Airline Optimisation

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Outline

1. Introduction
2. The basic fleet assignment model
3. Demonstration
4. Cost and profit calculation
5. Enhancement
6. Network and fleet planning
7. Time windows
8. Demand forecasting
9. Conclusion

Introduction

- Aim: minimise costs or maximise profit
- Basic fleet assignment
  - Task: allocate an aircraft to each flight leg
  - Operation research problem

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The basic fleet assignment model

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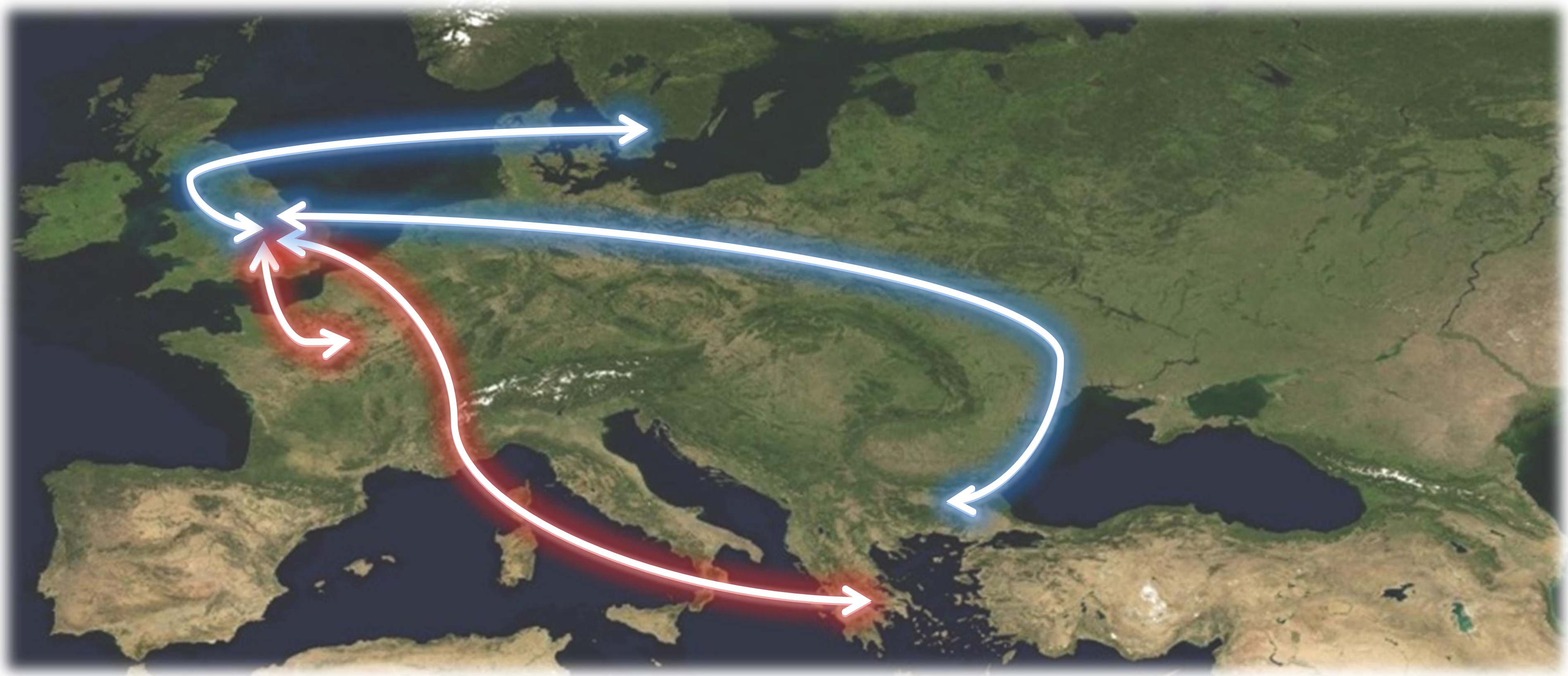
The basic fleet assignment model



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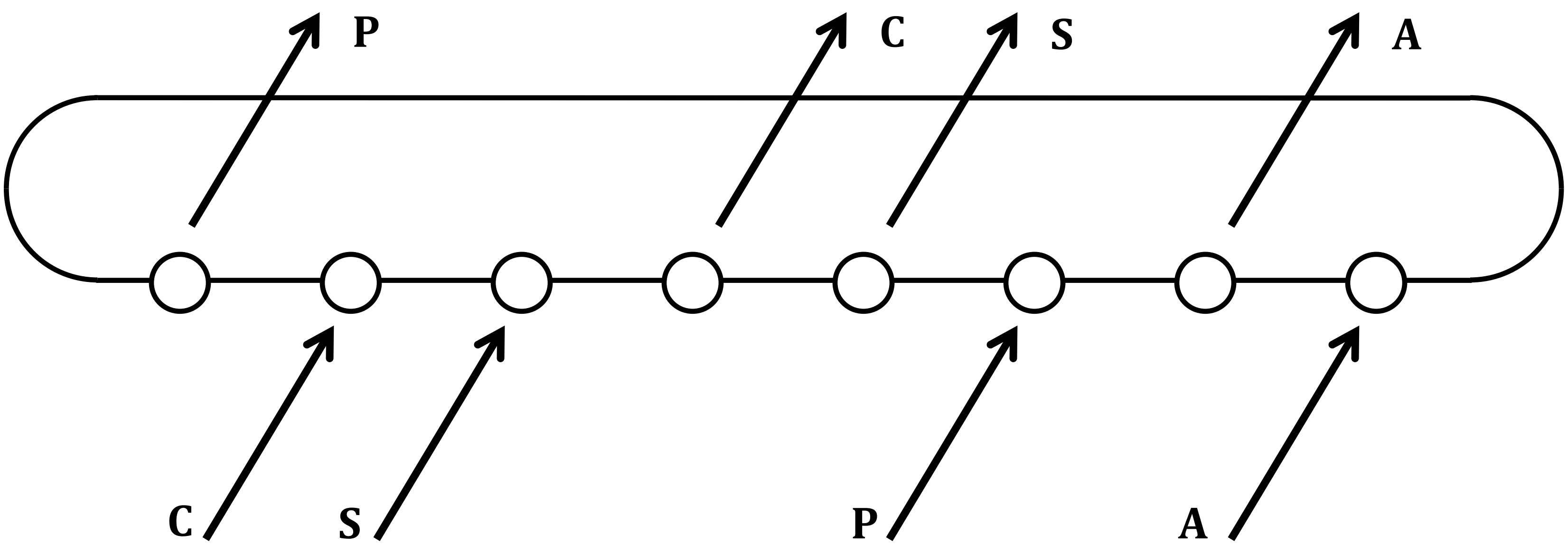
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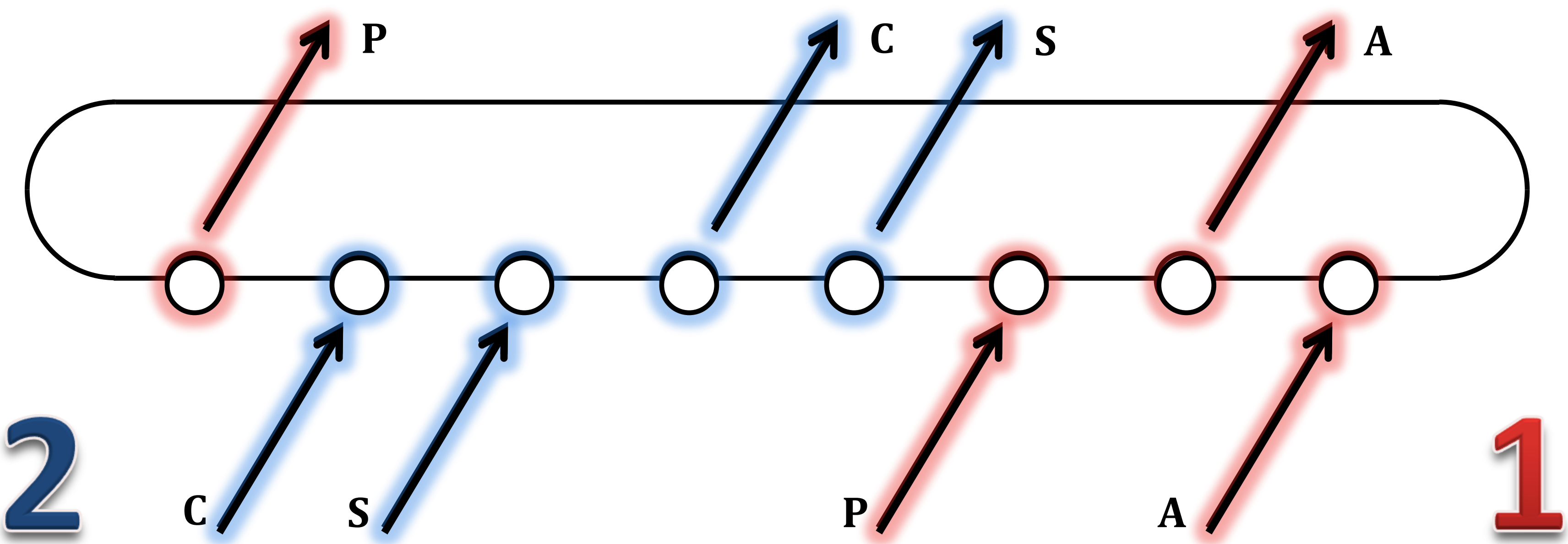
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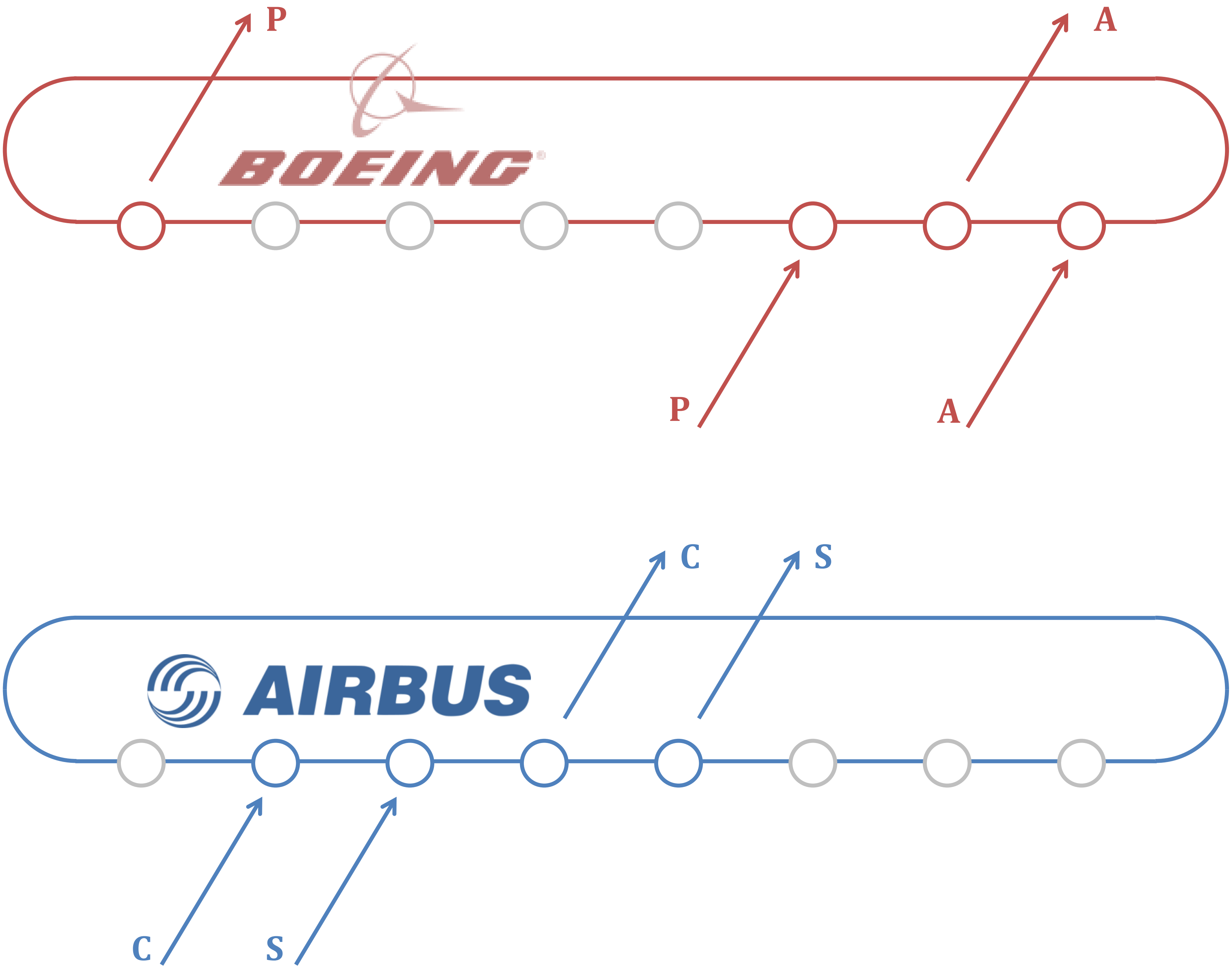
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Demonstration

Input

Flight schedule

ID	Dept.	Arr.	Fleet	Dept. time	Arr. time	Ready time
1	LHR	CDG	A340	360	480	505
1	LHR	CDG	B747	360	460	485
2	CDG	LHR	A340	700	920	945
2	CDG	LHR	B747	700	900	925
…	…	…	…	…	…	…

Fleet information

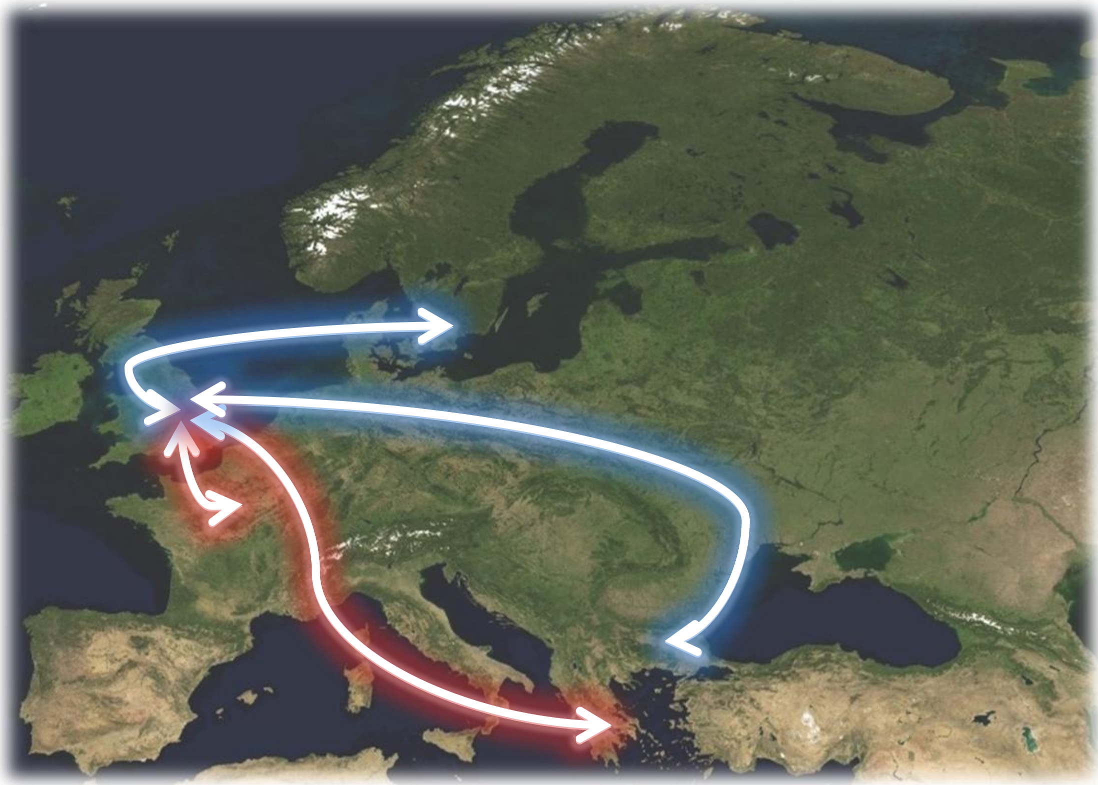
Aircraft	Code	Total	Seats	Weight (kg)	Consumption
Airbus 340	A340	2	308	177,800	10,980
Boeing 747	B747	1	344	396,890	12,788

Demonstration

Output

Fleet assignment

Flight Leg	Airbus	Boeing
LHR – CDG	0	1
CDG - LHR	0	1
LHR – CPH	1	0
CPH – LHR	1	0
LHR – SOF	1	0
SOF – LHR	1	0
LHR – ATH	0	1
ATH – LHR	0	1



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Cost and profit calculation

- Passenger demand data is confidential
- We sample Gamma distribution to generate demand
- Flight duration – Ticket price relationship

Passenger Demand

$$\mu \sim U(\min capacity, \max capacity)$$

$\sigma = Z\sqrt{\mu}$ where $Z \sim U(1,2)$

$$\Gamma(k, \theta)$$

$$\mu = k\theta$$

$$\sigma^2 = k\theta^2$$

Spill and Spill Cost

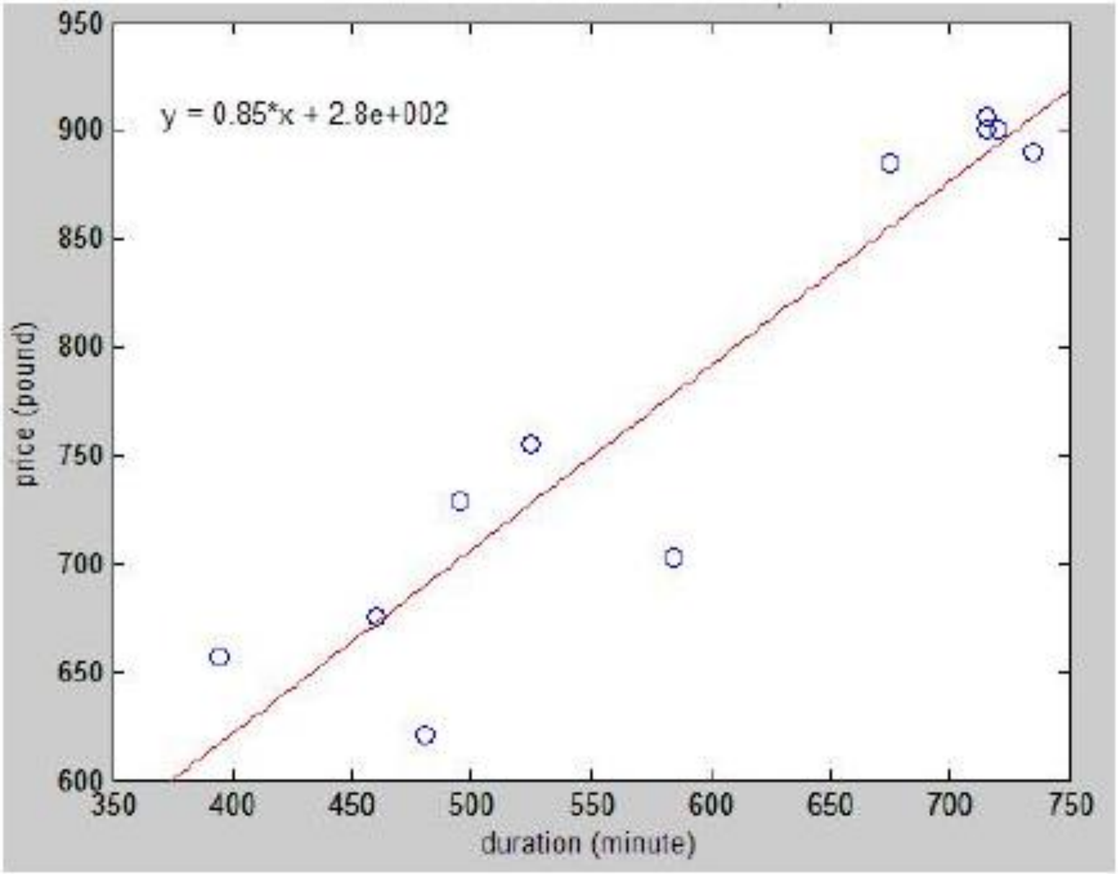
spill = max{demand – capacity, 0}

spill cost = spill * ticket price

Ticket Price

Price = 0.85 x Duration + 280

price in £ and duration in min



Cost and profit calculation

- Passenger demand data is confidential
- We sample Gamma distribution to generate demand
- Flight duration – Ticket price relationship

<u>For a single flight</u>:

cost = fuel consumption * duration * fuel price + landing fees * weight

total cost = cost + spill cost

revenue = min{capacity, demand} * ticket price

profit = revenue - cost

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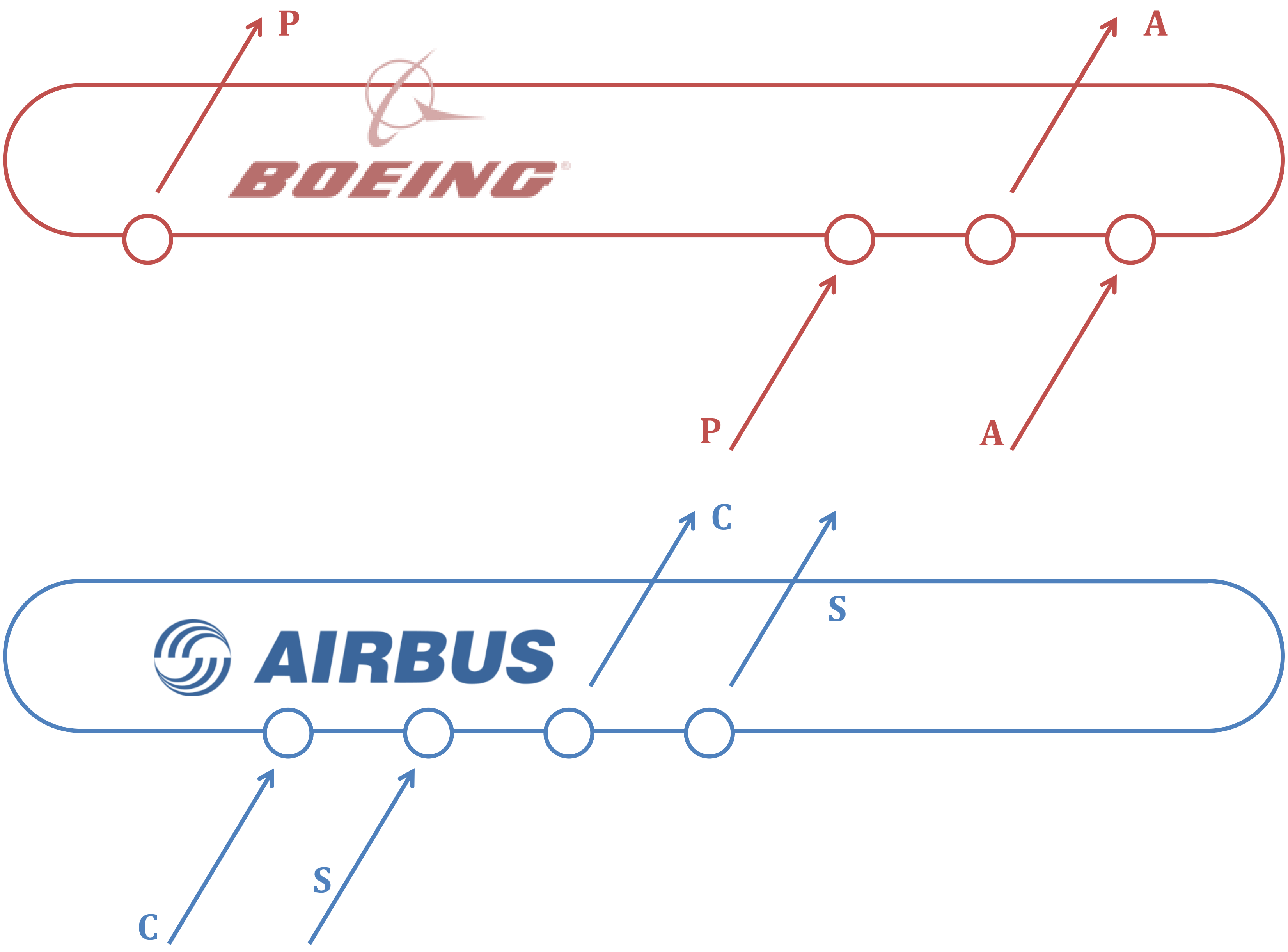
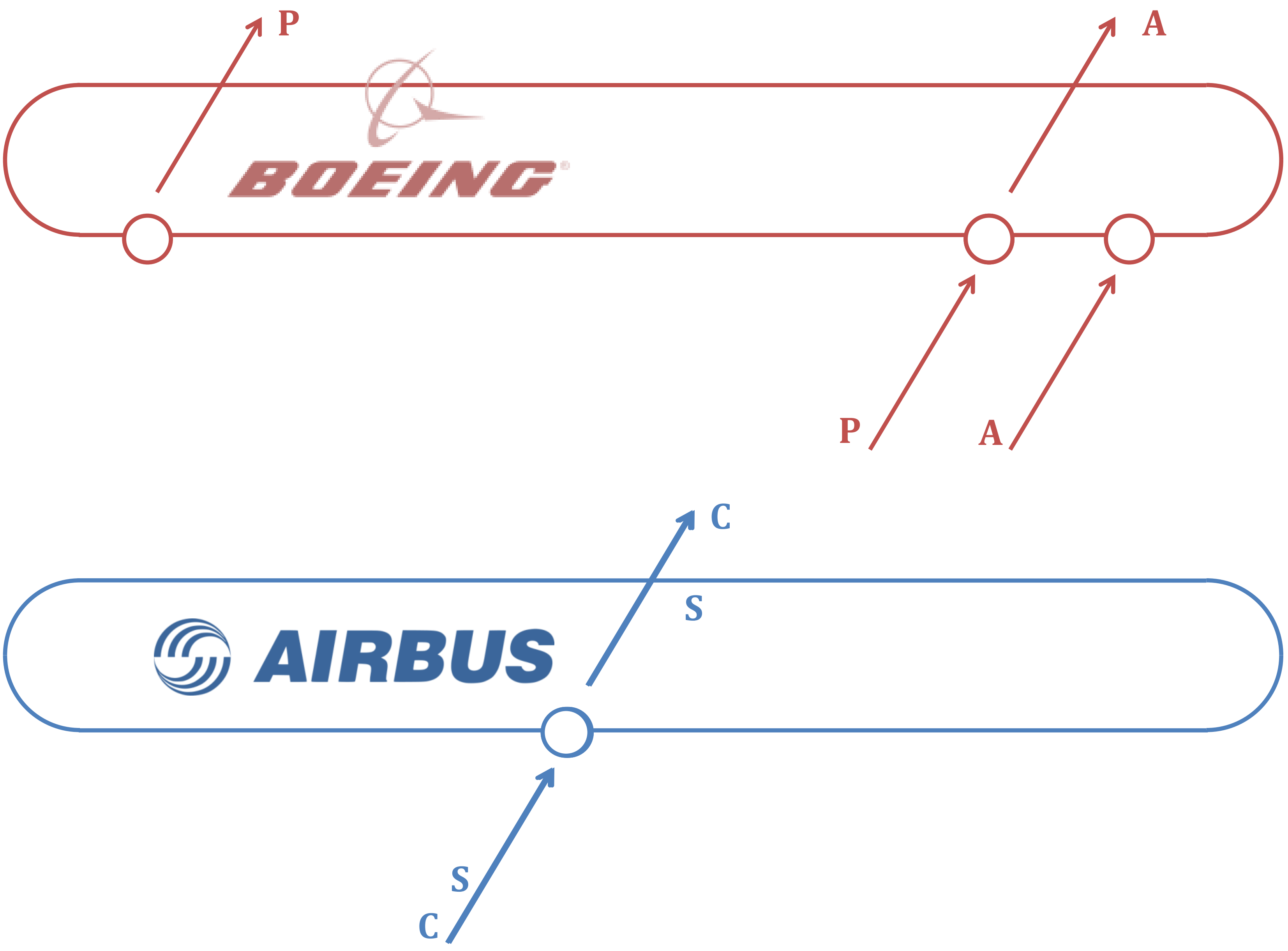
Aggregation



Aggregation



Aggregation

	Time reduction
Virgin	19.34 %
KLM	14.46 %

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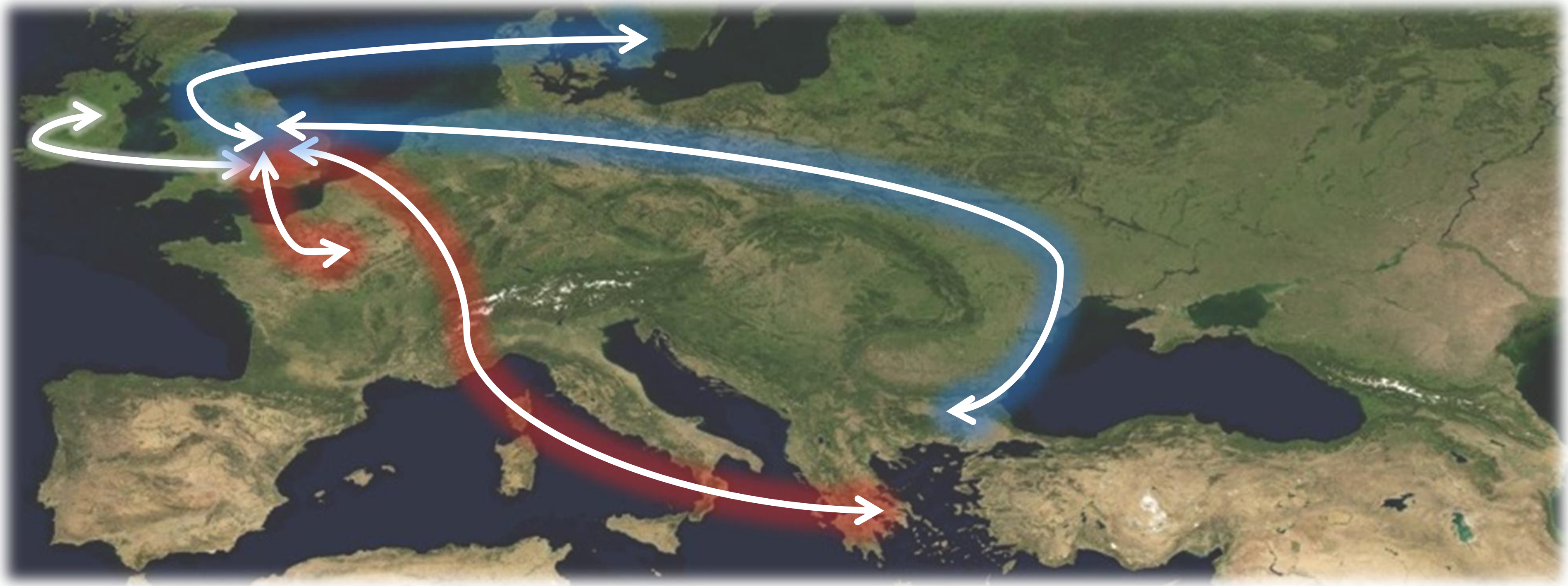
Network and fleet planning

Problem

Extra flight leg (Dublin).

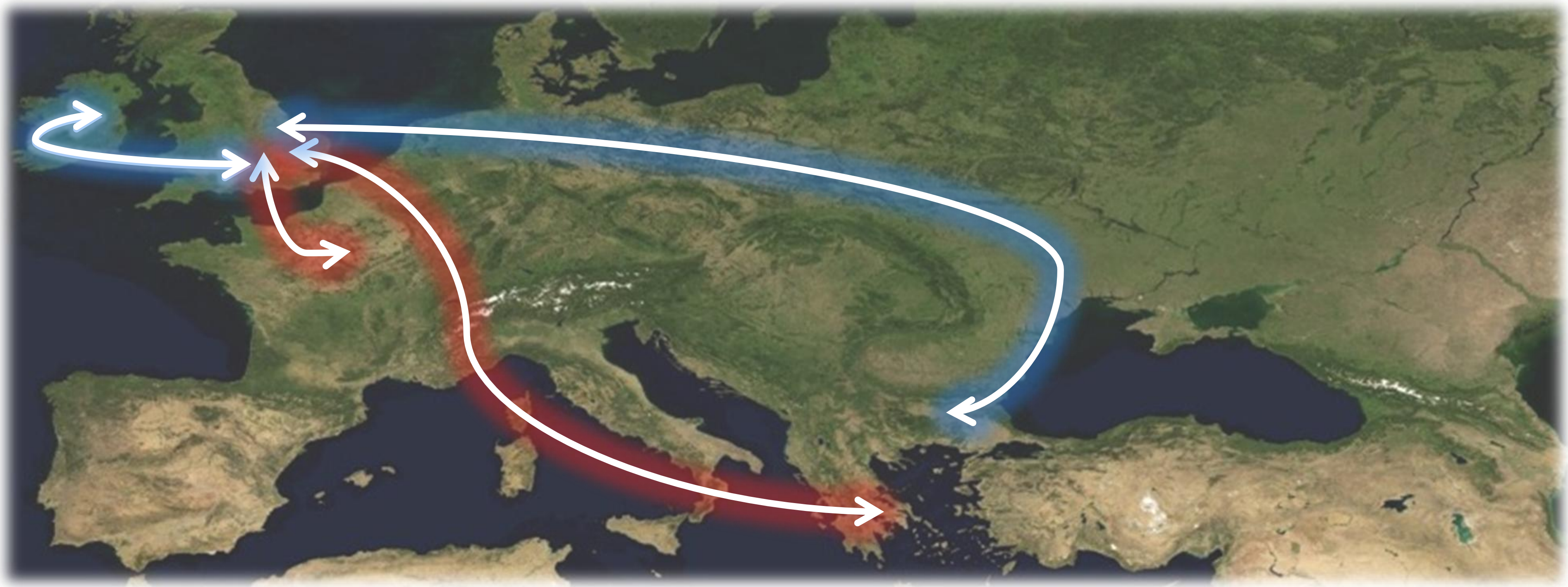
Not enough aircraft to service it.



Network planning

Idea #1

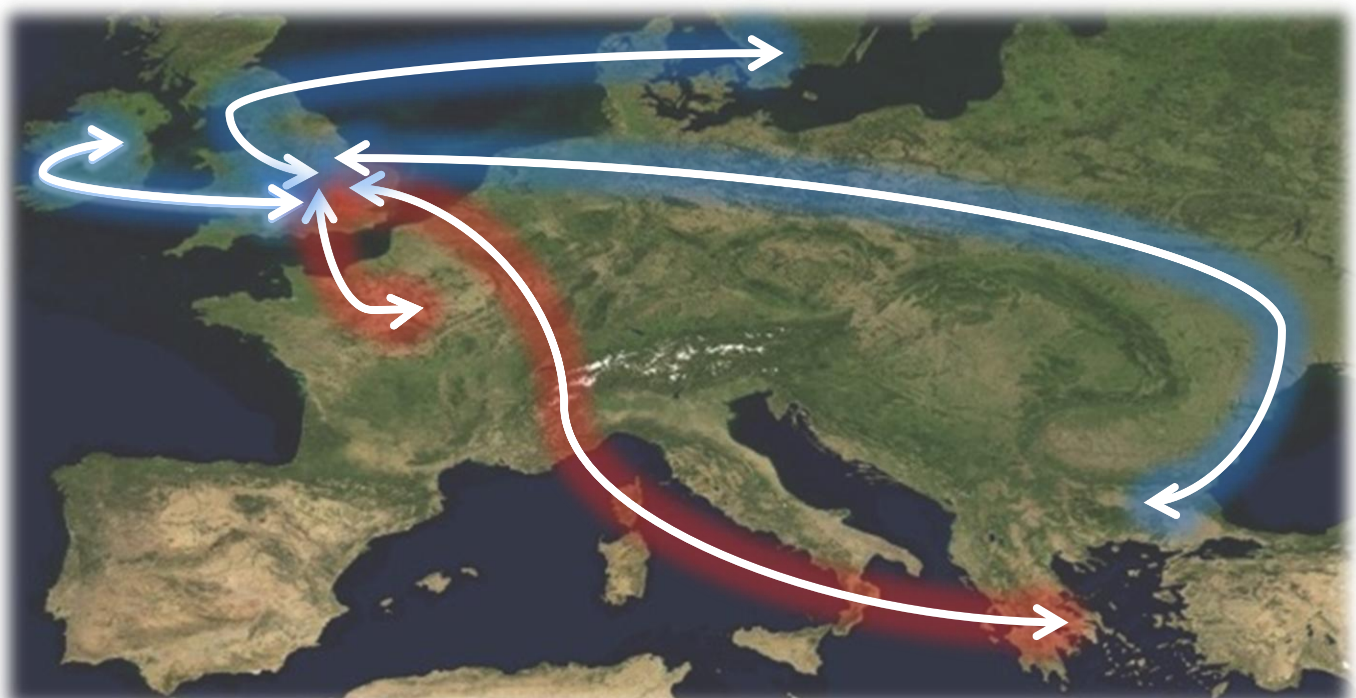
Keep the most profitable legs and eliminate the others.



Fleet planning

Idea #2

Buy additional aircraft.



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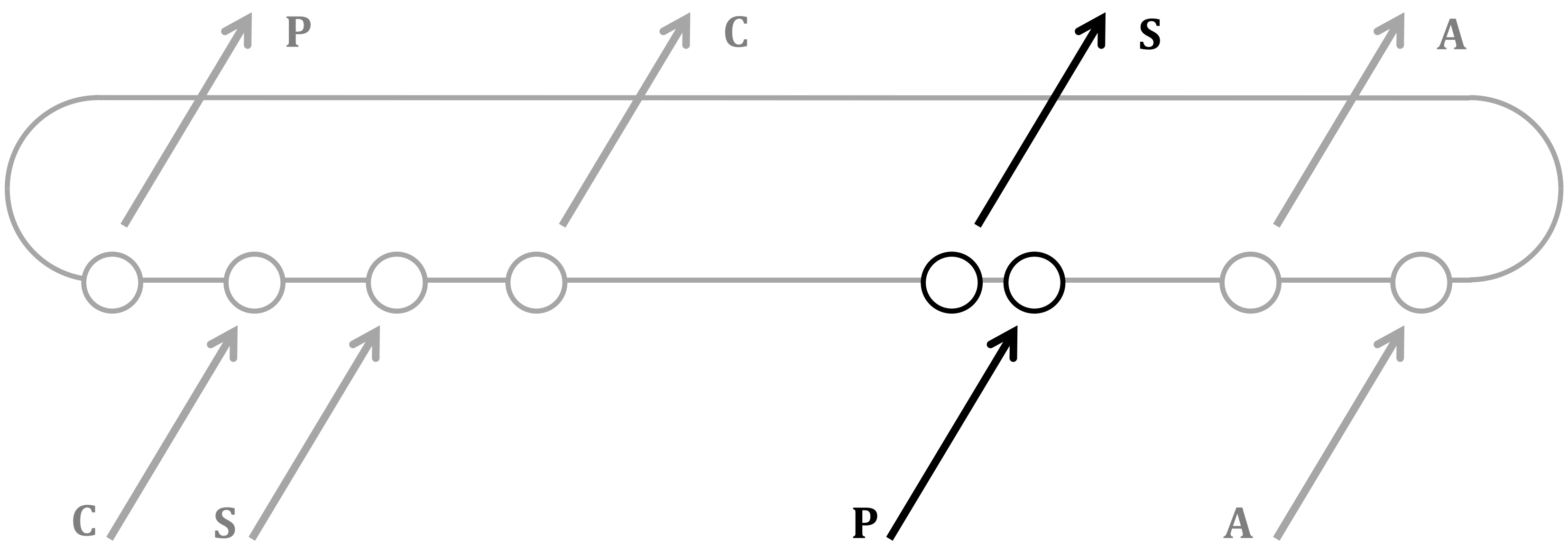
Time windows

Idea

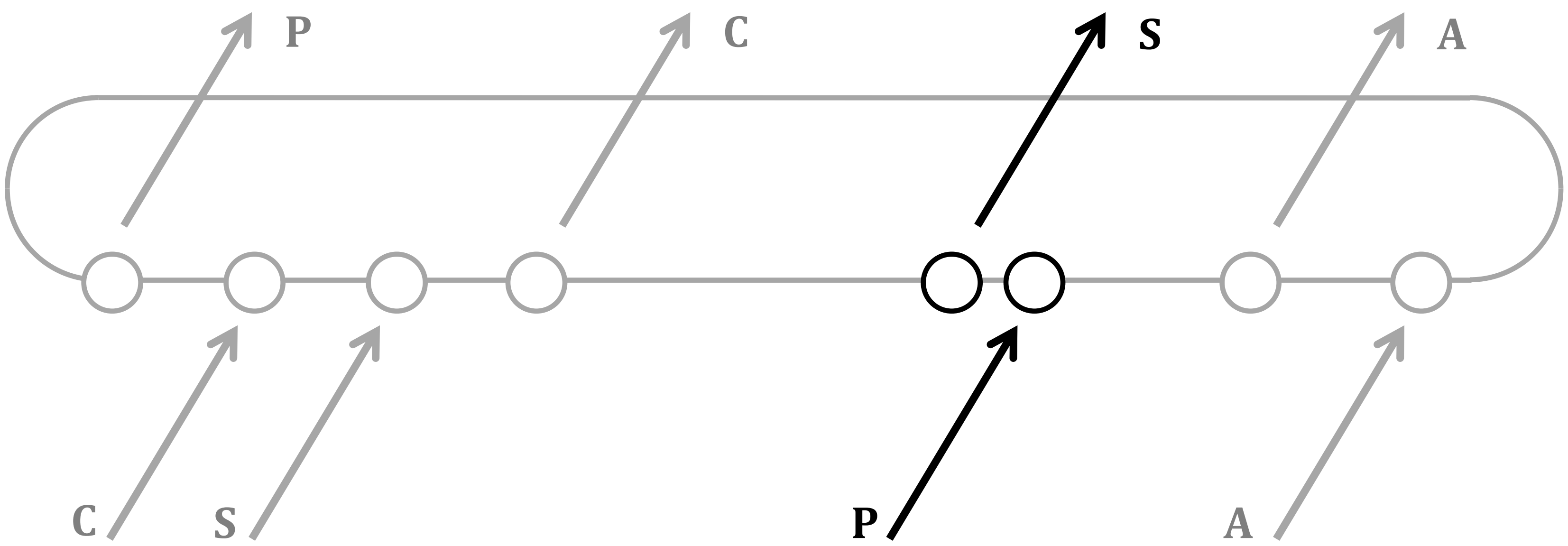
Assume a flight arrives from Paris less than 30 minutes after one leaves for Sofia



Time windows

Idea

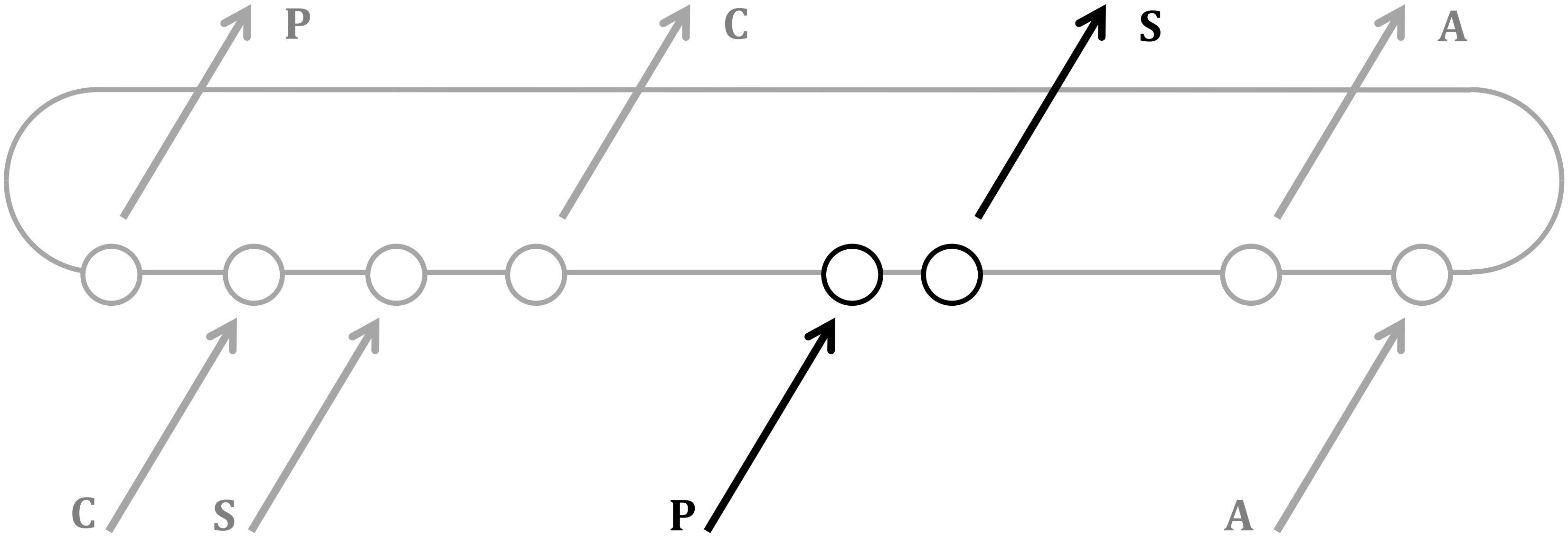
Some passengers from Paris might have been interested in a connection to Sofia.



Time windows

Idea

Slight schedule change ➔ increase demand for Sofia flight ➔ increase profit



Time windows

We used a 30-minute time window for Virgin, and a 20-minute time window for KLM.

	Cost decrease	Profit increase
Virgin	10.52	4.46
KLM	5.87	22.80

Yearly results in £Million.

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Demand Forecasting

R – real

E – expected

$$r = \frac{c_R(X_R) - c_R(X_E)}{c_R(X_R)}$$

	Cost increase	Profit decrease
Virgin	48.68	53.92
KLM	144.68	255.04

Yearly results in £Million.

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Conclusion

Virgin Airlines

Aggregation Time reduction
19.34 %

Time windows	
Cost decrease	Profit increase
10.52	4.46

Yearly results in £Million for a 30-minute time windows.

Bibliography

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Models

Basic fleet assignment

$$\min \sum_{i \in L} \sum_{f \in F} c_{fi} X_{fi} \tag{1}$$

$$\sum_{f} X_{fi} = 1, \ \forall i \in L, \tag{2}$$

$$\sum_{d} X_{fdot} + Y_{fot^{-}t} - \sum_{d} X_{fodt} - Y_{fott^{+}} = 0, \ \forall \{fot\} \in N, \tag{3}$$

$$\sum_{i \in O(f)} X_{fi} + \sum_{o \in C} Y_{fot_n t_1} \leq S(f), \ \forall f \in F, \tag{4}$$

$$Y_{fott^{+}} \geq 0, \ \forall \{fott^{+}\} \in N, \tag{5}$$

$$X_{fi} \in \{0, 1\}, \ \forall i \in L \text{ and } f \in F. \tag{6}$$

Models
Network planning

$$\min \sum_{i\in L}\sum_{f\in F}(-p_{fi})X_{fi} \tag{7}$$

$$\sum_{f} X_{fi} = Z_i, \ \forall i \in L, \tag{8}$$

$$\sum_{d} X_{fdot} + Y_{fot^-t} - \sum_{d} X_{fodt} - Y_{fott^+} = 0, \ \forall \{fot\} \in N, \tag{9}$$

$$\sum_{i\in O(f)} X_{fi} + \sum_{o\in C} Y_{fot_nt_1} \leq S(f), \ \forall f \in F, \tag{10}$$

$$Y_{fott^+} \geq 0, \ \forall \{fott^+\} \in N, \tag{11}$$

$$X_{fi} \in \{0,1\}, \ \forall i \in L \text{ and } f \in F, \tag{12}$$

$$Z_i \in \{0,1\}, \ \forall i \in L. \tag{13}$$

Models

Fleet planning

$$\min \sum_{i \in L} \sum_{f \in F} c_{fi} X_{fi} + \gamma S_p \tag{14}$$

$$\sum_{f} X_{fi} = 1, \ \forall i \in L, \tag{15}$$

$$\sum_{d} X_{fdot} + Y_{fot^{-}t} - \sum_{d} X_{fodt} - Y_{fott^{+}} = 0, \ \forall \{fot\} \in N, \tag{16}$$

$$\sum_{i \in O(f)} X_{fi} + \sum_{o \in C} Y_{fot_n t_1} \leq S(f), \ \forall f \in F \text{ and } S(p) = S_p, \tag{17}$$

$$Y_{fott^{+}} \geq 0, \ \forall \{fott^{+}\} \in N, \tag{18}$$

$$X_{fi} \in \{0, 1\}, \ \forall i \in L \text{ and } f \in F. \tag{19}$$

Models
Time windows

$$\min \sum_{i \in L} \sum_{f \in F} \sum_{u \in U} c_{fi} X_{fiu} \tag{20}$$

$$\sum_{f} \sum_{u} X_{fiu} = 1, \ \forall i \in L, \tag{21}$$

$$\sum_{d} X_{fdot_u} + Y_{fot_u^- t_u} - \sum_{d} X_{fodt_u} - Y_{fot_u t_u^+} = 0, \ \forall \{fot_u\} \in N, \tag{22}$$

$$\sum_{i \in O(f)} X_{fiu} + \sum_{o \in C} Y_{fot_{n_u} t_{1_u}} \leq S(f), \ \forall f \in F, \tag{23}$$

$$Y_{fot_u t_u^+} \geq 0, \ \forall \{fot_u t_u^+\} \in N, \tag{24}$$

$$X_{fiu} \in \{0, 1\}, \ \forall i \in L, f \in F \text{ and } u \in U. \tag{25}$$