The proof and the full definitions for Theorem 4.3

$\boxed{\text{Functions : Erase, Proc, Lin}}$

$\boxed{\text{Erase : Environment}}$

$\mathrm{Erase}(\emptyset) = \emptyset$

$\mathrm{Erase}(\Gamma, u{:}\alpha) = \mathrm{Erase}(\Gamma),$

$\mathrm{Erase}(\Gamma, u{:}\mathrm{begin}.\alpha) = \mathrm{Erase}(\Gamma), u{:}\mathrm{Erase}(\mathrm{begin}.\alpha)$

$\mathrm{Erase}(\Gamma, u{:}t) = \mathrm{Erase}(\Gamma), u{:}\mathrm{Erase}(t)$

$\mathrm{Erase}(\Delta) = \diamond$

$\mathrm{Erase}(\mathrm{unit}) = \mathrm{unit}$

$$\mathrm{Erase}(t \to \tau) = \begin{cases} \mathrm{Erase}(t) \to \mathrm{Erase}(\tau) & \text{if } \mathrm{Proc}(t \to \tau) \neq \emptyset \\ (\mathrm{Erase}(t) \to \mathrm{Erase}(\tau))^1 & \text{otherwise} \end{cases}$$

$$\mathrm{Erase}(\Pi x{:}\sigma.\tau) = \begin{cases} \mathrm{Erase}(\sigma) \to \mathrm{Erase}(\tau) & \text{if } \mathrm{Proc}(\Pi(x{:}\sigma)\tau) \neq \emptyset \\ (\mathrm{Erase}(\sigma) \to \mathrm{Erase}(\tau))^1 & \text{otherwise} \end{cases}$$

$\mathrm{Erase}(\mathrm{begin}.\alpha) = \mathrm{begin}.\mathrm{Erase}(\alpha)$

$\mathrm{Erase}(![\Pi(\tilde{x}{:}\tilde{\sigma})\tau];\alpha) = ![\mathrm{Erase}(\tau)];\mathrm{Erase}(\alpha)$

$\mathrm{Erase}(![\Pi(\tilde{x}{:}\tilde{\sigma})\sigma];\alpha) = ![\mathrm{Erase}(\sigma)];\mathrm{Erase}(\alpha)$

$\mathrm{Erase}(\oplus\{l_i{:}\alpha_i\}_{i\in I}) = \oplus\{l_i{:}\mathrm{Erase}(\alpha_i)\}_{i\in I}$

$\mathrm{Erase}(\mathrm{end}) = \mathrm{end}$

$\mathrm{Erase}(\mathrm{unit}) = \mathrm{unit}$

$\mathrm{Erase}(u) = u$

$\mathrm{Erase}(PQ) = \mathrm{Erase}(P)\mathrm{Erase}(Q)$

$\mathrm{Erase}(\lambda(x{:}t).P) = \lambda(x{:}\mathrm{Erase}(t)).\mathrm{Erase}(P)$

$\mathrm{Erase}(\lambda(x{:}\sigma).P) = \lambda(x{:}\mathrm{Erase}(\sigma)).\mathrm{Erase}(P)$

$\mathrm{Erase}(0) = 0 \quad \mathrm{Erase}(P \mid Q) = \mathrm{Erase}(P) \mid \mathrm{Erase}(Q)$ $\boxed{\text{CONT}}$

$\mathrm{Erase}((\nu a{:}\sigma)P) = (\nu a{:}\mathrm{Erase}(\sigma))\,\mathrm{Erase}(P)$

$\mathrm{Erase}((\nu K)P) = (\nu K)\,\mathrm{Erase}(P)$

$\mathrm{Erase}(!u(x).P) = !u(x).\,\mathrm{Erase}(P)$

$\mathrm{Erase}(\bar{u}(x).P) = \bar{u}(x).\,\mathrm{Erase}(P)$

$\mathrm{Erase}(K(\hat{x}{:}\hat{\sigma}, y{:}t).P) = K(y{:}\mathrm{Erase}(t)).\,\mathrm{Erase}(P)$

$\mathrm{Erase}(\bar{K}\langle\hat{v}, V\rangle.P) = \bar{K}\langle\mathrm{Erase}(V)\rangle.\,\mathrm{Erase}(P)$

$\mathrm{Erase}(K(\hat{x}{:}\hat{\sigma}, y{:}\sigma).P) = K(y{:}\mathrm{Erase}(\sigma)).$

$\boxed{\mathrm{Proc}}$

$\mathrm{Proc}(\Delta) = \{u : \mathrm{Erase}(\alpha) \mid u{:}\alpha \in \Delta\}$

$\mathrm{Proc}(\mathrm{unit}) = \mathrm{unit}$

$\mathrm{Proc}(t \to \tau) = \mathrm{Proc}(\tau) \setminus \mathrm{Proc}(t)$

$\mathrm{Proc}(\Pi(x{:}\sigma)\tau) = \mathrm{Proc}(\tau) \setminus x$

$\boxed{\mathrm{Lin}}$ $\mathrm{Lin}(\Gamma) = \{x \mid x{:}(t)^1 \in \mathrm{Erase}(\Gamma)\}$

Theorem 4-3 (1)

(a) $\Gamma \vdash u : \sigma \Rightarrow$

$$\mathrm{Erase}(\Gamma) \setminus \mathrm{Proc}(\Gamma) ; \mathrm{Proc}(\{u : \sigma\}) ; \phi \vdash u : \mathrm{Erase}(\sigma)$$

(b) $\Gamma \vdash x : t \Rightarrow$

$$\mathrm{Erase}(\Gamma) \setminus \mathrm{Proc}(\Gamma) ; \phi ; \mathrm{Lin}(\{x : t\}) \vdash x : \mathrm{Erase}(t)$$

(c) $\Gamma \vdash P : \tau$

$$\Rightarrow \mathrm{Erase}(\Gamma) \setminus \mathrm{Proc}(\Gamma) ; \mathrm{Proc}(\tau) \setminus \Sigma ; \mathrm{Lin}(\Gamma \upharpoonright fv(P)) \vdash \mathrm{Erase}(P) \triangleright \mathrm{Erase}(\tau)$$

$$\Sigma = \{\mathrm{Proc}(t) \mid x : t \in \Gamma \upharpoonright S\}$$

$$S = \mathrm{Lin}(\Gamma \upharpoonright fv(P))$$

Write

$$= \mathrm{LinProc}(\Gamma \upharpoonright P)$$

Rule changed. (The conditions corresponded to the first system are added) R−1

$\Delta \cdot u_i : \sigma \Rightarrow fs(\Delta) \cap fs(\sigma) = \emptyset$

(Abs)

$$\frac{\Gamma, x:t \vdash P \triangleright \tau \qquad Proc(\tau) \neq \emptyset \Rightarrow x \in Lin(\Gamma \upharpoonright P)}{\Gamma \vdash \lambda(x:t).P \triangleright t \to \tau}$$

(Abs^N)

$$\frac{\Gamma, x:\sigma \vdash P \triangleright \tau \qquad x:\sigma \notin LinProc(\Gamma \upharpoonright P)}{\Gamma \vdash \lambda(x:\sigma).P \triangleright \Pi x:\sigma.\tau}$$

(App)

$$\frac{\Gamma \vdash P \triangleright (t \to \tau)^1 \qquad \Gamma \vdash Q \triangleright t \qquad Proc(t) = \emptyset \Rightarrow Lin(\Gamma \upharpoonright Q) = \emptyset}{\Gamma \vdash PQ \triangleright \tau}$$

(App^N)

$$\frac{\Gamma \vdash P \triangleright \Pi(x:\sigma)\tau \qquad \Gamma \vdash u:\sigma}{\Gamma \vdash Pu \triangleright \tau[u/x]} \qquad \sigma \equiv \alpha \Rightarrow u \notin Proc(Lin(\Gamma \upharpoonright P)) \cup Proc(\tau)$$

(Nil), (Par), (Weak), (New) Unchanged.

(New^k)

$$\frac{\Gamma, k_0:\alpha, k_1:\bar{\alpha} \vdash P \triangleright \Delta, k_0:\alpha, k_1:\bar{\alpha}}{\Gamma \vdash (\nu k)P \triangleright \Delta}$$

$k_0, k_1 \notin LinProc(\Gamma \upharpoonright P)$

(R-2)

(Acc)

$$\frac{\Gamma \vdash u : \mathrm{begin}.\overline{\alpha} \qquad \{k:\beta\} \not\subseteq \Delta \qquad \begin{array}{c} x \notin \mathrm{LinProc}(\Gamma \upharpoonright P) \\ \Gamma, x:\alpha \vdash P : \Delta, x:\alpha \end{array}}{\Gamma \vdash !u(x).P \triangleright \Delta . u:\mathrm{begin}.\overline{\alpha}}$$

(Req)

$$\frac{\Gamma \vdash u : \mathrm{begin}.\alpha \qquad \begin{array}{c} x \notin \mathrm{LinProc}(\Gamma \upharpoonright P) \\ \Gamma, x:\alpha \vdash P \triangleright \Delta, x:\alpha \end{array}}{\Gamma \vdash \overline{u}(x).P \triangleright \Delta . u:\mathrm{begin}.\alpha}$$

(Rec)

$$\frac{\begin{array}{c} \Gamma \vdash k : ?[\Pi(\tilde{x}:\tilde{\sigma})t];\alpha \\ \Gamma, \tilde{x}:\tilde{\sigma}, y:t \vdash P \triangleright \Delta, \tilde{x}:\tilde{\sigma}, k:\alpha \\ \mathrm{Proc}(t) \neq \emptyset \Rightarrow y \in \mathrm{Lin}(\Gamma \upharpoonright P) \quad \tilde{x}, k \notin \mathrm{LinProc}(\Gamma \upharpoonright P) \end{array}}{\Gamma \vdash k(\tilde{x}:\tilde{\sigma}, y:t).P \triangleright \Delta, k:?[\Pi(\tilde{x}:\tilde{\sigma})t];\alpha}$$

(Recc)

$$\frac{\begin{array}{c} \Gamma \vdash k : ?[\Pi(\tilde{x}:\tilde{\sigma})\sigma];\alpha \\ \Gamma, \tilde{x}:\tilde{\sigma}, y:\sigma \vdash P \triangleright \Delta, \tilde{x}:\tilde{\sigma}, y:\sigma, k:\alpha \\ k, \tilde{x}, y \notin \mathrm{LinProc}(\Gamma \upharpoonright P) \end{array}}{\Gamma \vdash k(\tilde{x}:\tilde{\sigma}, y:\sigma).P \triangleright \Delta, k:?[\Pi(\tilde{x}:\tilde{\sigma})t];\alpha}$$

(Send)

$$\frac{\Gamma \vdash k![\Pi(\tilde{x}:\tilde{\sigma})t];\alpha \quad \Gamma \vdash P \triangleright \Delta \quad \{k:\alpha\} \subseteq \Delta,\ \tilde{v}:\tilde{\alpha} \quad \Gamma \vdash V :: I[\tilde{v}/\tilde{x}] \quad \Gamma \vdash \tilde{v}:\tilde{\sigma}}{\Gamma \vdash \overline{k}\langle \tilde{v}, V\rangle.\ P \triangleright \Delta \cdot \tilde{v}:\tilde{\sigma}/k \cdot k:![\Pi(\tilde{x}:\tilde{\sigma})t];\alpha}$$

if $Proc(I[\tilde{v}/\tilde{x}]) \neq \phi \Rightarrow LinProc(\Gamma \restriction V) = 0$

$k \notin LinProc(\Gamma \restriction P) \cup LinProc(\Gamma \restriction V)$

(Sendc) is similar.

(Bra)

$$\frac{k \notin LinProc(\Gamma \restriction P) \quad \Gamma \vdash k:\&\{l_i:\alpha_i\}_{i\in I} \quad \Gamma \vdash P_i \triangleright \Delta, k:\alpha}{\Gamma \vdash k \triangleright \{l_i : P_i\}_{i\in I} : \Delta, k:\&\{l_i:\alpha_i\}_{i\in I}}$$

(Sel)

$$\frac{k \notin LinProc(\Gamma \restriction P) \quad \Gamma \vdash k:\oplus\{l_i:\alpha_i\}_{i\in I} \quad \Gamma \vdash P \triangleright \Delta, k:\alpha_i}{\Gamma \vdash k \triangleleft l_i.\ P \triangleright \Delta, k:\oplus\{l_i:\alpha_i\}_{i\in I}}$$

Formulation Rule

$$\frac{\Gamma \vdash t : tp \quad \Gamma \vdash \tau : tp \quad Proc(t) \supset Proc(\tau)}{\Gamma \vdash t \to \tau : tp}$$

$$\frac{\Gamma, x : \sigma \vdash \tau : TP \qquad Proc(\{x : \sigma\}) \supset Proc(\tau)}{\Gamma \vdash \Pi x : \sigma . \tau : TP}$$

$$\frac{\Gamma \vdash \Delta : TP \qquad u \notin dom(\Delta) \quad \Gamma \vdash \sigma : TP}{\Gamma \vdash \Delta \cdot u : \sigma : TP}$$

$$\sigma_i = \alpha_i \Rightarrow x \in Proc(\tau)$$

$$\frac{\Gamma, \tilde{x} : \tilde{\sigma} \vdash \tau : TP \qquad \Gamma \vdash \alpha : TP}{\Gamma \vdash ![\Pi(\tilde{x} : \tilde{\sigma})\tau] ; \alpha}$$

This restricted system satisfies Subject Reduction Theorem.

$\boxed{\text{Proof}}$: (a), (b) are obvious.

(c)

$$\boxed{(\text{Base})}\quad \frac{\Gamma \vdash Env}{\Gamma \vdash () : unit}$$

$\Rightarrow Env(\Gamma) \setminus Proc(\Gamma) ; \phi ; \phi \vdash () : unit$

By (Unit)

$\boxed{(\text{Abs}_H)}$

$$\frac{\Gamma, x : t \vdash P \triangleright \tau}{\Gamma \vdash \lambda(x : t).P \triangleright t \to \tau}$$

By (IH)

$Erase(\Gamma) \setminus Proc(\Gamma), x : Erase(t) ;$

$Proc(\tau) \setminus \Sigma \quad (\Sigma = \{ Proc(t') \mid x : t' \in \Gamma \upharpoonright (Lin(\Gamma \upharpoonright fv(P)$

$Lin(\Gamma, x : t \upharpoonright fv(P)) \vdash Erase(P) \triangleright Erase(\tau))$

$\boxed{\text{Case (A)}}$ $Erase(t) \neq (t')^1 \Rightarrow Proc(t) = \phi$

By (Abs)

$Erase(\Gamma) \setminus Proc(\Gamma) ; \underset{\substack{\| \\ Proc(\tau)}}{Proc(t \to \tau)} \setminus \Sigma$

$\leftarrow$ this unchanged ~~since~~

$; Lin(\Gamma \upharpoonright fv(\lambda(x).P)) \vdash Erase(\lambda(x : t).P)$

$\triangleright Erase(t) \to Erase(\tau)$

$\boxed{\text{Case B}}$ $t = (t')^1$

We note $x \in Lin(\Gamma, x{:}t \restriction fv(P))$

Then $Lin(\Gamma \restriction fv(\lambda(x).P))$

$= Lin(\Gamma, x{:}t \restriction fv(P)) \setminus x$

We also note

$\Sigma = \{ Proc(t'') \mid y{:}t'' \in (\Gamma \restriction Lin(\Gamma \restriction fv(\lambda(x).P))) \}$

$\Sigma' = \{ Proc(t'') \mid y{:}t'' \in (\Gamma, x{:}t \restriction Lin(\Gamma, x{:}t \restriction fv(P))) \}$

$=$

$\Sigma \cup Proc(t)$

And $Proc(\tau) \setminus (\Sigma \cup Proc(t))$

$= (Proc(t) \setminus Proc(\tau)) \setminus \Sigma$

$= Proc(t \to \tau) \setminus \Sigma$

Thus we have

$Erase(\Gamma); Proc(t \to \tau) \setminus \Sigma; S \setminus x \vdash \lambda(x{:}t).P$

$\triangleright Erase(t) \to Erase(\tau))$

as desighered.

In both cases, if $Proc(t \to \tau) \neq \phi$

Then we apply (Subs) to obtain

$$(Erase(t) \to Erase(\tau))^1 .$$

(Abs^N)

$$\frac{\Gamma, x:\sigma \vdash P \triangleright \tau}{\Gamma \vdash \lambda(x:\sigma). P \triangleright \Pi x:\sigma . \tau}$$

The case $\sigma = begin.\alpha$ is similar to the case A in (Abs^H).

We show the case $\sigma = \alpha$.

$\Gamma, x:\alpha \vdash P \triangleright \tau$

$\overset{(IH)}{\Longrightarrow}$ $Erase(\Gamma); Proc(\tau) \setminus \Sigma ; S \vdash Erase(P) \triangleright Erase(\tau)$

since $x : Erase(\alpha) \in Proc(\tau)$

and $x : Erase(\alpha) \notin \Sigma$ (By condition)

and $Lin(\Gamma, x:\alpha \restriction fv'(P)) = Lin(\Gamma \restriction fv(P))$

Hence $Erase(\Gamma); Proc(\tau) \setminus \Sigma \setminus x ; S \vdash \lambda(x : Erase(\sigma)) \triangleright Erase(P)$

$\triangleright \Pi x:\sigma . \tau$

(APP^H) $$\frac{\Gamma \vdash P \triangleright (t \to \tau)^1 \quad \Gamma \vdash Q \triangleright t}{\Gamma \vdash PQ \triangleright \tau}$$

We have to take care of the case

$Erase(t) \neq (t')^1$

By definition, $Proc(t) = \phi$

Also by (Condition) $Lin(\Gamma \upharpoonright fv(Q)) = \phi$

Hence $Proc(t) \setminus \Sigma = \phi$

$E(\Gamma); \Sigma_1; S_1 \vdash Erase(P) \triangleright Erase(t \to \tau)$

$\Downarrow$ Since (Subs) can apply at the last step.

$E(\Gamma); \Sigma_1; S_1 \vdash Erase(P) \triangleright Erase(t) \to Erase(\tau)$

(Subs) $E(\Gamma); \Sigma_1; S_1 \vdash Erase(P) \triangleright (Erase(t) \to Erase(t))^1$

$$\frac{E(\Gamma); \phi; \phi \vdash Erase(Q) \triangleright Erase(t)}{E(\Gamma); \Sigma_1; S_1 \vdash Erase(PQ) \triangleright Erase(\tau)}$$

(App^N) The Interesting Case is $\sigma = \alpha$.

$$\Gamma \vdash P \triangleright \Pi(x:\alpha)\tau$$

$$\Gamma \vdash K \triangleright \alpha \quad \Rightarrow E(\Gamma); \{K : E(\alpha)\}; \phi \vdash K \triangleright E(\alpha))$$

(IH)

$$E(\Gamma); \underset{\sim\sim\sim}{Proc(\Pi(x:\alpha)\tau)\backslash\Sigma}; S \vdash P \triangleright E(\Pi(x:\alpha)\tau))$$

$$\equiv E(\Gamma); (Proc(\tau)\backslash\{x : E(\alpha)\})\backslash\Sigma; S \vdash P \triangleright E(\alpha) \to E(\tau))$$

with $S = Lin(\Gamma \upharpoonright fv(P))$

$$\Sigma = \{Proc(\tau'') \mid y : \tau'' \in (\Gamma \upharpoonright_{Lin} fv(P))\}$$

Note by the revised application rule

$$K : E(\alpha) \notin Pro(\tau) \cup \Sigma$$

Hence $\sim\sim\sim$ and $\{K : \alpha\}$ is disjoint

and $Proc(\tau[K/x])\backslash\Sigma$

$$= \{K : E(\alpha)\} \cup \sim\sim\sim$$

Hence $E(\Gamma); Proc(\tau[K/x])\backslash\Sigma; S \vdash E(PK) \triangleright E(\tau[K/x])$

as required.

In Process, only interesting case is (Out) 9.

(Out)

$\Gamma \vdash k : ![\Pi(\tilde{x} : \tilde{\sigma})\tau] ; \alpha$

$\Gamma \vdash P \triangleright \Delta$

$\Gamma \vdash v_i : \sigma_i [\tilde{v}/\tilde{x}]$ $\quad \{k : \alpha\} \subseteq \Delta, \tilde{v} : \tilde{\alpha}$

$\Gamma \vdash V : \tau [\tilde{v}/\tilde{x}]$ $\quad \Gamma \vdash \tilde{v} : \tilde{\sigma}$ $\quad \tilde{x} \subseteq fv(\ldots)$

$k \notin LinProc(\Gamma \lceil PV)$

if $Proc(\tau[\tilde{v}/\tilde{x}]) \neq \phi \Rightarrow LinProc(\Gamma \lceil V) = 0$

$$\frac{}{\Gamma \vdash \overline{k}\langle \tilde{v}, V \rangle . P \triangleright \Delta, \tilde{v} : \tilde{\sigma}/k, k ![\Pi(\tilde{x} : \tilde{\sigma})\tau] ; \alpha}$$

We prove the case $\{v_i : \sigma_i\} \neq k : \alpha$

$\Gamma \vdash V : \tau [\tilde{v}/\tilde{x}]$

Note
In this case, this is a linear function

$E(\Gamma) ; \underline{Proc(\tau[\tilde{v}/\tilde{x}]) \setminus \Sigma_V} ; Lin(\Gamma \lceil V)$ $\vdash E(V) : E[\tau \ldots]$

$\Sigma_V = LinPro(\Gamma \lceil V)$
$\not\ni k$ (By condition)

$\exists \Delta' . \{k : E(\alpha)\}, \Delta' = Proc(\tau[\tilde{v}/\tilde{x}]) \setminus \Sigma_V$

By the formulation rule.

$E(\Gamma) ; Proc(\Delta) \setminus \Sigma_P ; Lin(\Gamma \lceil P) \vdash$

$Lin(\Gamma \lceil P) \cap Lin(\Gamma \lceil V) = \phi$ (By dependency formulation rule) $\quad E(P) \triangleright \Delta(\Delta)$

Hence we can apply to obtain

$$\frac{}{E(\Gamma);\ Proc(\Delta\cdot k:![\Pi(\tilde{x}:\tilde{\sigma})\tau])\setminus Proc(\Sigma_V \cup \Sigma);\ Lin(\Gamma\lceil PV) \vdash E(\bar{k}\langle\tilde{v},V\rangle, P) \triangleright \Diamond}$$

as required.

Theorem 4.3 (2) is rather obvious from (1). We shall outline how to translate it

First, since P is initial (Definition can be found in Section 2.1)

$$\Gamma ; \emptyset ; \emptyset \vdash P \triangleright \diamond$$

This is an intial process.

First, w.o.l.g. we think the normal form

$$P \equiv (\nu \tilde{a} : \tilde{\sigma})(P_1 | \dots \ | P_n)$$

hence we start from the transtation of

$$\Gamma, \underline{\tilde{a}} : \tilde{\sigma} ; \phi ; \phi \vdash P_i \triangleright \diamond$$

a_i is written as $\underline{a_i}$ since if, for example

$$P_i \equiv \bar{b} \langle \ulcorner \bar{a} \langle V \rangle \urcorner \rangle . P_i'$$

then it is transtated as

$$P_i \ \bar{b} \langle a, \ulcorner \bar{a} \langle V \rangle \urcorner \rangle . P_i$$

Apart from this restricted shared names, dependent names are needed for

(1) session channels (since they are restricted by server/client)

(2) variables for channels (both shared and session).

Then we construct the function $\langle\!\langle\ \rangle\!\rangle$

$$\langle\!\langle \Gamma, \Sigma, P \rangle\!\rangle = \langle \Gamma', \Delta', P' \rangle$$

Nil $\langle\!\langle \Gamma, \emptyset, 0 \rangle\!\rangle = \langle \Gamma', \emptyset, \mathbf{0} \rangle$

Output $\langle\!\langle \Gamma, \Sigma_V, V \rangle\!\rangle = \langle \Gamma'_V, \Delta_V, W \rangle$

$\langle\!\langle \Gamma, \Sigma_P, P \rangle\!\rangle = \langle \Gamma'_1, \Delta_P, P' \rangle$

$\Gamma' \vdash \underline{\tilde{a}}:\tilde{\sigma},\ \tilde{k}:\tilde{\alpha},\ \tilde{x}:\tilde{\sigma}'$

and $\tilde{a}, \tilde{k}, \tilde{x} \in fv(W) \cup fn(W)$

$\Rightarrow \langle\!\langle \Gamma, (\Sigma_V, \Sigma_P) \backslash k{:}\alpha,\ b\langle V \rangle . P \rangle\!\rangle$

$\Rightarrow \langle \Gamma', (\Delta_V, \Delta_P) \backslash k,\ \bar{b}\langle \tilde{a}\tilde{k}\tilde{x}, W \rangle . P' \rangle$

etc. then the theorem is mechanically proved by induction.

$\square$