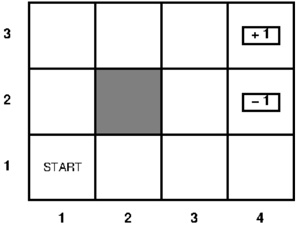
These properties allow us to make plans.
E.g., plans with determistic agents: as we know
$P([x, y]_t \mid (x, y)_{t-1}, [x-1, y]_{t-1}) = 1$

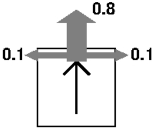
Lets make plans then





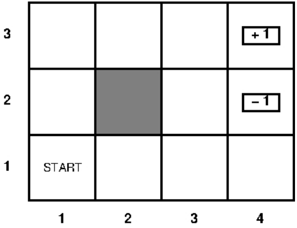
Lets make plans then

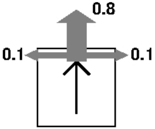




$\{Up, Down, Left, Right\}$ to denote the intended directions.

Lets make plans then

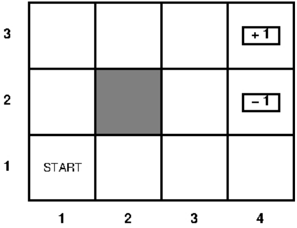


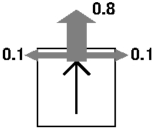


$\{Up, Down, Left, Right\}$ to denote the intended directions.

So $[Up, Down, Up, Right]$ is going to be the plan that, **from the starting state**, executes the moves n the specified order.

Makings plans





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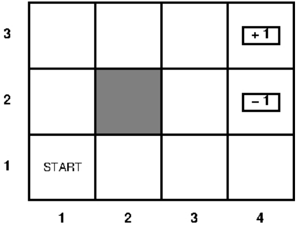
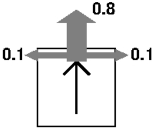
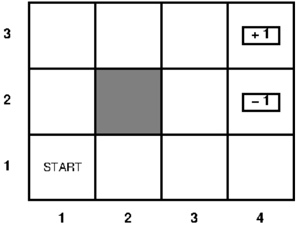
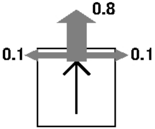




Goal: get to $+1$

Makings plans

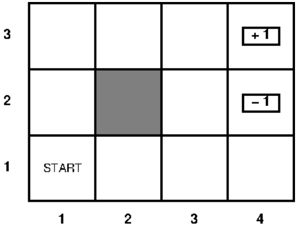


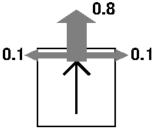


Goal: get to $+1$

Consider the plan $[\mathit{Up}, \mathit{Up}, \mathit{Right}, \mathit{Right}, \mathit{Right}]$:

Makings plans



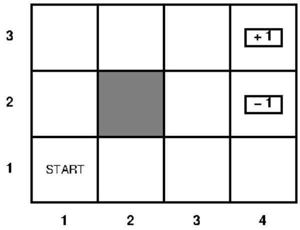


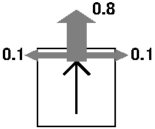
Goal: get to $+1$

Consider the plan $[Up, Up, Right, Right, Right]$:

- With deterministic agents, it gets us to $+1$ with probability 1.

Makings plans



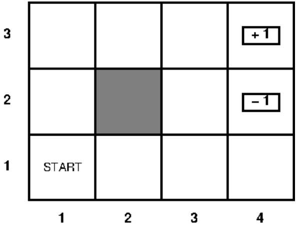


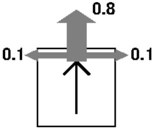
Goal: get to $+1$

Consider the plan $[Up, Up, Right, Right, Right]$:

- With deterministic agents, it gets us to $+1$ with probability 1.
- But now?

Makings plans





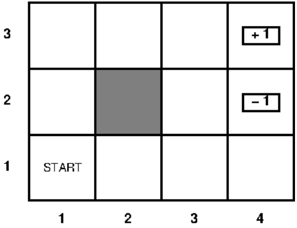
Goal: get to $+1$

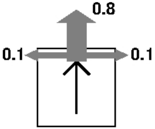
Consider the plan $[Up, Up, Right, Right, Right]$:

- With deterministic agents, it gets us to $+1$ with probability 1.
- But now?

What's the probability that $[Up, Up, Right, Right, Right]$ gets us to $+1$?

Makings plans





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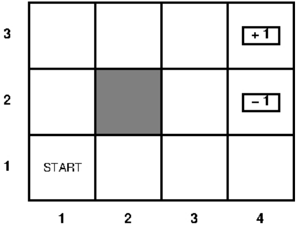

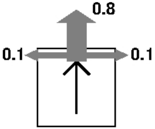

- It's not 0.8^5!

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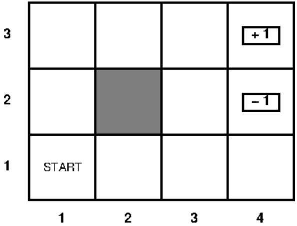

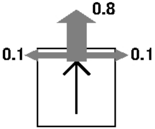

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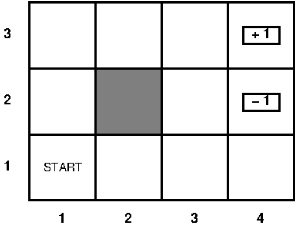
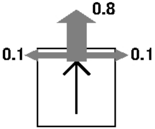
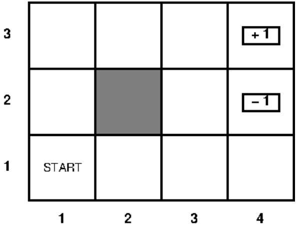
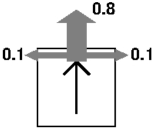




- It's not 0.8^5!
- 0.8^5 is the probability that we get to $+1$ actually using the intended plan $[Up, Up, Right, Right, Right]$
- $0.8^5 = 0.32768$: this means that we do not even get there 1 time out of 3.

Makings plans





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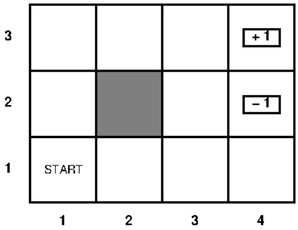

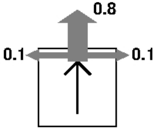

- There is a small chance of [*Up*, *Up*, *Right*, *Right*, *Right*] accidentally reaching the goal by going the other way round!

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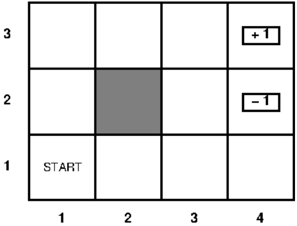

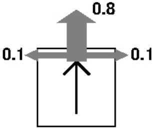

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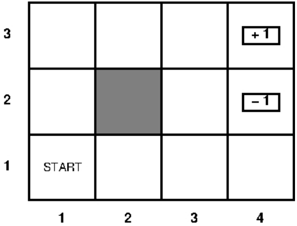
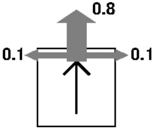
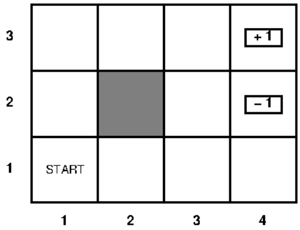
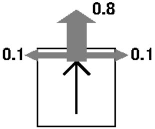




- There is a small chance of $[Up, Up, Right, Right, Right]$ accidentally reaching the goal by going the other way round!
- The probability of this to happen is $0.1^4 \times 0.8 = 0.00008$
- So the probability that $[Up, Up, Right, Right, Right]$ gets us to $+1$ is $0.32768 + 0.00008 = 0.32776$

Makings plans





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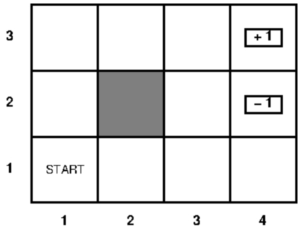

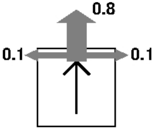

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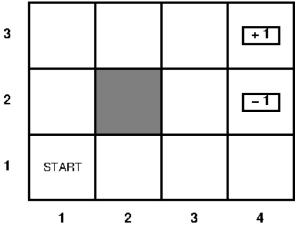
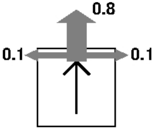
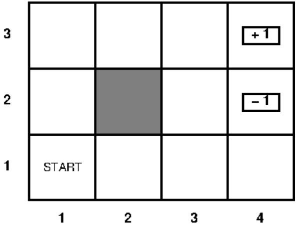
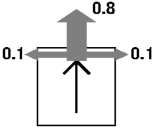




- In this case, the probability of accidental successes doesn't play a significant role. However it might very well, under different decision models, rewards, environments etc.
- 0.32776 is still less than $\frac{1}{3}$, so we don't seem to be doing very well.

Rewards





We introduce a utility function

$$r : S \to \mathbb{R}$$

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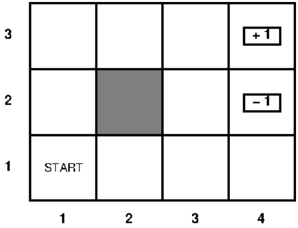

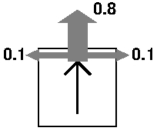

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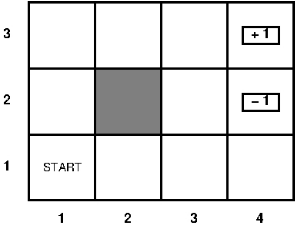

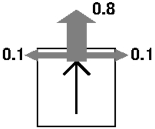

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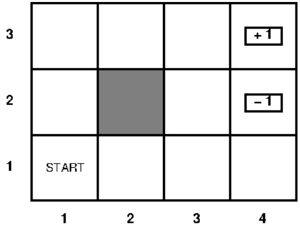

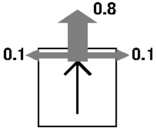

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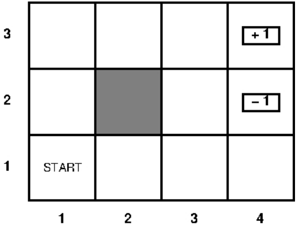

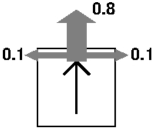

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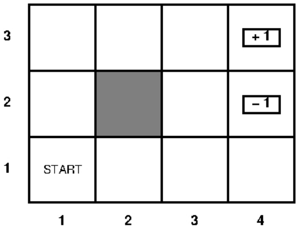

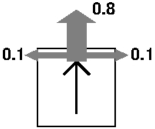

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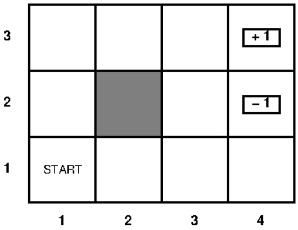

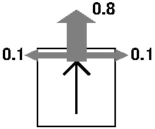

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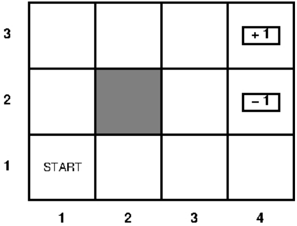
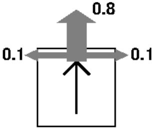
utility and **expected utility** used as general terms applied to actions, states, sequences of states etc. - denoted u

Rewards





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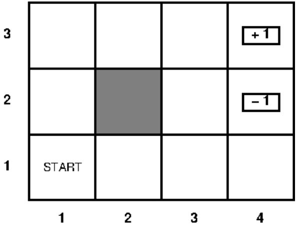

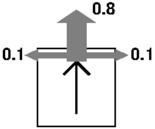

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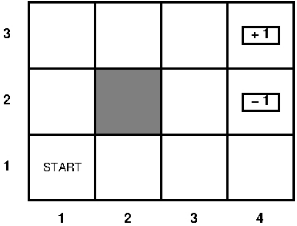

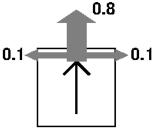

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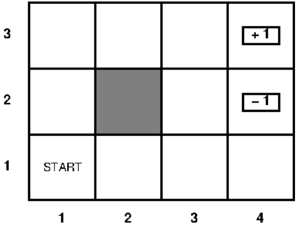

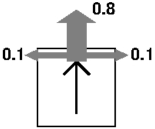

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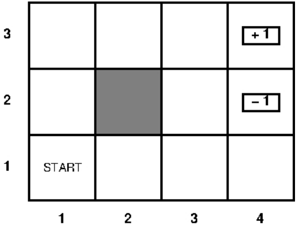

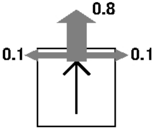

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IT DEPENDS on how we are going to put rewards together!

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$$[r, r_0, r_1, r_2, \ldots] \succ [r, r_0', r_1', r_2', \ldots] \Leftrightarrow [r_0, r_1, r_2, \ldots] \succ [r_0', r_1', r_2', \ldots]$$

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where $\gamma \in [0, 1]$ *is the* **discount factor**

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- Variants: hyperbolic discounting

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$$u[s_1, s_2, \ldots] = \sum_{t=0}^{\infty} \gamma^t r(s_t) \leq \sum_{t=0}^{\infty} \gamma^t \mathbf{r} = \frac{\mathbf{r}}{1-\gamma}$$

Markov Decision Process

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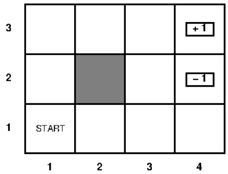
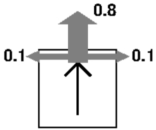
- with a Markovian transition model
- and with discounted (possibly additive) rewards

MDPs formally





Definition

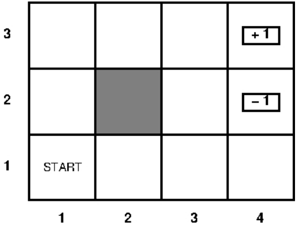
States $s \in S$, actions $a \in A$

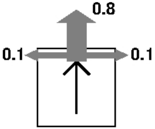
Model $P(s'|s,a)$ = probability that a in s leads to s'

Reward function $R(s)$ (or $R(s,a)$, $R(s,a,s')$) =

$$\begin{cases} -0.04 & \text{(small penalty) for nonterminal states} \\ \pm 1 & \text{for terminal states} \end{cases}$$

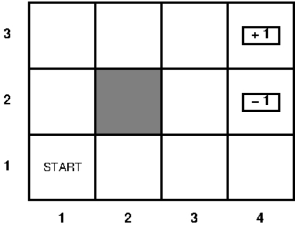
Value of plans

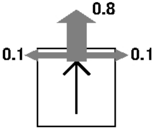




The utility of executing a plan p from state s is given by:

Value of plans

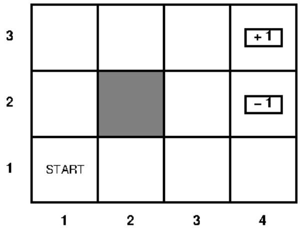


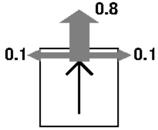


The utility of executing a plan p from state s is given by:

$$v^p(s) = E[\sum_{t=0}^{\infty} \gamma^t r(S_t)]$$

Value of plans





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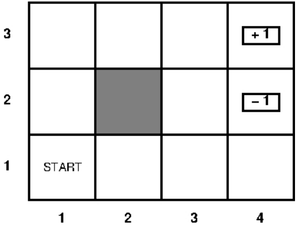
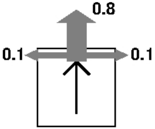
Where S_t is a random variable and the expectation is wrt to the probability distribution over state sequences determined by s and p.

Value of plans





- Calculate the utility of the sequences you can actually perform, times the probability of reaching them.

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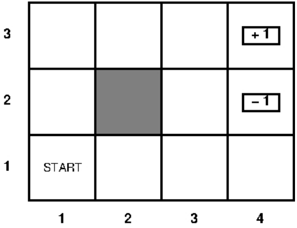

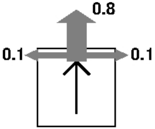

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- Add these numbers

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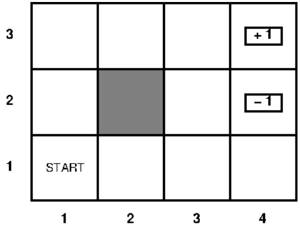
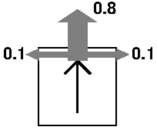
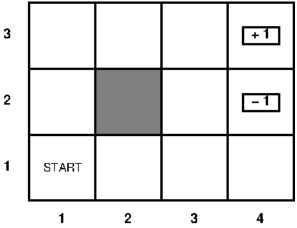
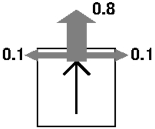




- Calculate the utility of the sequences you can actually perform, times the probability of reaching them.
- Add these numbers
- Forget about the rest

Value of plans





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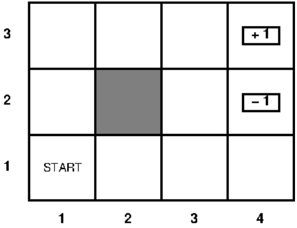
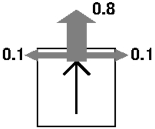
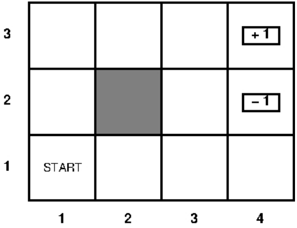
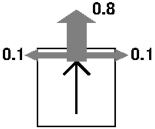




For instance the plan $[Up, Up]$ can generate sequences

Value of plans





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- $([1,1],[2,1],[3,1])$ with probability 0.8^2

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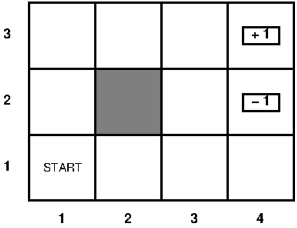

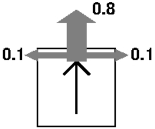

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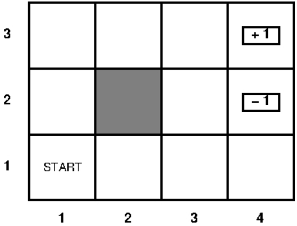
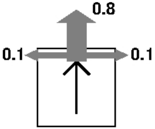
- $([1,1],[2,1],[2,1])$ with probability $2 \times 0.8 \times 0.1$ (collisions)

Value of plans





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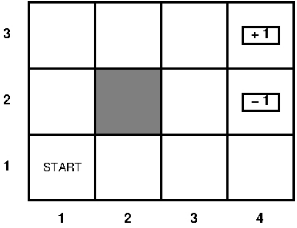
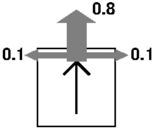
Value of plans





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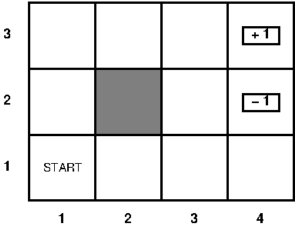

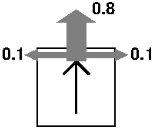

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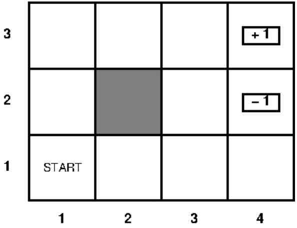

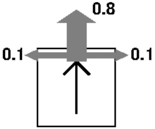

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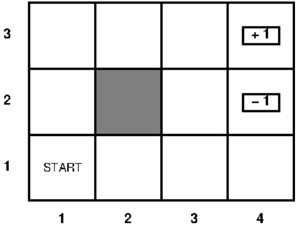

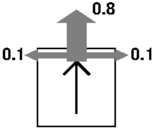

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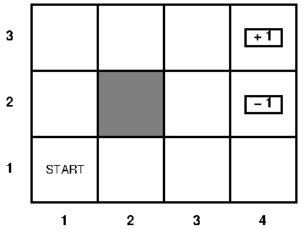

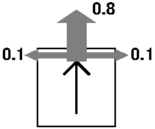

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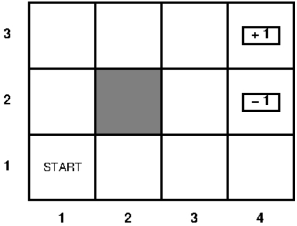

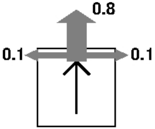

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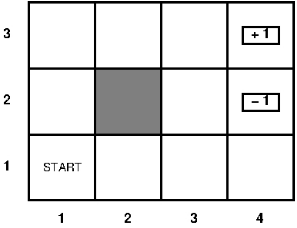

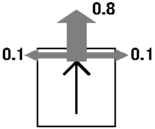

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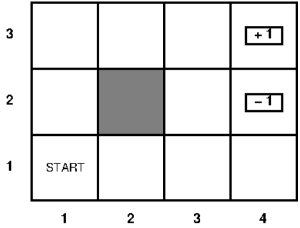
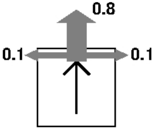
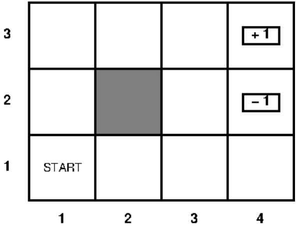


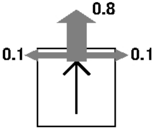


For instance the plan $[Up, Up]$ can generate sequences

- $([1,1],[1,2],[1,1])$ with probability 0.1^2
- $([1,1],[1,2],[1,3])$ with probability 0.1^2
- for a total of nine sequences

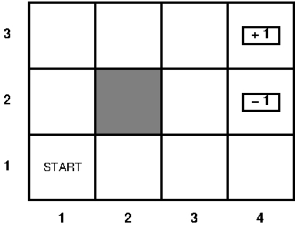
Value of plans

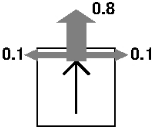




Adding utility and summing up, we have that the expected utility is -0.08

Value of plans





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To be expected, because no matter how we proceed, we are making two steps and at each step getting -0.04 of reward.

Plans vs Policies

- We have looked at a finite sequence of actions. But why should the agent stop after, say, five steps, if she can reach the terminal states in a few steps?

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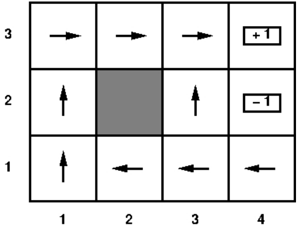
A **policy** is a specification of moves at each decision point

A policy



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We want the **optimal** policy:

$$\pi^*_s = \operatorname*{argmax}_{\pi} v^{\pi}(s)$$

A remarkable fact

Theorem

With discounted rewards and infinite horizons
$\pi^*_s = \pi^*_{s'}$, *for each* $s' \in S$

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Theorem

With discounted rewards and infinite horizons
$\pi^*_s = \pi^*_{s'}$, *for each* $s' \in S$

Idea: Take π^*_a and π^*_b. If they both reach a state c, because they are both optimal, there is no reason why they should disagree. So π^*_c is identical for both. But then they behave the same at all states!

Optimal policies

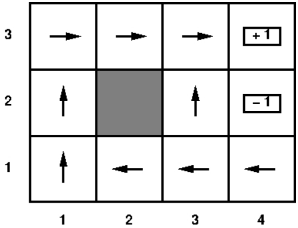
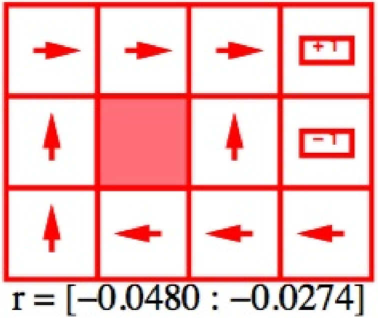


Figure: Optimal policy when state penalty $R(s)$ is –0.04:

Risk and reward



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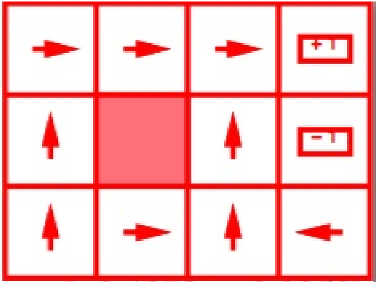
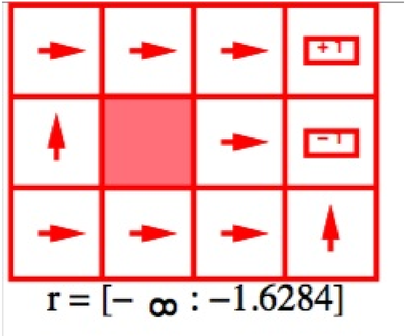


$r = [-0.4278 : -0.0850]$

Risk and reward



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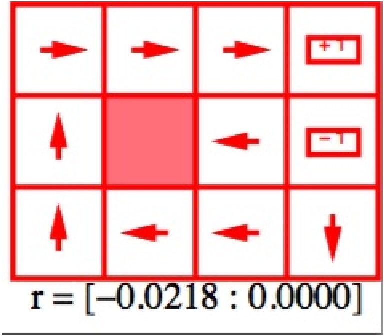

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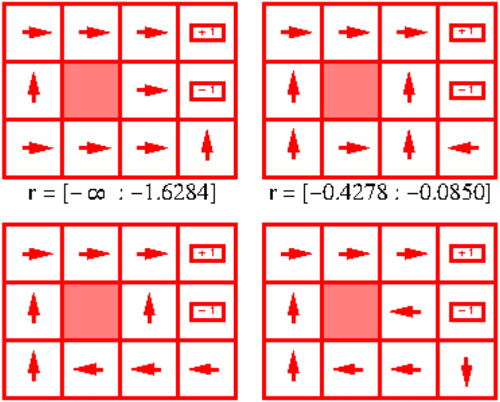

r = [−∞ : −1.6284]   r = [−0.4278 : −0.0850]

r = [−0.0480 : −0.0274]   r = [−0.0218 : 0.0000]

To be continued

- Next Tuesday we are going to finish the slides on MDPs