Recursion (Extension discussed in Sec 5) Rec-1

Honda, Vasconcelos, Kubo [ESOP 98.]
Yoshida and Vasco [SecRet 07]

has

$$\mathsf{def}\ X(k) = k![1]; X\langle k\rangle \ \mathsf{in}$$
$$k \triangleright [\mathsf{true}: 0 \ ;\ \mathsf{false}: X\langle k\rangle]$$

Where k has a type

$$\mu t.[\mathsf{true}: \mathsf{end}\ \&\ \mathsf{false}: ![\mathsf{nat}].t]$$

which appears in many practice.

In HOπ, we can represent this recursion by the standard letrec as

$$\mathsf{letrec}\ y = \lambda x.\ x![1]; y$$
$$\mathsf{in}\ \ k\ [\mathsf{true}: 0\ ;\ \mathsf{false}: y k]$$

Syntax

letrec $y = M$ in N

Type

$\alpha ::= \ldots\ \mu t.\alpha \mid t$

where t is a type variable.

Structure Rule

letrec $y = M$ in $0 \equiv 0$

(νk) ! letrec $z = N$ in M

$\equiv$ letrec $z = N$ in $(\nu k) M$

$k \notin \{z\} \cup fn(M) \cup fv(M)$

(νa) letrec $z = N$ in M similar

letrec $y_1 = M_1$ in letrec $y_2 = M_2$ in N

$\equiv$ letrec $y_2 = M_2$ in letrec $y_1 = M_1$ in N

$y_i \notin fv(M_j)$

! letrec $y = M$ in $(P \mid Q)$

$\equiv$ (! letrec $y = M$ in $P) \mid Q$

$y \notin fv(Q)$

letrec $y = N$ in $M \equiv M$ if $y \notin fv(M)$

letrec $y = V$ in $y \equiv V$ $y \notin fv(V)$

Reduction

$$\text{letrec } y = \lambda x.N \text{ in } E[yV] \quad \text{letrec } y = \lambda x.N \text{ in } E[N[V/y]]$$

$$\frac{M \to M'}{\text{letrec } y = V \text{ in } M \to \text{letrec } y = V \text{ in } M'}$$

$$\frac{N \to N'}{\text{letrec } y = N \text{ in } M \to \text{letrec } y = N' \text{ in } M}$$

Typing

$$\frac{\Gamma, y : (t \to \tau)^1 ; \phi ; \phi \vdash P \triangleright t \to \tau \qquad \Gamma, y : (t \to \tau)^1 ; \Sigma ; S \vdash Q \triangleright \tau'}{\Gamma ; \Sigma ; S \vdash \text{letrec } y = P \text{ in } Q : \tau'}$$

Proof

The only interesting case is the reduction of the axiom

$$\frac{\Gamma,\ y:(t\to\tau)^1;\ \phi\ ;\ \phi\ \vdash\ \lambda x.N:(t\to\tau)^1 \qquad \Gamma,\ y:(t\to\tau)^1;\ \Sigma\ ;\ S\ \vdash\ yV:\tau}{\Gamma\ ;\ \Sigma\ ;\ S\vdash \text{letrec}\ \ y=\lambda x.N\ \text{in}\ yV}$$

$$\longrightarrow\ \text{letrec}\ \ y=\lambda x.N\ \text{in}\ N[V/x]$$

1 $\Gamma,\ y:(t\to\tau)^1,\ x:t\ ;\ \phi\ ;\ S'\vdash N:\tau$ $\quad S'=\{x\}$ or ϕ

OR 2 $\Gamma,\ y:(\alpha\to\tau)^1;\ x:\alpha\ ;\ \phi\ \vdash N:\tau$ $\quad t=\alpha$

$\Gamma;\Sigma;S'\vdash V:t$

Case 1 $\quad \Gamma,\ y:(t\to\tau)^1;\ \Sigma\ ;\ S,\underbrace{S'_{\setminus x}}_{=\phi}\vdash N[V/x]:\tau$

case 2 $\quad \Gamma,\ y:(\alpha\to\tau)^1;\ k:\alpha\ ;\ \phi\vdash N[k/x]:\tau$

$V=k$

$\Sigma=\{k:\alpha\}$

$\therefore\ \ \Gamma;\Sigma;S\vdash \text{letrec}\ \ y=\lambda x.N\ \text{in}\ N[V/x]$

as desiered

For Channel-Dependency Typing System,

we just have

$$\frac{\Gamma, y : t \to \mathcal{I} \vdash \lambda x. N : t \to \mathcal{I} \qquad \Gamma, y : t \to \mathcal{I} \vdash P : \Delta}{\Gamma \vdash \text{letrec } y = \lambda x. N \text{ in } P : \mathcal{I}}$$

$$\frac{\Gamma, y : \Pi(x:6)\mathcal{I} \vdash \lambda x. N : \Pi(x:6)\mathcal{I} \qquad \Gamma, y : \Pi(x:6)\mathcal{I} \vdash P : \mathcal{I}}{\Gamma \vdash \text{letrec } y = \lambda x. N \text{ in } P : \mathcal{I}}$$

and the proof is standard by induction.