Link-based Analysis on Large Graphs

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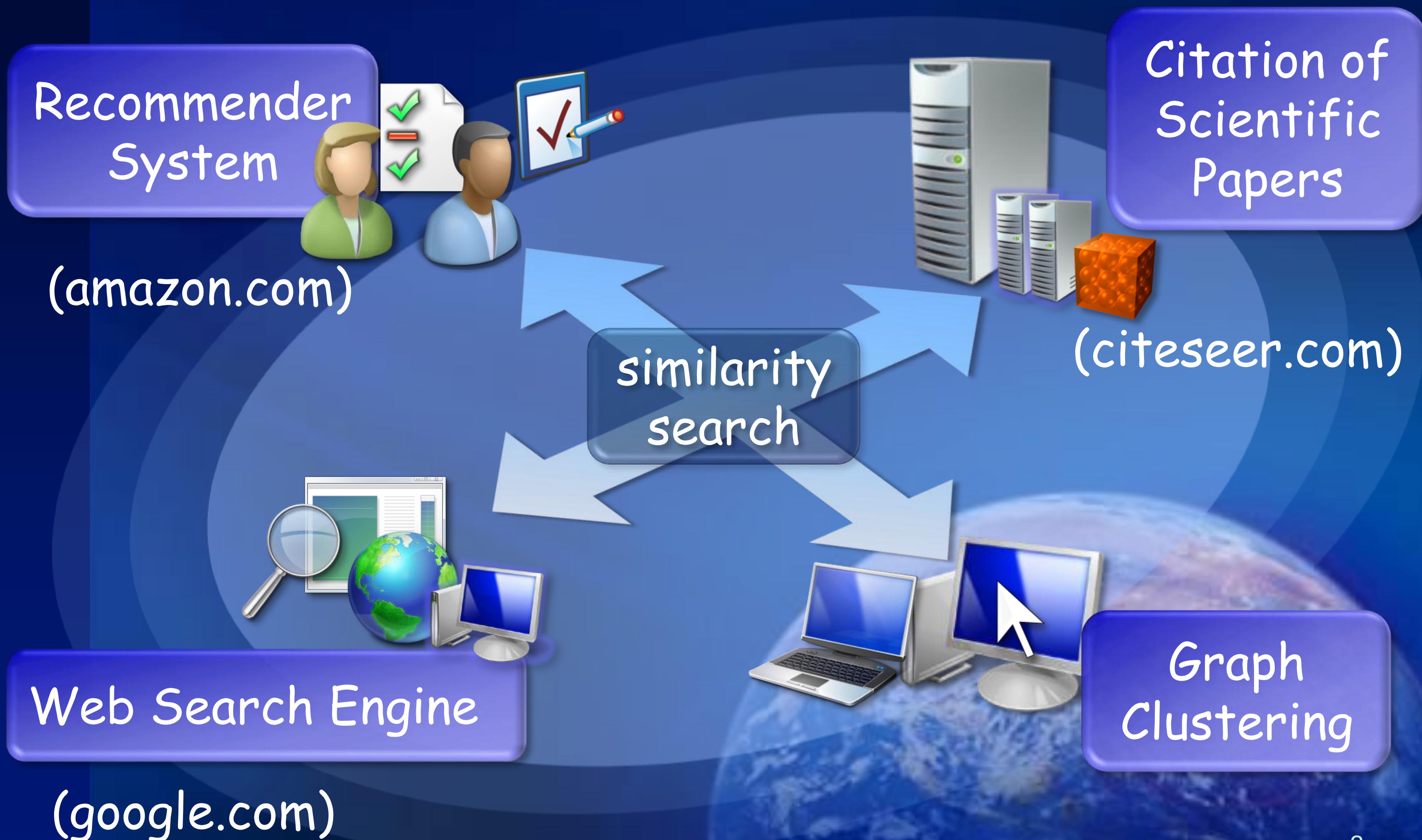
Overview

1. *Introduction* ✓
2. **Problem Definition**
3. **Optimization Techniques**
4. **Experimental Results**

1. Introduction

- Many applications require a measure of “similarity” between objects.



Existing Similarity Measures

- Textual-Content Similarity (text-based)
  - Vector-cosine similarity, Pearson correlation in IR , …
- Structural-Context Similarity (link-based)
  - PageRank : A page's authority is decided by its neighbors' authorities.
  - HITS :
    - "a good hub" --- a page that pointed to many other pages
    - "a good authority" --- a page that was linked by many hubs
  - SimRank : similar objects are referenced by similar objects.
  - LinkFusion : reinforcement assumption
  - Penetrating-Rank : entities are similar if
    - (1) they are referenced by similar entities;
    - (2) they reference similar entities.

What is SimRank?

- The similarity in a domain can be modeled as graphs.

  [*vertices* → *objects* , *edges* → *relationships*]

- SimRank is an important similarity measure which exploits the relationships between vertices on web graphs. (Glen Jeh & Jennifer Widom ,2002)

- Basic intuition:
  - Two objects are similar if their neighbors are similar. (the recursive definition)
  - Objects are maximally similar to themselves. (the base case)

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SimRank Equation

- Definition 1 (SimRank similarity)

  Let $s : V^2 \rightarrow [0, 1]$ be a similarity function on G^2

  - if a = b, ➔ s (a, b) = 1,
  - if I(a) or I(b) = ∅, ➔ s (a, b) = 0,
  - otherwise: $$s(a,b) = \frac{C}{|I(a)| \cdot |I(b)|} \cdot \sum_{i=1}^{|I(a)|} \sum_{j=1}^{|I(b)|} s(I_i(a), I_j(b))$$
    - C is a decay factor btw. 0 & 1
    - symmetric : s(a,b) = s(b,a)

Similarity btw. a & b is the average similarity btw. in-neighbors of a and in-neighbors of b.

Naïve SimRank Computation

- Iterative Paradigm:

$$S_0(a,b) = \begin{cases} 0 & a \neq b \\ 1 & a = b \end{cases}$$

$$s_{k+1}(a,b) = \frac{C}{|I(a)| \cdot |I(b)|} \cdot \sum_{i=1}^{|I(a)|} \sum_{j=1}^{|I(b)|} s_k(I_i(a), I_j(b)), \quad k = 0,1,\cdots$$

- (monotonicity) $S_k(a, b) \nearrow S(a, b)$ as $k \to \infty$.
- (symmetry) $S(a, b) = S(b, a)$.
- (stability) $S(a, b)$ is independent of $S_0(a, b)$.
- (complexity) Time : $O(Kn^2d^2)$, Space : $O(n^2)$,

  where d is the average of $|I(\cdot)|$ over all nodes.

Existing Techniques for SimRank Optimization

- Deterministic Method [PVLDB 08]
  - (to compute $s(\cdot,\cdot)$ iteratively for finding a fixed point)

$$s_{k+1}(a,b) = \frac{C}{|I(a)| \cdot |I(b)|} \cdot \sum_{i=1}^{|I(a)|} \sum_{j=1}^{|I(b)|} s_k(I_i(a), I_j(b)), \quad k = 0,1,\cdots$$

  - Advantage: accurate
  - Disadvantage: high time complexity $O(Kn^3)$
- Probabilistic Method [WWW 05]
  - (to estimate $s(\cdot,\cdot)$ stochastically by using Monte-Carlo)

$s(a,b) = E\,(c^{T(a,b)})$, where

$T(a,b)$: the first meeting time btw. a & b

  - Advantage: scalable (linear time)
  - Disadvantage: low similarity quality

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Motivation

- The computational time, which has been reduced to $O(Kn^3)$ by [PVLDB08], is still rather costly for practical purposes.
- Optimization for SimRank storage space has not been addressed in scientific literature yet.
- The accuracy estimate $\epsilon = c^k$ in [PVLDB08] is solely based on the empirical inductive method and, therefore, is not preferable.

Our Contributions

- A matrix representation and a storage scheme for SimRank model has been introduced.
  - to reduce space from $O(n^2)$ to $O(m + n)$
  - to improve time from $O(n^3)$ to $O(\min\{n \cdot m, n^r\})$ in the worst case, where $r \le \log_2 7$
- Optimization techniques for minimizing the matrix bandwidth have been developed.
  - to improve I/O efficiency
- A successive over-relaxation method has been showed.
  - to accelerate the rate of convergence

3.1 Matrix Representations for SimRank Model

- Let $S = (s_{i,j}) \in R^{n \times n}$ be a SimRank matrix, where
  $s_{i,j}$ = the SimRank value btw. vertices i and j.
  $P = (p_{i,j}) \in N^{n \times n}$ be an adjacency matrix , where
  $p_{i,j}$ = # of edges from vertices i to j.
- SimRank in matrix notation

O(n^3) for matrix multiplication

$$\begin{cases} S^{(0)} = I_n \\ S^{(k+1)} = c \cdot \boxed{Q \cdot S^{(k)} \cdot Q^T} \vee I_n \quad (k = 0,1,\cdots) \end{cases}$$

$$s_{k+1}(a,b) = \frac{c}{|I(a)| \cdot |I(b)|} \cdot \sum_{i=1}^{|I(a)|} \sum_{j=1}^{|I(b)|} s_k(I_i(a), I_j(b))$$

$$= c \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{p_{i,a}}{\sum_i p_{i,a}} \right) \cdot s_{i,j}^{(k)} \cdot \left(\frac{p_{j,b}}{\sum_j p_{j,b}} \right), \quad k = 0,1,\cdots$$

$$S^{(k+1)} = c \cdot Q \cdot S^{(k)} \cdot Q^T \quad (a \neq b)$$

3.1 Matrix Representations for SimRank Model (cont.)

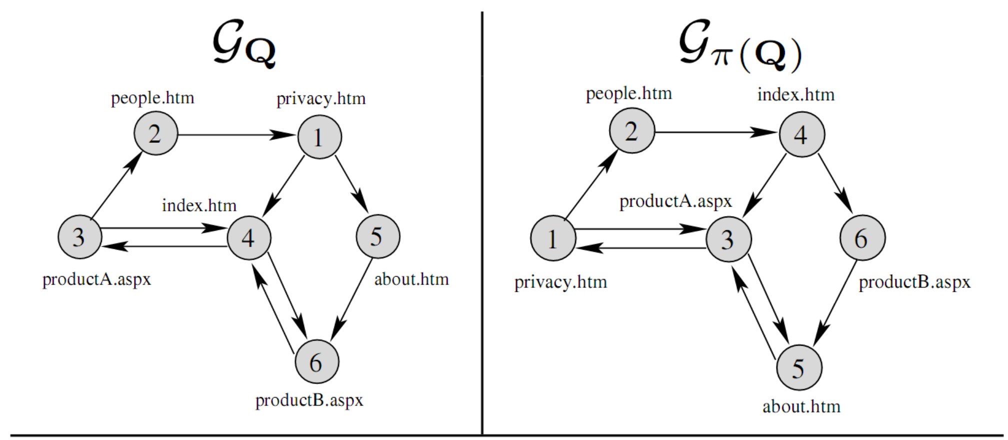
- For dense graphs
  - Fast matrix multiplication algorithms can be applied to speed up the SimRank computation.
    - Strassen Algorithm: $O(n^r)$, where $r = \log_2 7$
    - Coppersmith-Winograd Algorithm: $O(n^{2.38})$
- For sparse graphs
  - Compressed Sparse Row (CSR) are used to represent Q due to its high compression ratio.
  - CSR has $O(m \cdot n)$ time with the space $O(m+n)$.

3.2 Permuted SimRank Iterative Approach

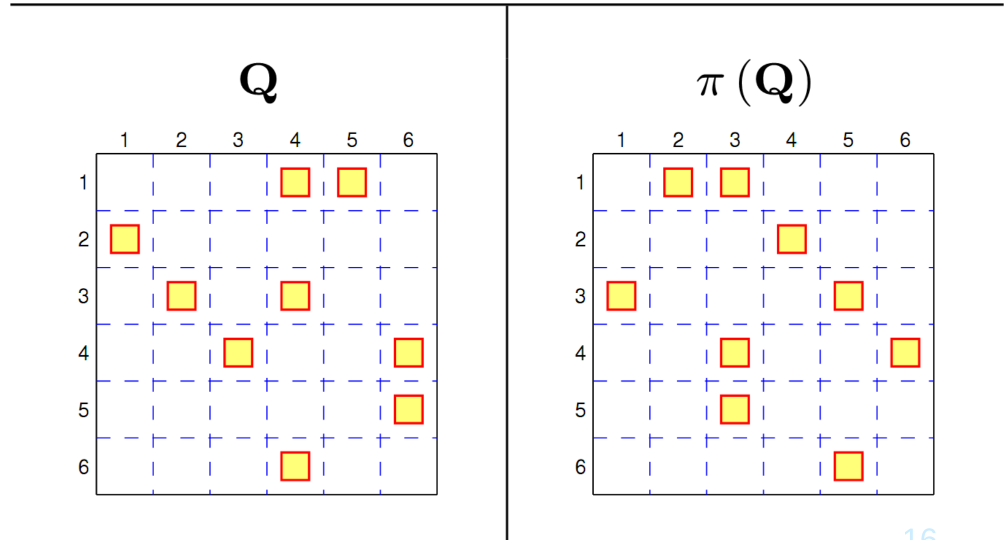
- The permutation method allows improving I/O efficiency for SimRank computation.
- The main idea involves 2 steps:
  - Reversed Cuthill-McKee (RCM) algorithm for non-symmetric matrix is introduced for finding an optimal permutation while reordering the matrix Q during the precomputation phase.
  - Permuted SimRank iterative equation is developed for reducing the matrix bandwidth for SimRank computation.

3.2 Permuted SimRank Iterative Approach (cont.)

- The permutation π can be thought of as a bijection between the vertices of the labeled graph G_Q and $G_{\pi(Q)}$.
- $\beta(\pi(Q)) \leq \beta(Q)$.
- We extend the original RCM to the directed graph by adding "the mate Q^T" and apply RCM to $Q + Q^T$.



$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 1 & 3 & 6 & 5 \end{pmatrix} \begin{matrix} node\ v_i \\ \pi(v_i) \end{matrix}$$



3.2 Permuted SimRank Iterative Approach (cont.)

- Permuted SimRank Equation
  - Let π be an arbitrary permutation with an induced permutation matrix Θ. For a given graph *G*, SimRank similarity score can be computed as

$$S^{(k)} = \pi^{-1}(\hat{S}^{(k)})$$

where

$$\begin{cases} \hat{S}^{(0)} = I_n \\ \hat{S}^{(k+1)} = c \cdot \pi(Q) \cdot \hat{S}^{(k)} \cdot \pi(Q)^T \vee I_n, \quad k = 0,1,\cdots \end{cases}$$

- For the computation to be I/O efficient, Q needs to be preordered during the precomputation phrase.

3.3 Successive Over-relaxation (SOR) SimRank Algorithm

- SOR can be used for computing $S^{(k)}$ to effectively exhibit faster rate of convergence.
- SOR SimRank Equation:
  - Let $Q = (q_{i,j}) \in R^{n \times n}$, $S^{(k)} = (s_1^{(k)} \; s_2^{(k)} \; \ldots \; s_n^{(k)})$, where $s_j^{(k)}$ is the j-th column vector of $S^{(k)}$, then

$$s_i^{GS\,(k+1)} = c \cdot Q \cdot \Big(\sum_{j<i} q_{i,j} \cdot s_j^{(k)} + \sum_{j>i} q_{i,j} \cdot s_j^{(k+1)}\Big) \vee I_n$$

$$s_i^{SOR\,(k+1)} = (1-\omega) \cdot s_i^{SOR\,(k)} + \omega \cdot s_i^{GS\,(k+1)}$$

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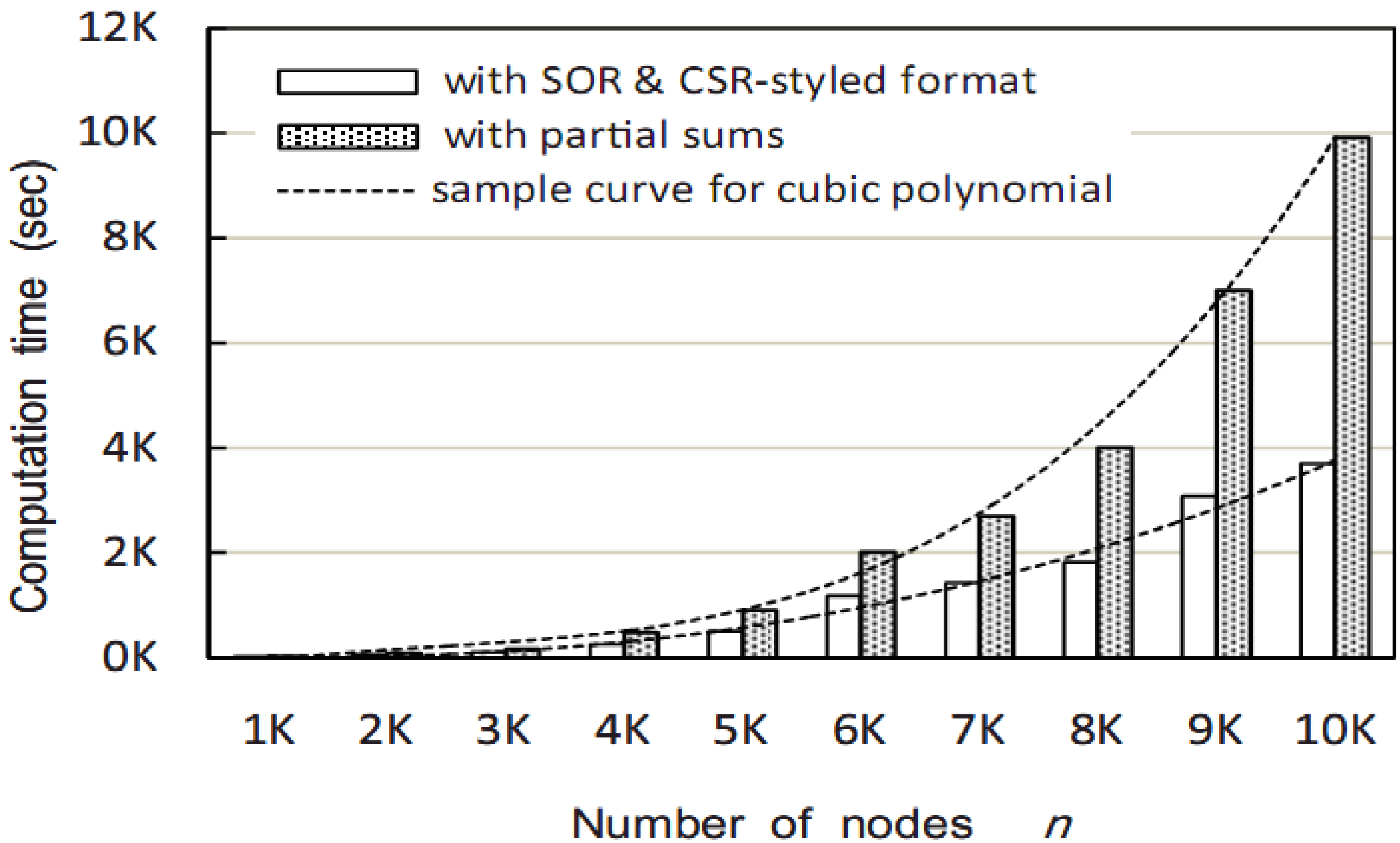
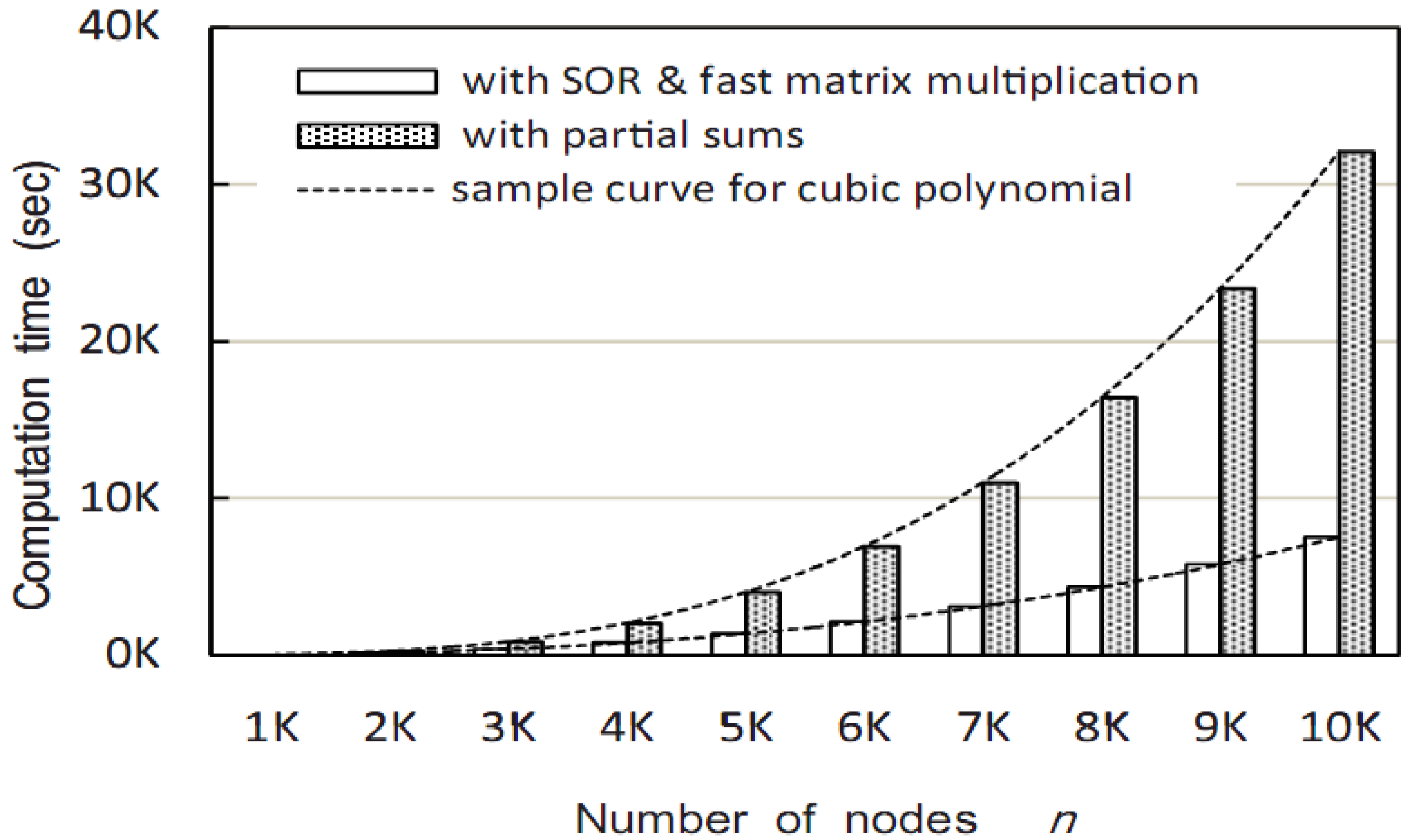
4 Experimental Evaluation

- Experimental Setup
  - Hardware
    - 2.0GHz Pentium(R) Dual-Core / 2GB RAM
    - Windows Vista OS / Visual C++ 6.0
  - Data Sets
    - Synthetic
      - graph with an average of 8 links per page.
      - 10 sample adjacency matrices from 1K to 10K with $\xi \sim$uniform[0; 16] out-links on each row.
    - Real-life
      - Wikipedia (3.2M articles with 110M intra-wiki links Oct. '07)
      - We choose the relationship : "a category contains an article to be a link from the category to the article".
  - Parameter Settings
    - $c = 0.8$, $\omega = 1.3$, $\epsilon = 0.05$

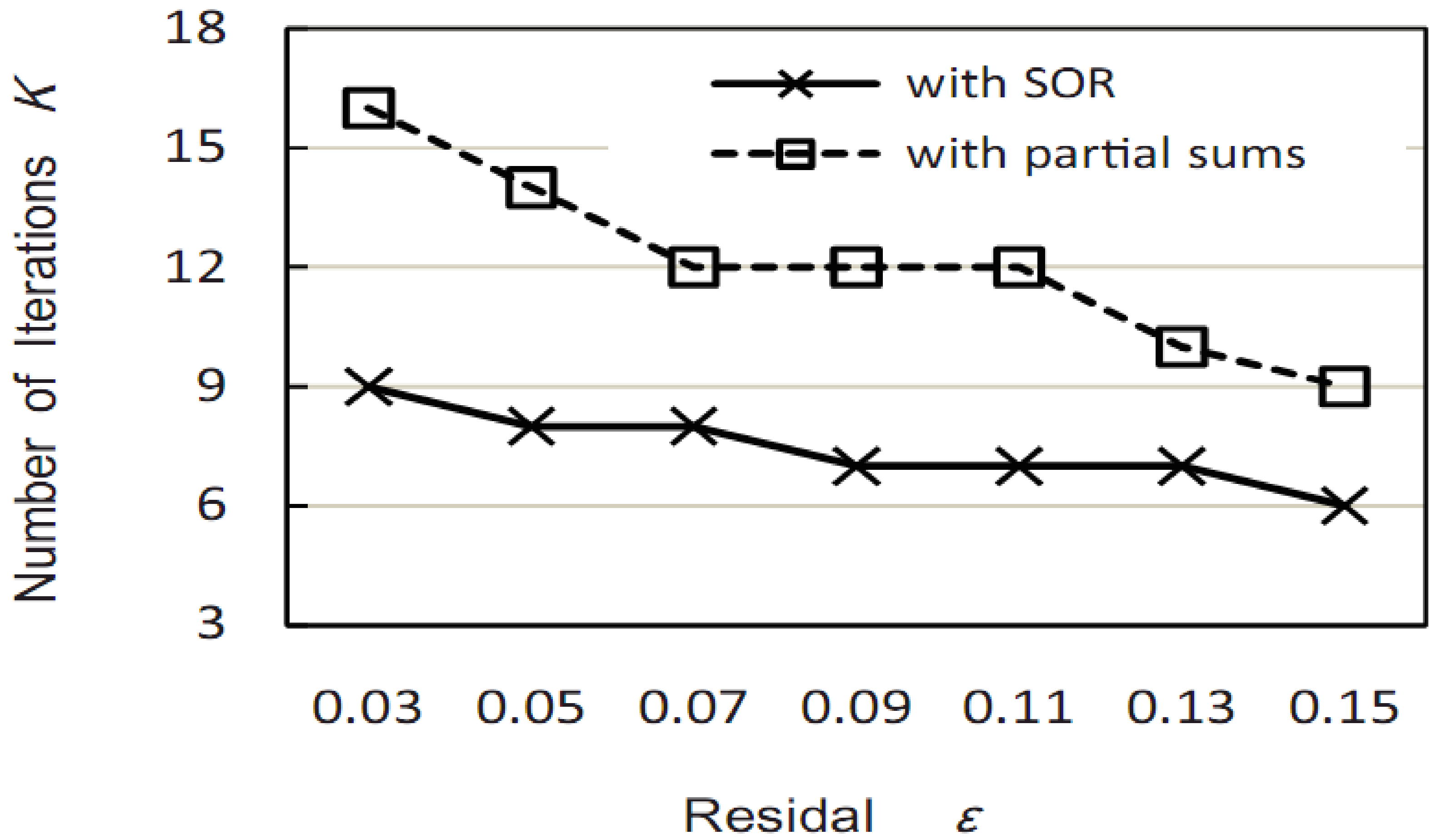
Experimental Results



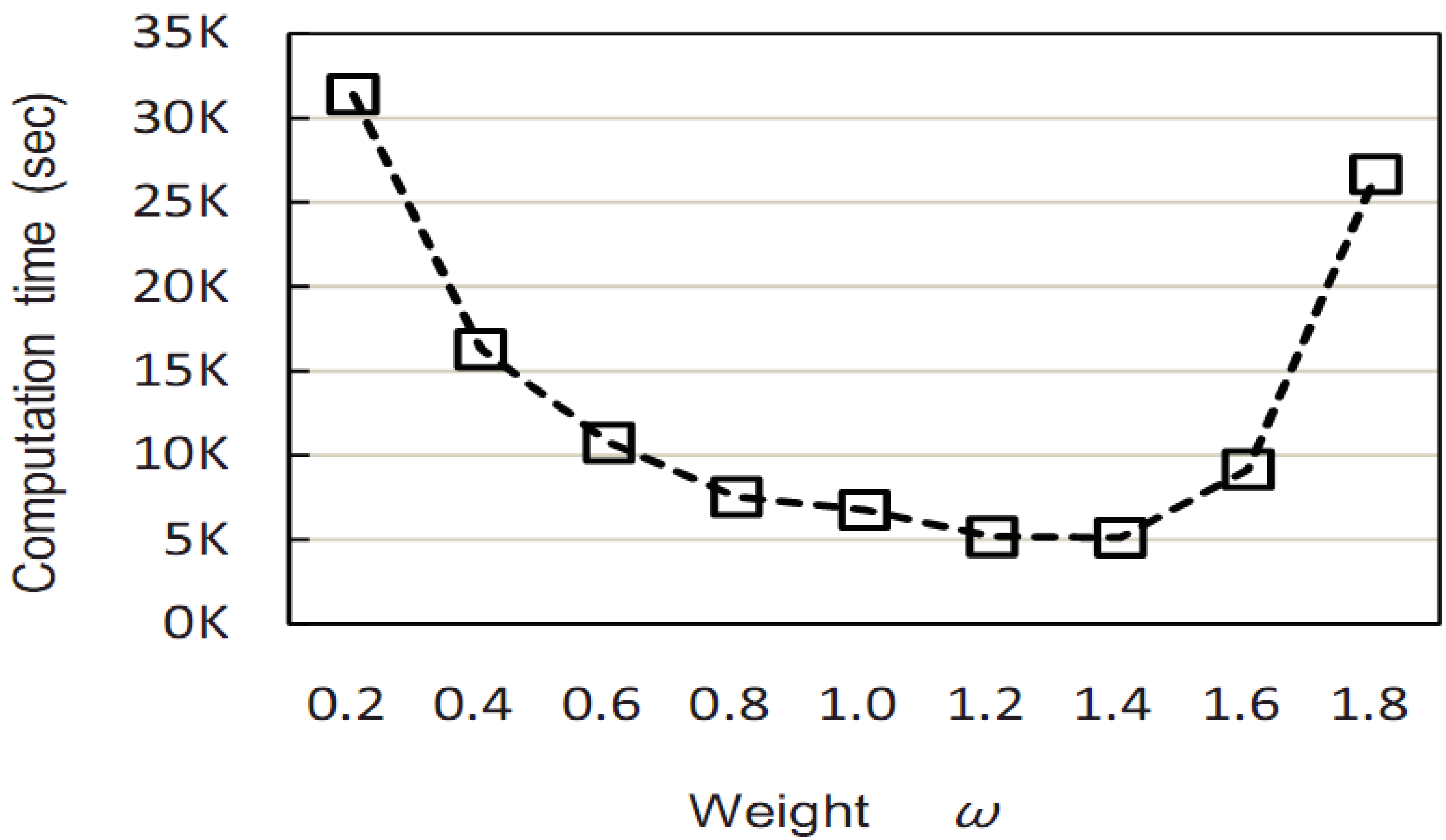
(a) Time Efficiency on Sparse Graphs



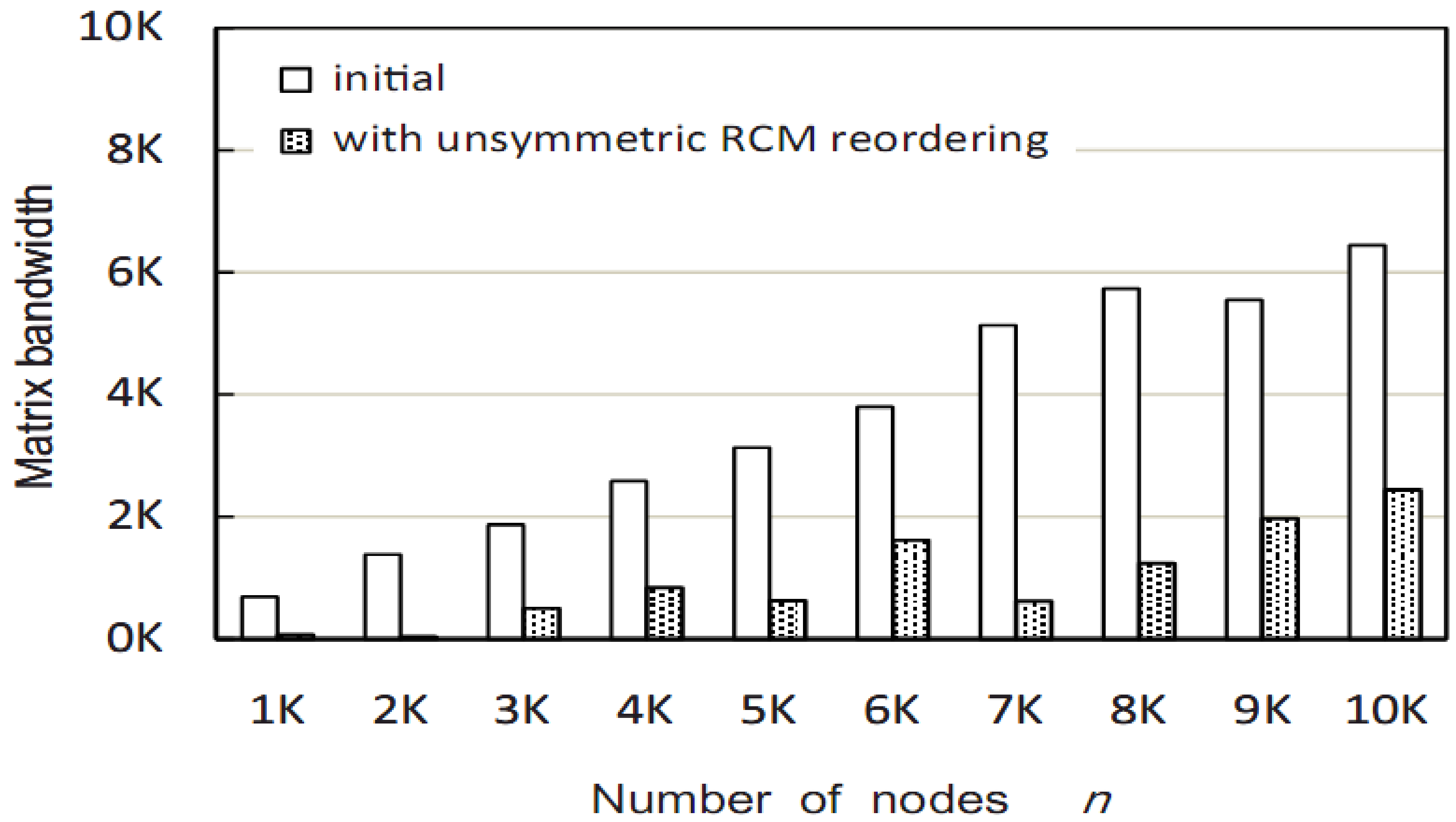
(b) Time Efficiency on Dense Graphs



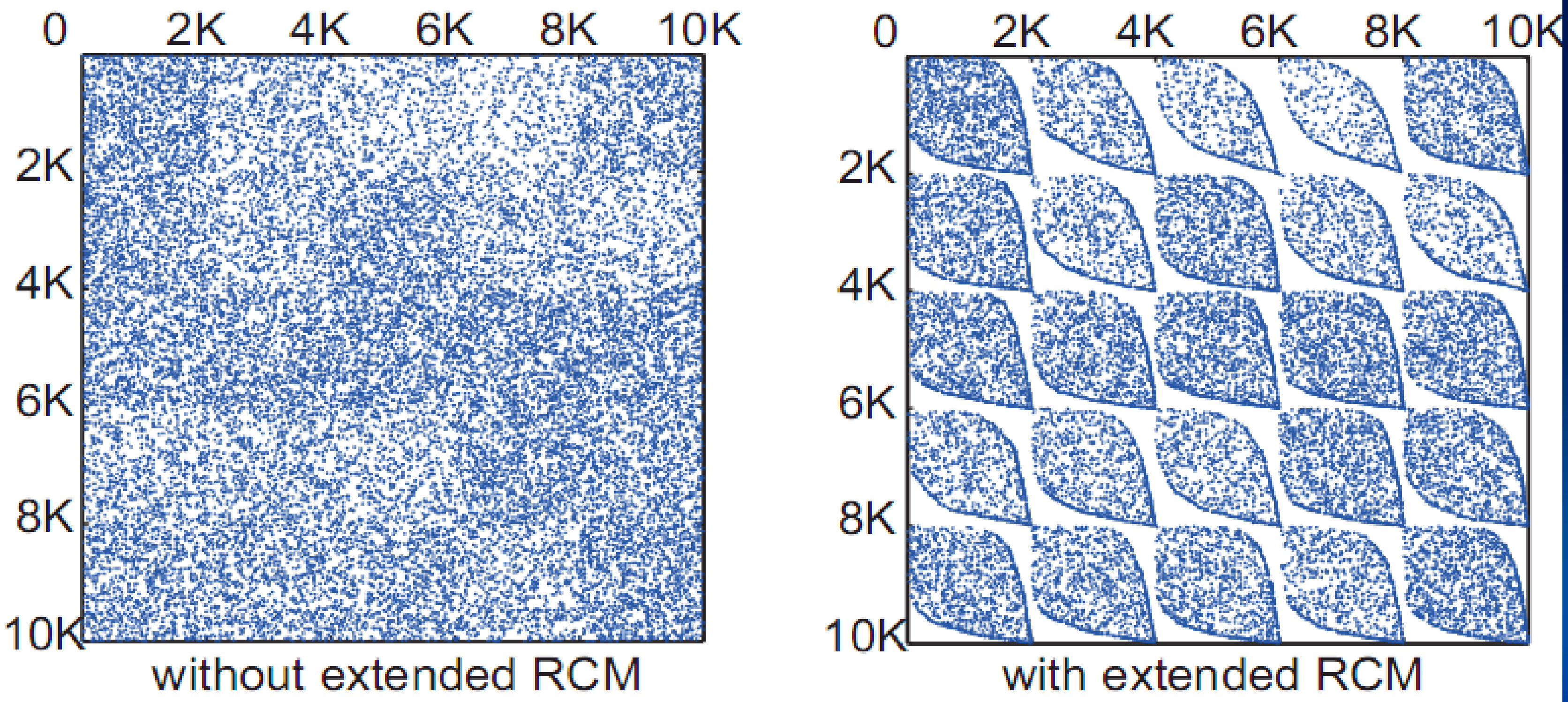
(c) Convergence Rate ($n = 10K, \omega = 1.3$)



(d) Relaxation Factor ($n = 10K, \varepsilon = 0.05$)



(e) I/O Efficiency on Sparse Graphs



(f) Extended RCM ($m = 5,496,208$)


Thank You !

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5. *Conclusions*

Conclusions

- We formalized the SimRank equation in matrix notations.
- We investigated optimization issues for SimRank computation.
  - A compressed storage scheme for sparse graphs is adopted for reducing the space from $O(n^2)$ to $O(n + m)$.
  - A fast matrix multiplication for dense graphs is used for improving the time from $O(n^2 \cdot d)$ to $O(\min\{n \cdot m, n^r\})$, $r \le \log_2 7$.
  - A permuted SimRank iteration was developed in combination of the extended RCM algorithm to achieve its I/O efficiency.
  - A SOR method has been showed to significantly speed up the convergence rate of the SimRank iteration.
- Our experimental evaluations on synthetic and real-life data sets demonstrate the efficiency of our methods.