Rational Agents

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Introduction to Artificial Intelligence
2nd Part

What you have seen

You have seen procedures for computational problem-solving:

- searching
- learning
- planning

What we will be looking at

An agent, a mathematical entity acting in a simple world

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  - Imperfect and incomplete information
  - Quantifying uncertainty, attaching probabilities
  - Going for uncertain outcomes, calculating expected utility

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- Able to take decisions under uncertainty
    - Imperfect and incomplete information
    - Quantifying uncertainty, attaching probabilities
    - Going for uncertain outcomes, calculating expected utility
- Able to update his (or her) beliefs when confronted with new information (learning)

What is rationality?

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Nobel Prize Winner
Economics

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Agents (not only humans) *can* be rational!

The lectures one by one

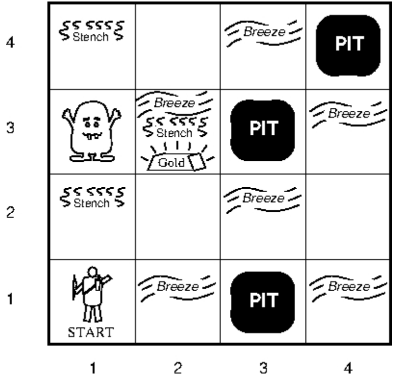
- Logical Agents I
- Logical Agents II
- An Uncertain World
- Making Sense of Uncertainty
- Making (Good) Decisions
- Making Good Decisions in time
- Learning from Experience I
- Learning from Experience II

Logical Agents I

The main reference

Stuart Russell and Peter Norvig
Artificial Intelligence: a modern approach
Chapters 7-9

The Wumpus World



Agents

Sensors	Breeze, Glitter, Smell
Actuators	Turn L/R, Go, Grab, Release, Shoot, Climb
Rewards	1000 escaping with gold, -1000 dying, -10 using arrow, -1 walking
Environment	• Squares adjacent to Wumpus are smelly • Squares adjacent to pit are breezy • Glitter iff gold is in the same square • Shooting kills Wumpus if you are facing it • Shooting uses up the only arrow • Grabbing picks up gold if in same square • Releasing drops the gold in same square

Knowledge base

- A set of sentences representing what the agent thinks about the world.
  - "I am in [2,1]"
  - "I am out of arrows"
  - "I smell Wumpus"
  - "I'd better not go forward"
- We interpret it as what the agent *knows*, but it can very well work for what the agent *believes*.

Updating the knowledge base

- What we TELL the knowledge base
- What we ASK the knowledge base

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   return action
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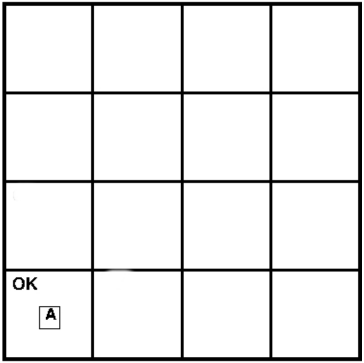
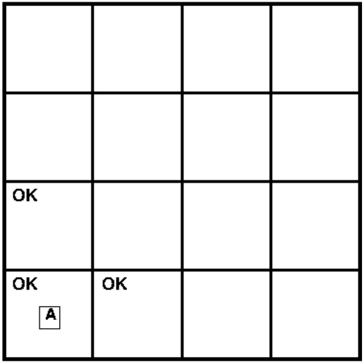
Rational explorations

- The starting state...



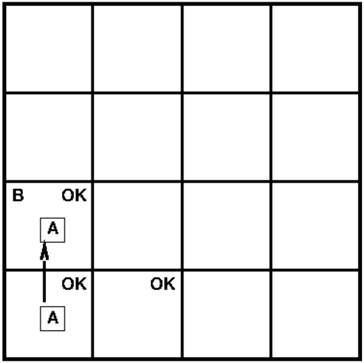
Rational explorations

- and what we know.



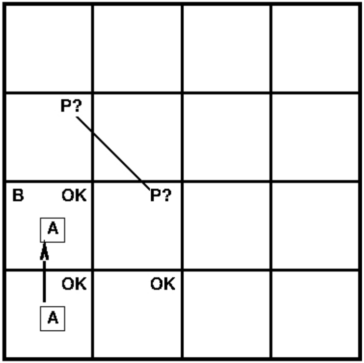
Rational explorations

- B stands for Breeze



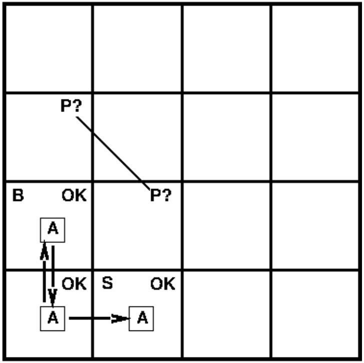
Rational explorations

- Where is the pit?
- We are ruling out one square!



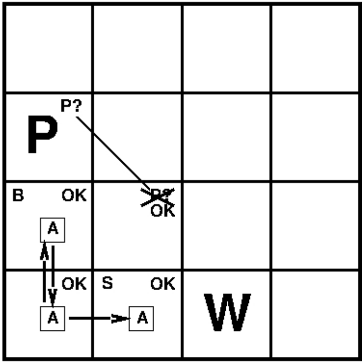
Rational explorations

- S stands for smell
- What do we know?



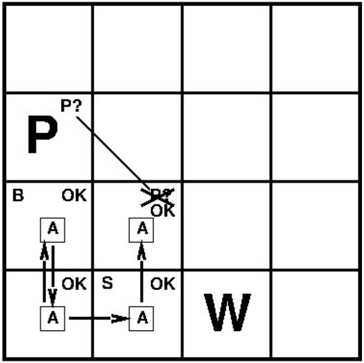
Rational explorations

- Logic is the key



Rational explorations

- The further we go the more we know



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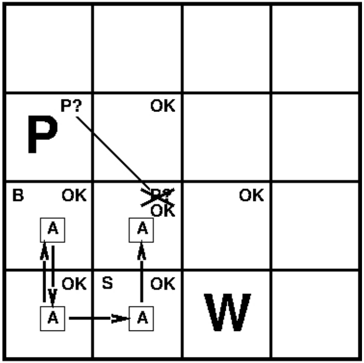
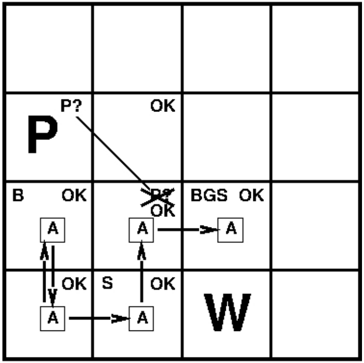


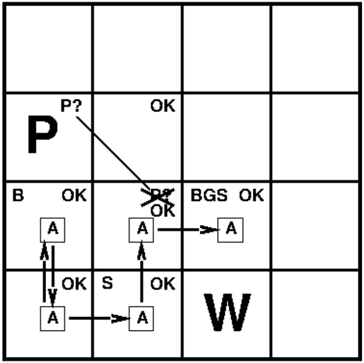
Rational explorations

- Gold!



Rational explorations

- We know the way out
- Game over



Reasoning in the Wumpus World

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

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"Pits cause breezes in adjacent squares"

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"Pits cause breezes in adjacent squares"

$$\begin{aligned} B_{1,1} &\Leftrightarrow (P_{1,2} \vee P_{2,1}) \\ B_{2,1} &\Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1}) \end{aligned}$$

"A square is breezy **if and only if** there is an adjacent pit"

Expressivity: at what cost?

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Propositional Logic lacks expressive power

First order logic

- Massive increase of expressivity
- But there are costs, e.g., decidability
- We will see how to exploit the gains while limiting the costs

FOL KB

- We can encode the KB at each particular time point using FOL

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  - Percept (at given time), e.g., $Percept([Stench, Breeze, Glitter], 5)$ or $Percept([None, Breeze, None], 3)$

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  - Starting Knowledge Base, e.g., $\neg AtGold(0)$

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  - Axioms from knowledge to knowledge, e.g., $\forall\, t \;\; AtGold(t) \wedge Action(Grab, t) \;\Rightarrow\; Holding(Gold, t+1)$

Describing the world

Perception $\forall s, b, t \;\; Percept([s, b, Glitter], t) \Rightarrow AtGold(t)$

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Location $At(Agent, s, t)$

Decision-making $\forall t \;\; AtGold(t) \Rightarrow Action(Grab, t)$

Internal reflection $\forall t \;\; AtGold(t) \wedge \neg Holding(Gold, t) \Rightarrow Action(Grab, t)$, do we have gold already? (notice we cannot observe if we are holding gold, we need to track it)

Describing the world

Adjacent squares

$$\forall x, y, a, b \;\; Adjacent([x, y], [a, b]) \quad \Leftrightarrow$$
$$(x = a \land (y = b - 1 \lor y = b + 1) \lor$$
$$(y = b \land (x = a - 1 \lor x = a + 1))$$

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"A square is breezy **if and only if** there is an adjacent pit"

$$\forall s \;\; , Breezy(s) \;\Leftrightarrow\; \exists r \, (Adjacent(r, s) \wedge Pit(r))$$

Describing the world

- We can go on and describe plans, causal rules, etc.
- But let's do some reasoning now

Facts and Knowledge Bases

"Richard the Lionheart is a king"

Facts and Knowledge Bases

"Joffrey Baratheon is a king"

Telling and Asking

$Tell(KB, King(Joffrey))$

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$Tell(KB, Person(Jaime))$

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$Tell(KB, \forall x \;\; King(x) \;\Rightarrow\; Person(x))$

$Ask(KB, \exists x Person(x))$ is there a person?

$Askvar(KB, Person(x))$ who is a person?

$Askvar$ returns a list of **substitutions**: $\{x/Joffrey\}, \{x/Jaime\}$

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$S\sigma = Smarter(Tyrion, Joffrey)$

$Askvar(KB, S)$ returns some/all σ such that $KB \models S\sigma$

Unification

$$\forall x \;\; King(x) \land Greedy(x) \;\Rightarrow\; Evil(x)$$

$$King(Joffrey)$$

$$\forall y \;\; Greedy(y)$$

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$\theta = \{x/Joffrey, y/Joffrey\}$ works

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p	q	θ
$Knows(Joffrey, x)$	$Knows(Joffrey, Sansa)$	
$Knows(Joffrey, x)$	$Knows(y, Sansa)$	
$Knows(Joffrey, x)$	$Knows(y, Mother(Joffrey))$	
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$Knows(Joffrey, x)$	$Knows(y, Mother(Joffrey))$	$\{y/Joffrey, x/Mother(Joffrey)\}$
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$Knows(Joffrey, x)$	$Knows(y, Mother(Joffrey))$	$\{y/Joffrey, x/Mother(Joffrey)\}$
$Knows(Joffrey, x)$	$Knows(x, Sansa)$	$fail$

Standardising apart

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Standardising apart eliminates overlap of variables, e.g., $Knows(z_{17}, Sansa)$

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Definite clause:
disjunction of literals, **exactly** one of which positive

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$$\frac{p_1{}', \;\; p_2{}', \;\ldots, \;\; p_n{}', \;\; (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta} \qquad \text{where } p_i{}'\theta = p_i\theta \text{ for all } i$$

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Assuming all variables are universally quantified...

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Assuming all variables are universally quantified...

$p_1{}'$ is $King(Joffrey)$	p_1 is $King(x)$
$p_2{}'$ is $Greedy(y)$	p_2 is $Greedy(x)$
θ is $\{x/Joffrey, y/Joffrey\}$	q is $Evil(x)$
$q\theta$ is $Evil(Joffrey)$	

Soundness of GMP

Need to show that

$$p_1', \ldots, p_n', (p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models q\theta$$

provided that $p_i'\theta = p_i\theta$ for all i
Lemma: If φ is definite clause, then $\varphi \models \varphi\theta$ by Universal Instantiation.

1. $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \models (p_1 \wedge \ldots \wedge p_n \Rightarrow q)\theta = (p_1\theta \wedge \ldots \wedge p_n\theta \Rightarrow q\theta)$
2. $p_1', \ldots, p_n' \models p_1' \wedge \ldots \wedge p_n' \models p_1'\theta \wedge \ldots \wedge p_n'\theta$
3. From 1 and 2, $q\theta$ follows by ordinary Modus Ponens

Coming next

- Making sound and efficient inferences
- Where to start?
- How to go on?