Knowledge Representation (II)

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Introduction to Artificial Intelligence

The main reference

Stuart Russell and Peter Norvig
Artificial Intelligence: a modern approach
Chapter 9

Today's class

- Start with a knowledge base and try to prove something interesting
- Various methods for doing so, with different computational properties

Artificial Intelligence and Law

The formalization of legislation and the development of computer systems to assist with legal problem solving provide a rich domain for developing and testing artificial-intelligence technology.

Marek Sergot, Fariba Sadri, Robert Kowalski, (and others)
The British Nationality Act as a logic program
Communications of the ACM, 1986

An informal knowledge base

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

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Prove that Colonel West is a criminal

Example knowledge base contd.

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Example knowledge base contd.

... it is a crime for an American to sell weapons to hostile nations:

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Example knowledge base contd.

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$\forall x \;\; Missile(x) \wedge Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$

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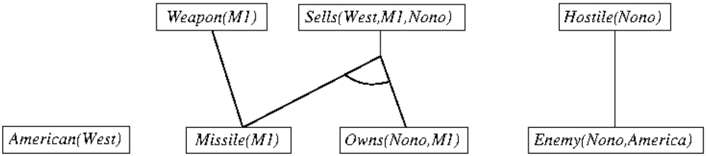
$Enemy(Nono, America)$

Forward chaining proof

$\boxed{American(West)}$ $\boxed{Missile(M1)}$ $\boxed{Owns(Nono,M1)}$ $\boxed{Enemy(Nono,America)}$

Forward chaining proof



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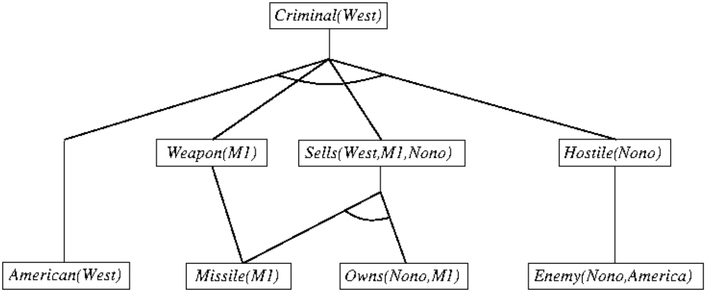
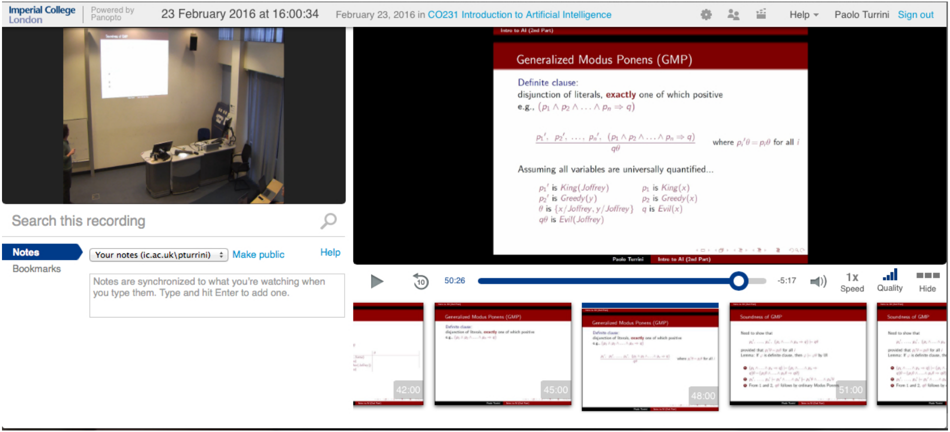


Footage from Tuesday



GMP: recall...

Definite clause:
disjunction of literals, **exactly** one of which positive
e.g., $(p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)$

$$\frac{p_1{}',\;\; p_2{}',\; \ldots,\; p_n{}',\;\; (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{q\theta} \qquad \text{where } p_i{}'\theta = p_i\theta \text{ for all } i$$

Assuming all variables are universally quantified...

$p_1{}'$ is $King(Joffrey)$	p_1 is $King(x)$
$p_2{}'$ is $Greedy(y)$	p_2 is $Greedy(x)$
θ is $\{x/Joffrey, y/Joffrey\}$	q is $Evil(x)$
$q\theta$ is $Evil(Joffrey)$	

Forward chaining algorithm

function FOL-FC-ASK(KB, α) **returns** a substitution or *false*
inputs: *KB*, the knowledge base, a set of definite clauses
α, the (atomic) query
local variables: *new*, the new sentences inferred at each iteration

repeat until *new* is empty
  new $\leftarrow \{ \}$
  for each sentence *r* **in** *KB* **do**
    $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \leftarrow$ STANDARDIZE-APART(r)
    for each θ such that $(p_1 \wedge \ldots \wedge p_n)\theta = (p'_1 \wedge \ldots \wedge p'_n)\theta$
      for some $p'_1, \ldots, p'_n$ in *KB* **do**
      $q' \leftarrow q\theta$
      if q' does not already unify in *KB* or *new* **then**
        add q' to *new*
        $\phi \leftarrow$ UNIFY(q', α)
        if ϕ is not *fail* **then return** ϕ
  add *new* to *KB*
return *false*

First iteration

On the first iteration...

$$American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

First iteration

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$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

has unsatisfied premises

First iteration

$$\forall x \;\; Missile(x) \wedge Owns(Nono, x) \;\Rightarrow\; Sells(West, x, Nono)$$

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is satisfied with $\{x/M_1\}$ and $Sells(West, M_1, Nono)$ is added

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$$Enemy(x, America) \Rightarrow Hostile(x)$$

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First iteration

So in total we have added...

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$Sells(West, M_1, Nono)$

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$Sells(West, M_1, Nono)$

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So in total we have added...

$Sells(West, M_1, Nono)$

$Weapon(M_1)$

$Hostile(Nono)$

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On the second iteration...

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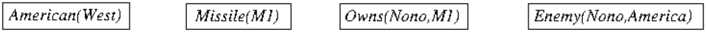
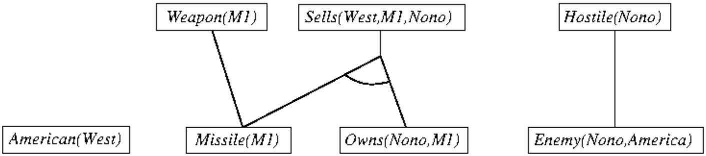
Yay! :)

Forward chaining proof



Forward chaining proof



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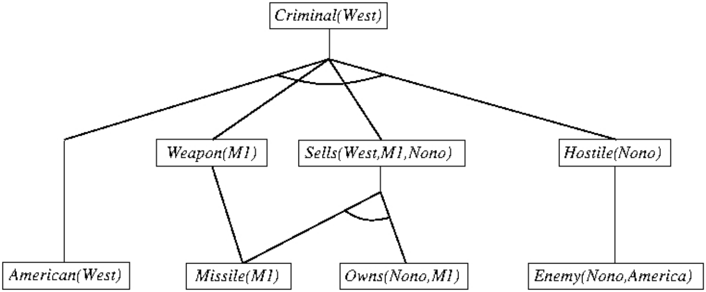

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- Sound (because of soundness of GMP)

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Forward chaining is widely used in deductive databases

Backward chaining example

$\boxed{Criminal(West)}$

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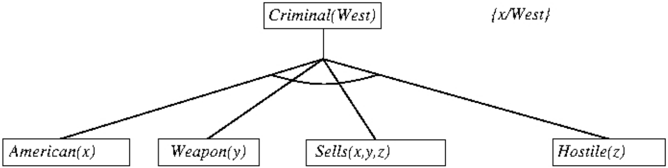

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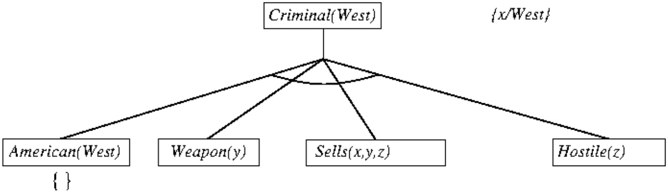

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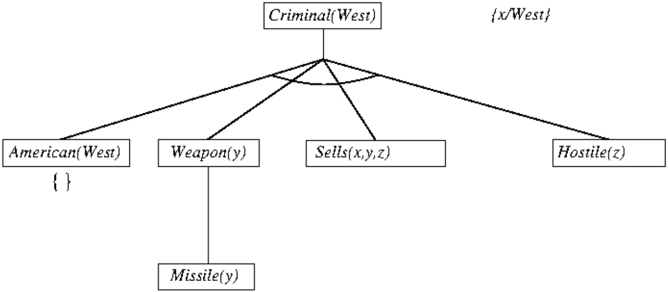

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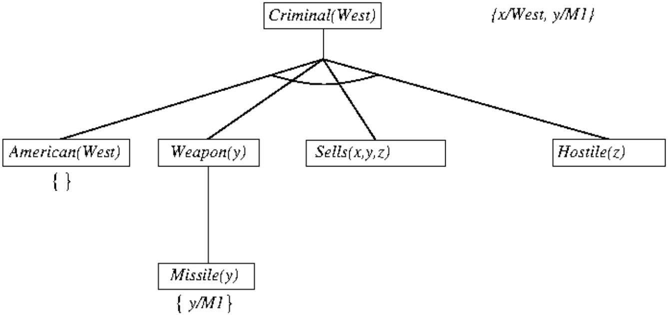

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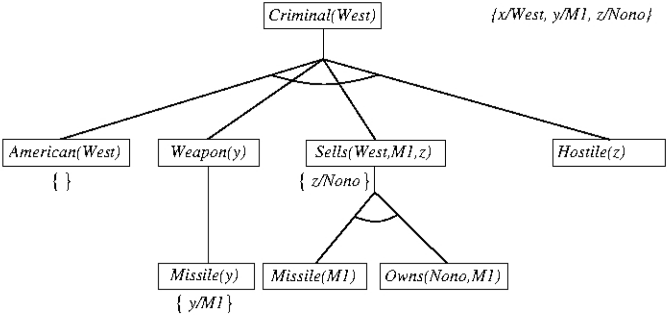

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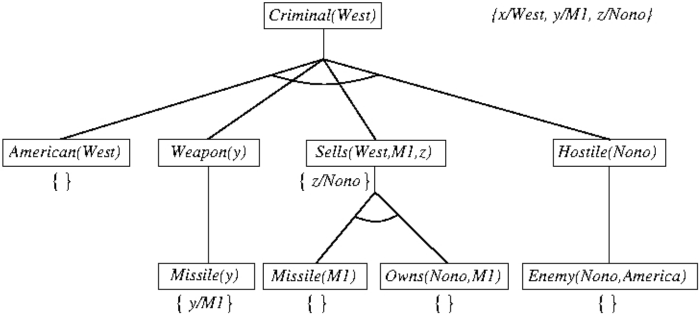


Backward chaining algorithm

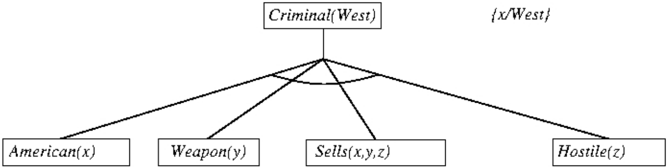
function FOL-BC-ASK($KB, goals, \theta$) **returns** a set of substitutions
  inputs: KB, a knowledge base
    $goals$, a list of conjuncts forming a query (θ already applied)
    θ, the current substitution, initially the empty substitution $\{\,\}$
  local variables: $answers$, a set of substitutions, initially empty

  if $goals$ is empty **then return** $\{\theta\}$
  $q' \leftarrow \textsc{First}(goals)\theta$
  for each sentence r **in** KB
    where $\textsc{Standardize-Apart}(r) = (\,p_1 \wedge \ldots \wedge p_n \Rightarrow q)$
    and $\theta' \leftarrow \textsc{Unify}(q, q')$ succeeds
   $new\_goals \leftarrow [\,p_1, \ldots, p_n | \textsc{Rest}(goals)]$
    $answers \leftarrow \text{FOL-BC-ASK}(KB, new\_goals, \textsc{Compose}(\theta', \theta)) \cup$
$answers$
  return $answers$

Backward chaining example

$$\boxed{Criminal(West)}$$

Backward chaining example



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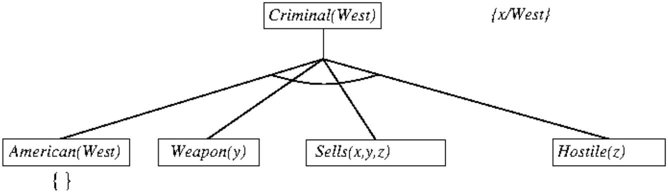

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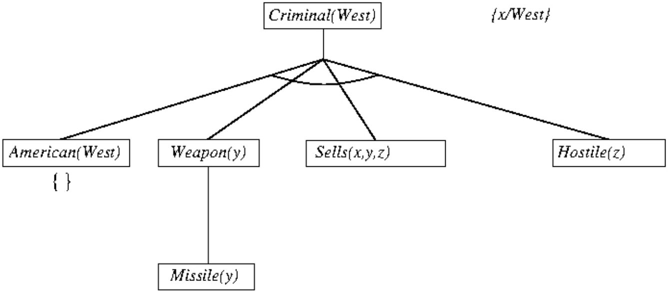

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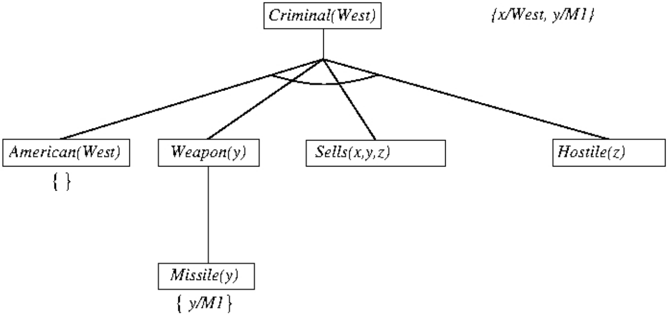

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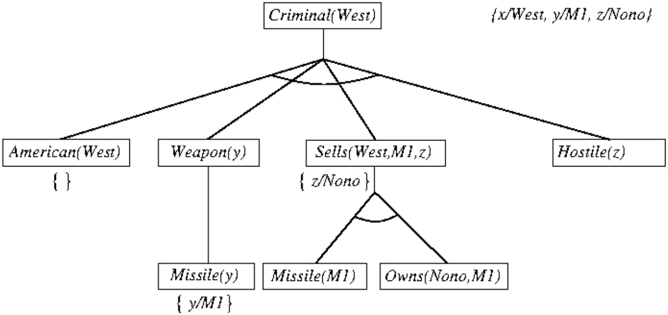

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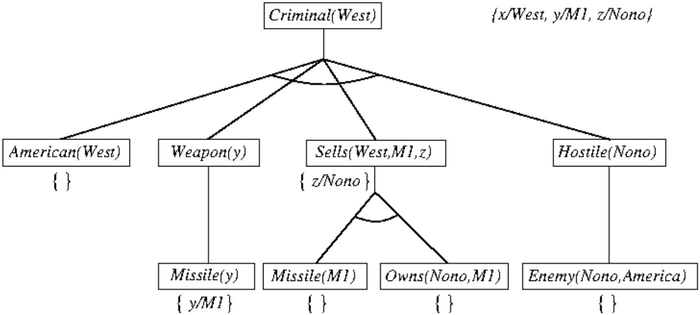

Properties of backward chaining

Depth-first recursive proof search

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- space is linear in size of proof

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Widely used for logic programming

Resolution

Full first-order version:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{(\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)\theta}$$

where $\textsc{Unify}(\ell_i, \neg m_j) = \theta$.

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For example,

$$\neg Rich(x) \vee Unhappy(x)$$

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Apply resolution steps to $CNF(KB \wedge \neg\alpha)$

Conversion to CNF

Everyone who dislikes pizzas is disliked by someone:

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2. Move $\neg$ inwards: $\neg\forall x\varphi \ \equiv \exists x\ \neg\varphi$, $\ \neg\exists x\varphi \ \equiv \forall x\ \neg\varphi$:

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Conversion to CNF contd.

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$$\forall x \;\; [Pizza(F(x)) \wedge \neg Dislikes(x, F(x))] \vee Dislikes(G(x), x)$$

Conversion to CNF contd.

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Conversion to CNF contd.

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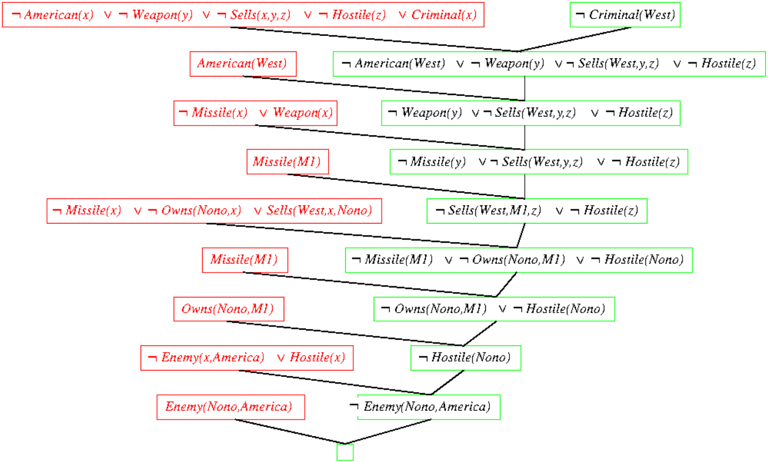
6. Distribute $\wedge$ over $\vee$:

$$[Pizza(F(x)) \vee Dislikes(G(x), x)] \wedge$$

$$\wedge [\neg Dislikes(x, F(x)) \vee Dislikes(G(x), x)]$$

Resolution proof: definite clauses



What we have seen

Inference methods with different structure and properties

- Forward-chaining (deductive databases)
- Backward-chaining (logic programming)
- Resolution (full-first order logic)

Coming next

- Knowledge and uncertainty: what if we don't know exactly?
- How to handle uncertain information:
  - representation
  - reasoning

Appendix: Universal Instantiation

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$$\frac{\forall v \;\; \alpha}{\alpha(\{v/g\})}$$

for any variable v and ground term g

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$$\begin{array}{l} King(Joffrey) \wedge Greedy(Joffrey) \Rightarrow Evil(Joffrey) \\ King(Aerys) \wedge Greedy(Aerys) \Rightarrow Evil(Aerys) \\ King(Father(Joffrey)) \wedge Greedy(Father(Joffrey)) \Rightarrow Evil(Father(Joffrey)) \\ \quad \vdots \end{array}$$

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E.g., $\exists x \;\; Crown(x) \wedge OnHead(x, John)$ yields

$$Crown(C_1) \wedge OnHead(C_1, John)$$

provided C_1 is a **new** constant symbol, called a Skolem constant

Appendix: Existential instantiation contd.

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the new KB is logically equivalent to the old

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the new KB is **not** equivalent to the old,

but is satisfiable iff the old KB was satisfiable