

# Multiparty Session Types and Communicating Automata

FCT 2021: 23rd International Symposium on Fundamentals of Computation Theory



Nobuko Yoshida

Imperial College  
London

# Communications are Ubiquitous

- Increasingly, **communications** are the way to organise software and systems.
- Industry trend – programming languages with **explicit message-passing primitives**.



microservices



# Problems: Ambiguity

- Protocol descriptions are **ambiguous**
- **SMTP: simple mail transfer protocol**
  - They are written in English, often very long



RFC 821 August 1982  
Simple Mail Transfer Protocol

TABLE OF CONTENTS

<u>1.</u>	<u>INTRODUCTION .....</u>	<u>1</u>
<u>2.</u>	<u>THE SMTP MODEL .....</u>	<u>2</u>
<u>3.</u>	<u>THE SMTP PROCEDURE .....</u>	<u>4</u>
<u>3.1.</u>	<u>Mail .....</u>	<u>4</u>
<u>3.2.</u>	<u>Forwarding .....</u>	<u>7</u>
<u>3.3.</u>	<u>Verifying and Expanding .....</u>	<u>8</u>
<u>3.4.</u>	<u>Sending and Mailing .....</u>	<u>11</u>
<u>3.5.</u>	<u>Opening and Closing .....</u>	<u>13</u>
<u>3.6.</u>	<u>Relaying .....</u>	<u>14</u>
<u>3.7.</u>	<u>Domains .....</u>	<u>17</u>
<u>3.8.</u>	<u>Changing Roles .....</u>	<u>18</u>
<u>4.</u>	<u>THE SMTP SPECIFICATIONS .....</u>	<u>19</u>
<u>4.1.</u>	<u>SMTP Commands .....</u>	<u>19</u>
<u>4.1.1.</u>	<u>Command Semantics .....</u>	<u>19</u>
<u>4.1.2.</u>	<u>Command Syntax .....</u>	<u>27</u>
<u>4.2.</u>	<u>SMTP Replies .....</u>	<u>34</u>
<u>4.2.1.</u>	<u>Reply Codes by Function Group .....</u>	<u>35</u>
<u>4.2.2.</u>	<u>Reply Codes in Numeric Order .....</u>	<u>36</u>
<u>4.3.</u>	<u>Sequencing of Commands and Replies .....</u>	<u>37</u>
<u>4.4.</u>	<u>State Diagrams .....</u>	<u>39</u>
<u>4.5.</u>	<u>Details .....</u>	<u>41</u>
<u>4.5.1.</u>	<u>Minimum Implementation .....</u>	<u>41</u>
<u>4.5.2.</u>	<u>Transparency .....</u>	<u>41</u>
<u>4.5.3.</u>	<u>Sizes .....</u>	<u>42</u>

# Problems: Ambiguity

- Protocol descriptions are **ambiguous**
- **SMTP: simple mail transfer protocol**
  - They are written in English, often very long



## 3.1. MAIL

There are three steps to SMTP mail transactions. The transaction is started with a MAIL command which gives the sender identification. A series of one or more RCPT commands follows giving the receiver information. Then a DATA command gives the mail data. And finally, the end of mail data indicator confirms the transaction.

The first step in the procedure is the MAIL command. The <reverse-path> contains the source mailbox.

```
MAIL <SP> FROM:<reverse-path> <CRLF>
```

This command tells the SMTP-receiver that a new mail transaction is starting and to reset all its state tables and buffers, including any recipients or mail data. It gives the reverse-path which can be used to report errors. If accepted, the receiver-SMTP returns a 250 OK reply.

The <reverse-path> can contain more than just a mailbox. The <reverse-path> is a reverse source routing list of hosts and source mailbox. The first host in the <reverse-path> should be the host sending this command.

The second step in the procedure is the RCPT command.

```
RCPT <SP> TO:<forward-path> <CRLF>
```

This command gives a forward-path identifying one recipient. If accepted, the receiver-SMTP returns a 250 OK reply, and stores the forward-path. If the recipient is unknown the receiver-SMTP returns a 550 Failure reply. This second step of the procedure can be repeated any number of times.

# Problems: Concurrency Bugs

---

- Communications increase **concurrency bugs**
  - Survey of 4K users [golang.org]
  - Analysis of 6 large software systems [ASPLOS 19]

deadlock

channel errors

More than a half of concurrency bugs in Go are caused by communications.



The Go Gopher

# Problems: Concurrency Bugs

- Communications increase **concurrency bugs**
  - Survey of 4k users [golang.org]
  - Analysis of 6 large software systems [ASPLOS 19]

More than a half of concurrency bugs in Go are caused by communications.



## Session Types

- Prevent concurrency bugs.
- Can abstract, implement and manage communications as **Protocols**.
- **Clean, Cheap** and **Retrofittable**.

# Why Session Types, Why Now?

---

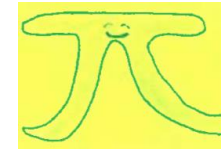
Significant academic and industry interests via fundamental breakthroughs

Milner,  
Honda, NY



Binary Session Types

ESOP'98



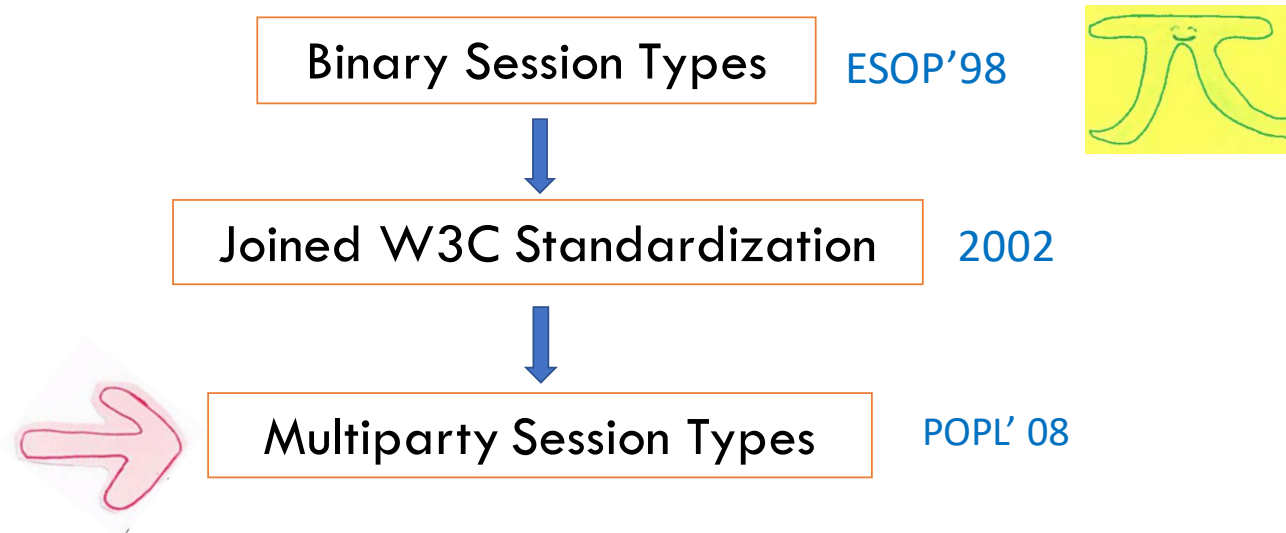
Joined W3C Standardization

2002

# Why Session Types, Why Now?

---

Significant academic and industry interests via fundamental breakthroughs

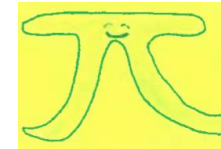


# Why Session Types, Why Now?

Significant academic and industry interests via fundamental breakthroughs

Binary Session Types

ESOP'98



Joined W3C Standardization

2002



Multiparty Session Types

POPL'08

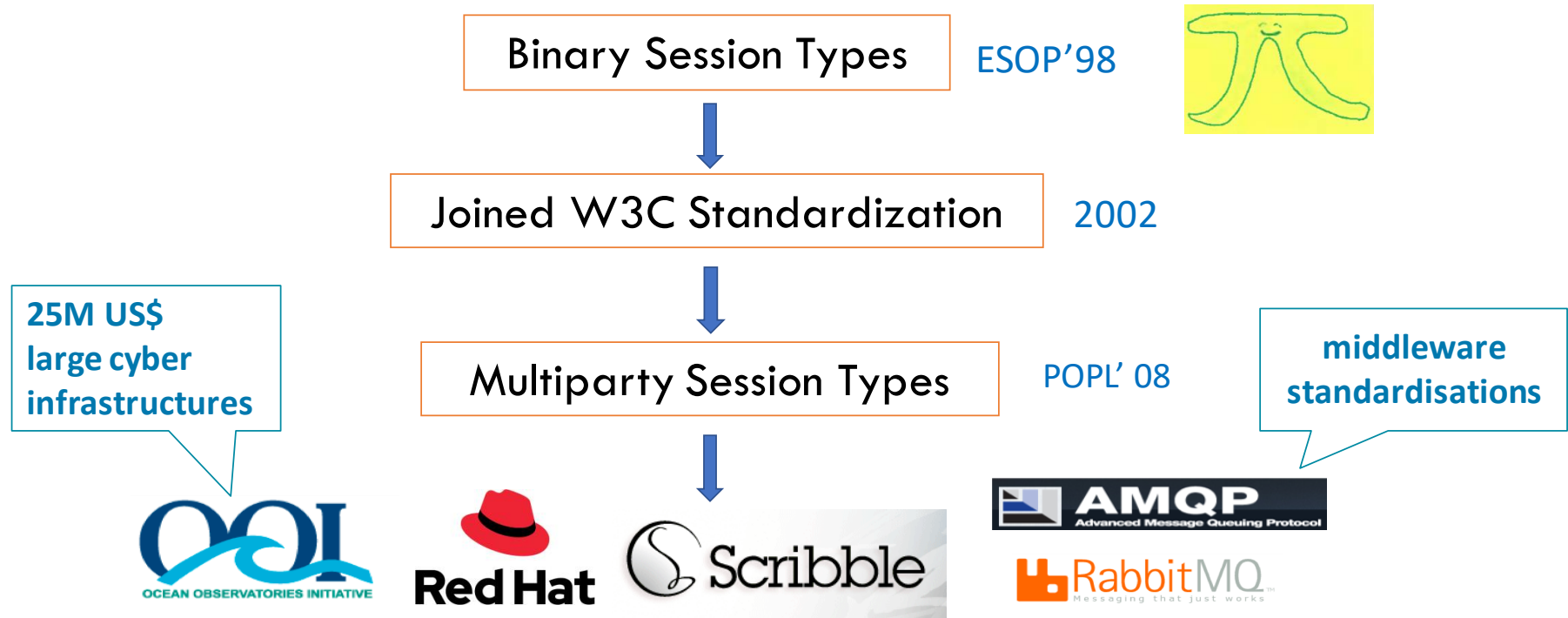


largest open source  
company in the world



# Why Session Types, Why Now?

Significant academic and industry interests via fundamental breakthroughs



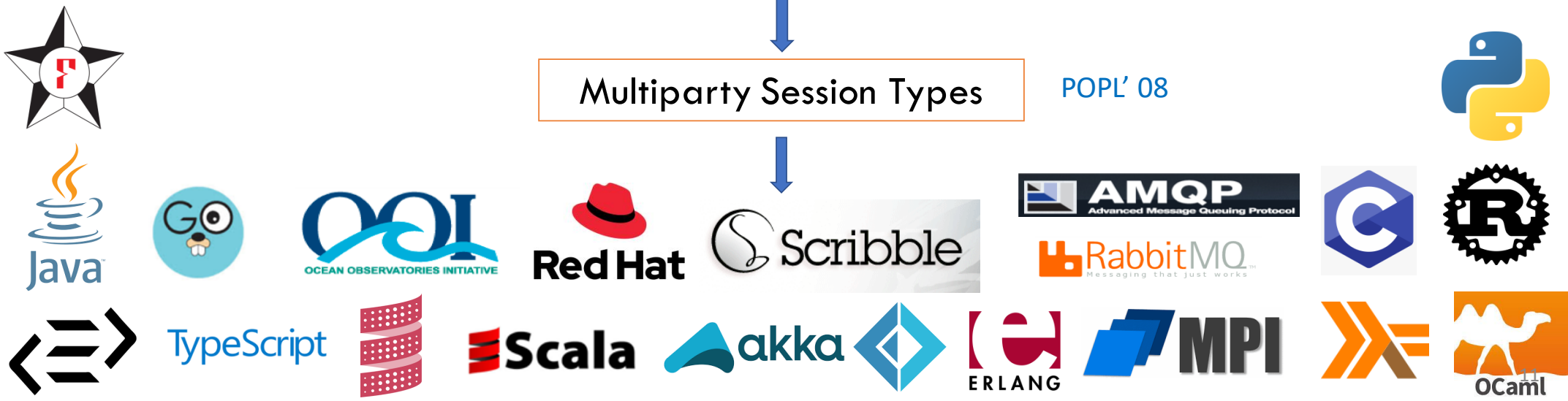
# Why Session Types, Why Now?

Significant academic and industry interests via fundamental breakthroughs

Binary Session Types ESOP'98 

Joined W3C Standardization 2002

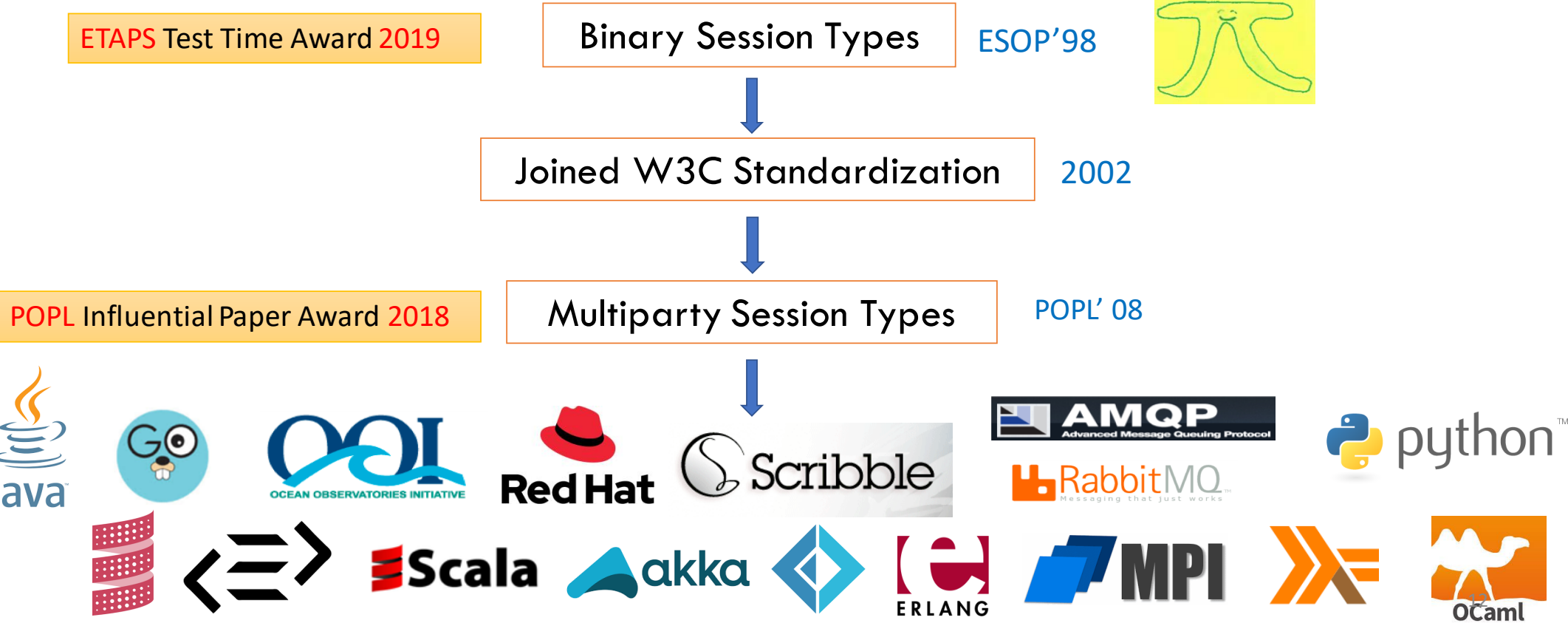
Multiparty Session Types POPL'08



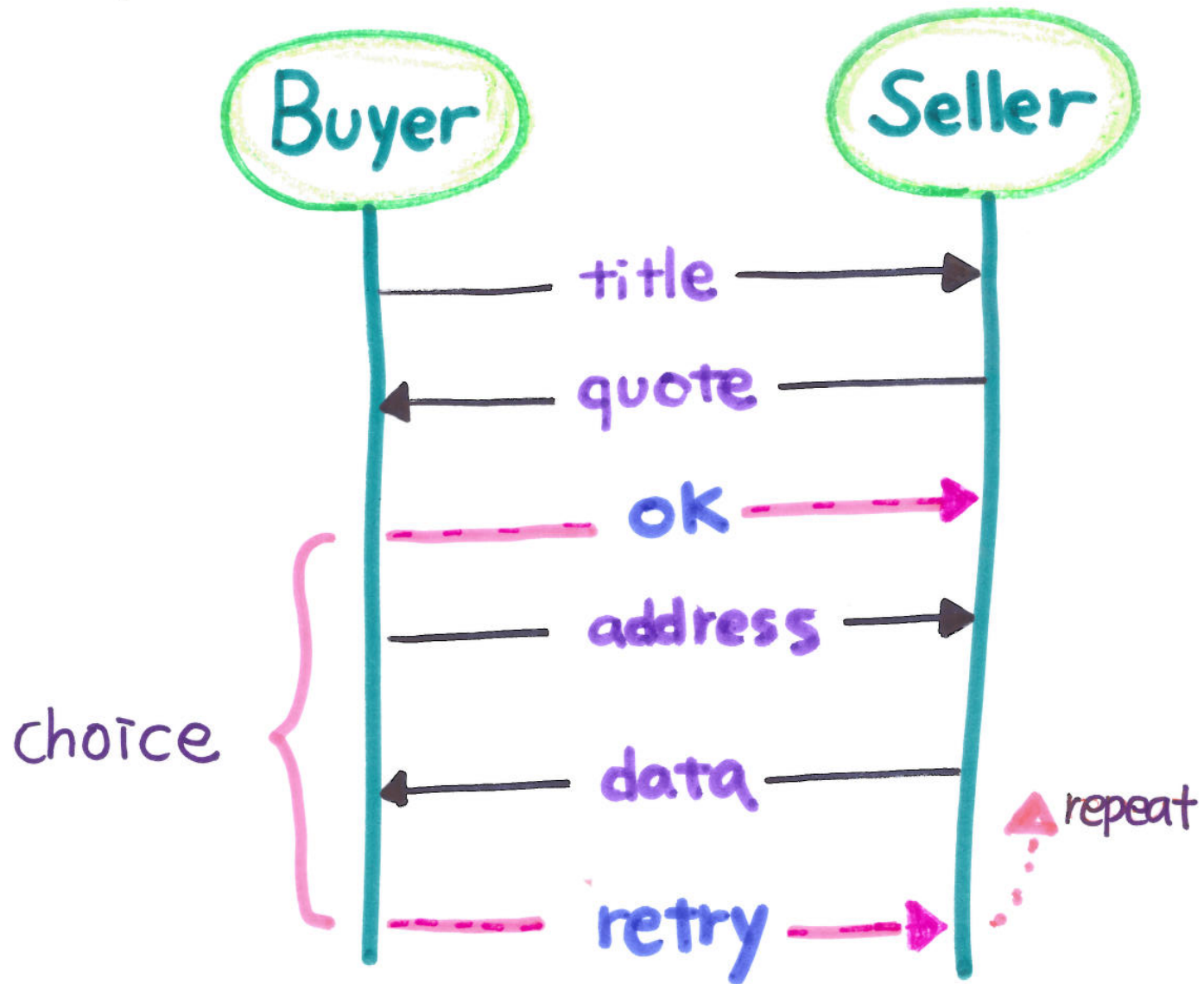
A collection of logos for various programming languages, frameworks, and organizations associated with session types. The logos include: a star with a red question mark, Java, TypeScript, OOI (Ocean Observatories Initiative), Red Hat, Scribble, AMQP (Advanced Message Queuing Protocol), RabbitMQ (Messaging that just works), Scala, akka, Erlang, MPI, OCaml, Python, and R.

# Why Session Types, Why Now?

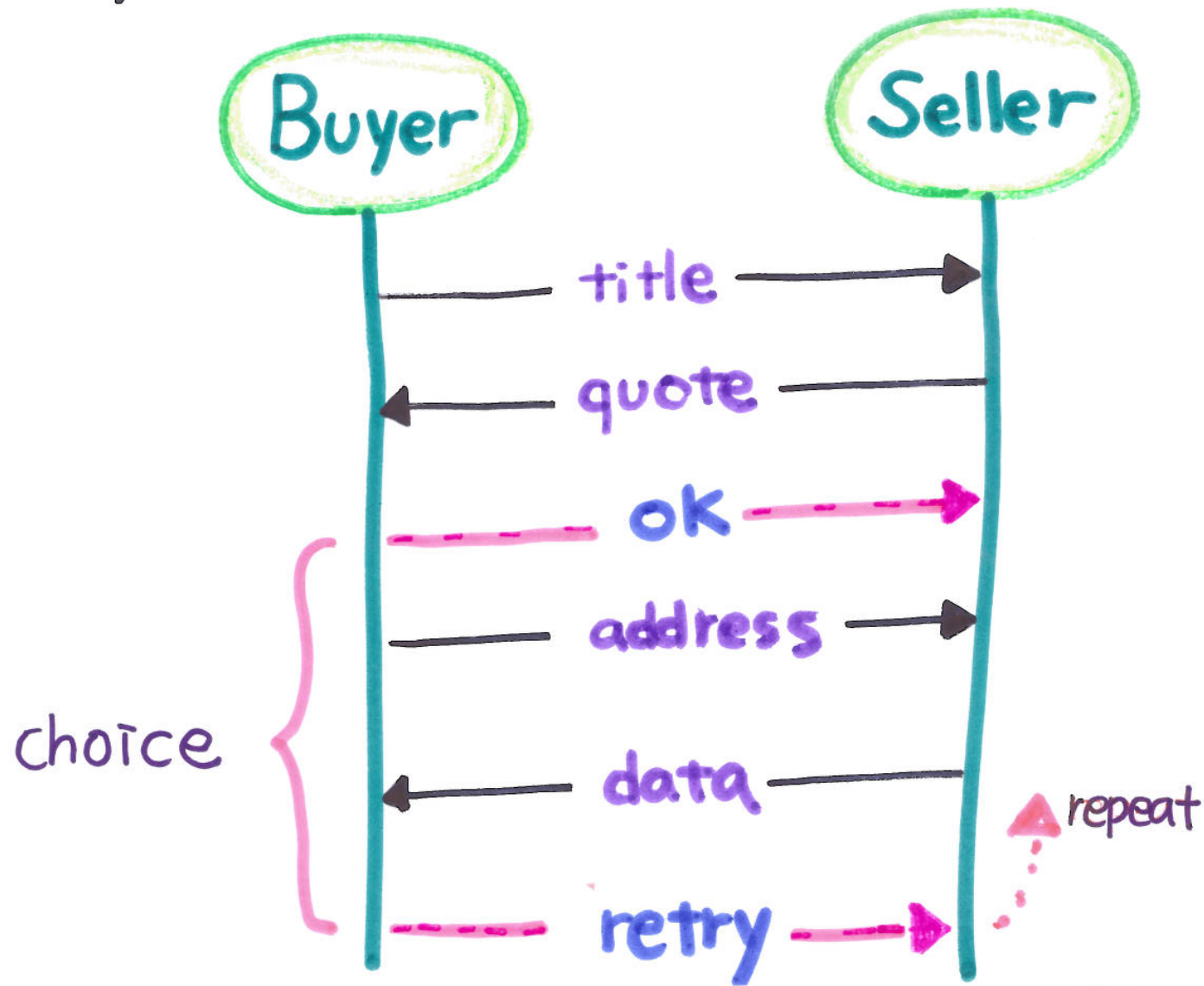
Significant academic and industry interests via fundamental breakthroughs



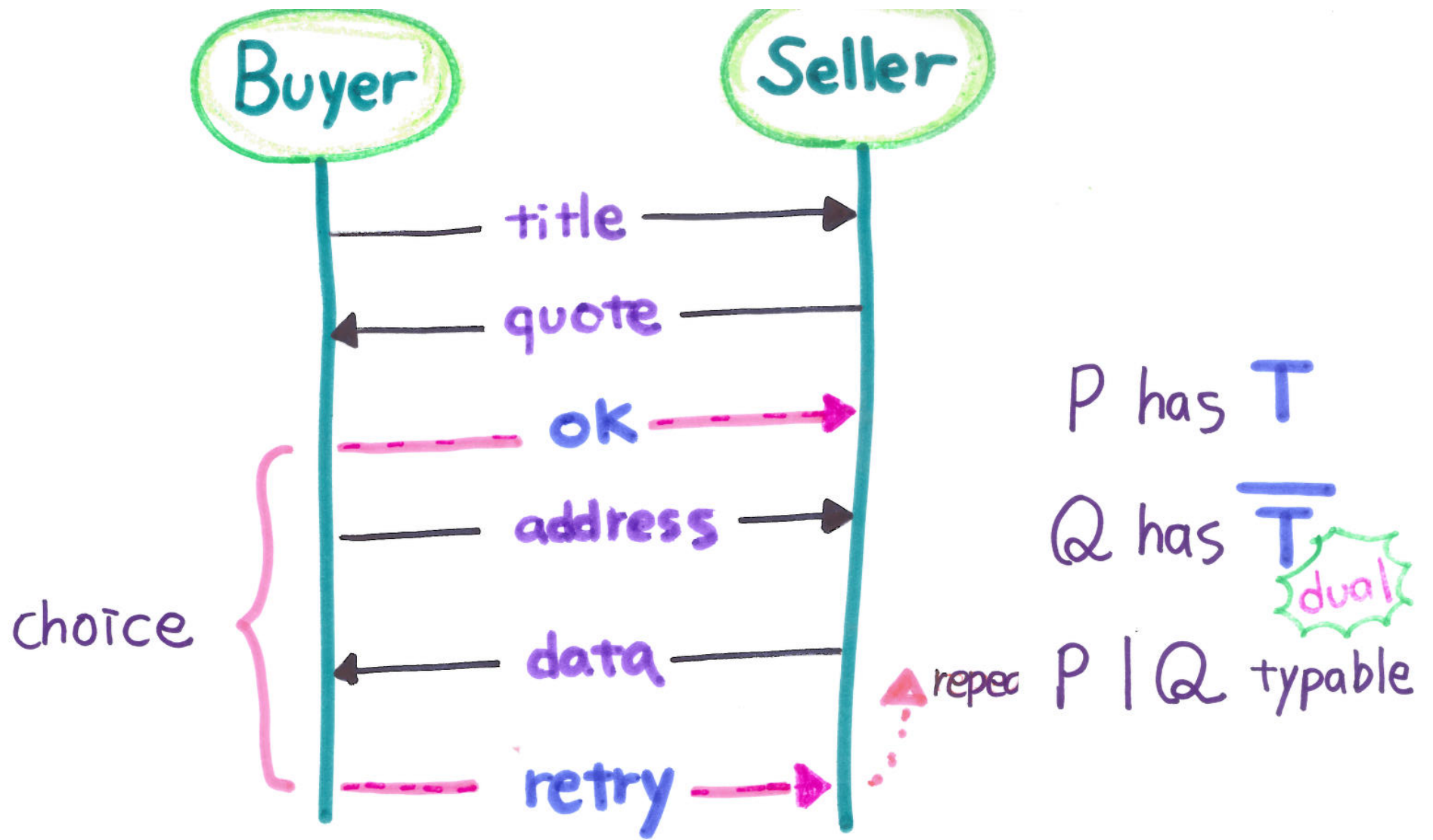
# Binary Session Types: Buyer - Seller Protocol



# Binary Session Types: Buyer - Seller Protocol



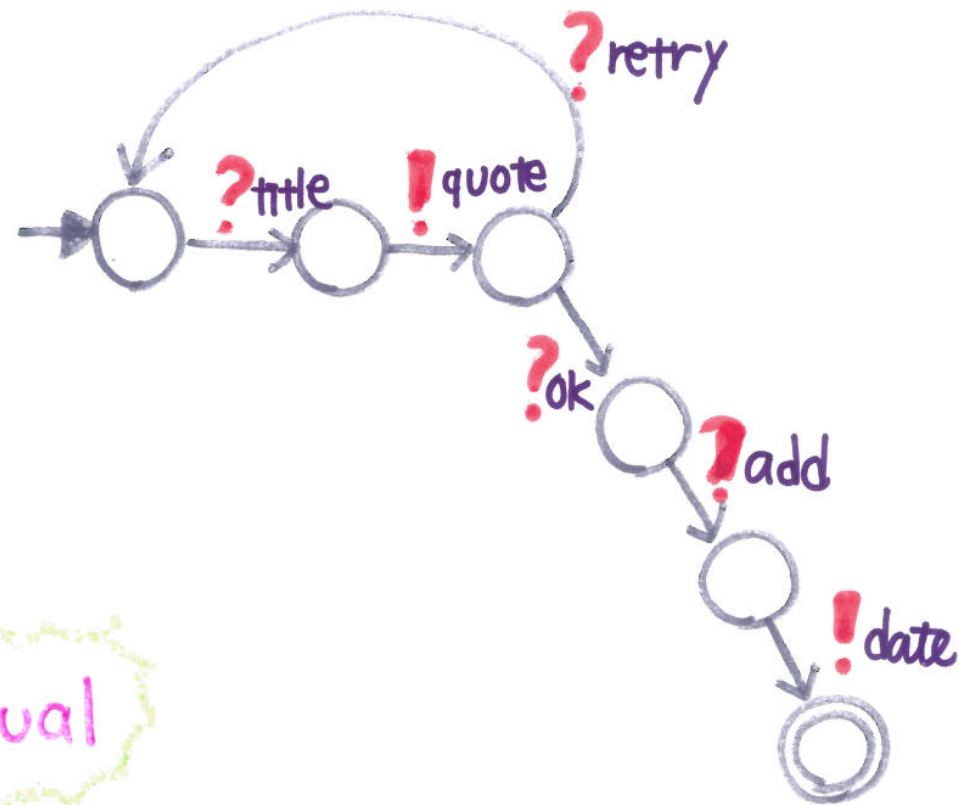
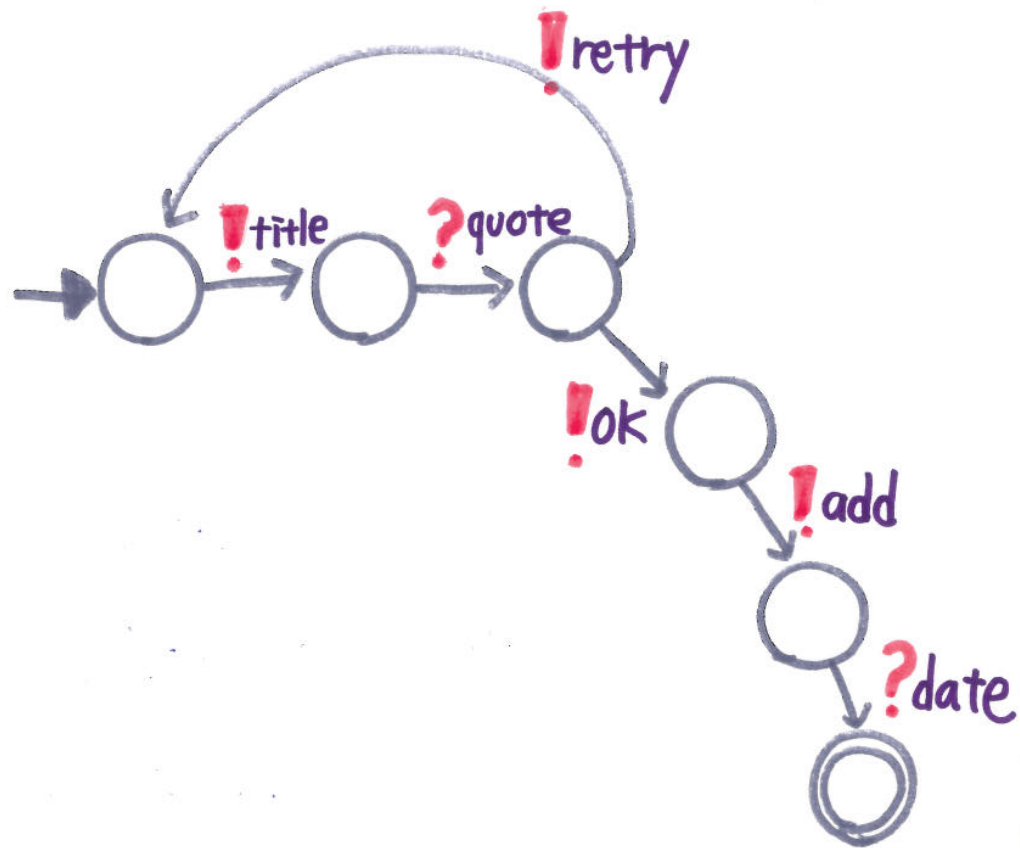
nt! Title ; ? Quote ; ! { ok: ! Add ; ? Date, retry: t }



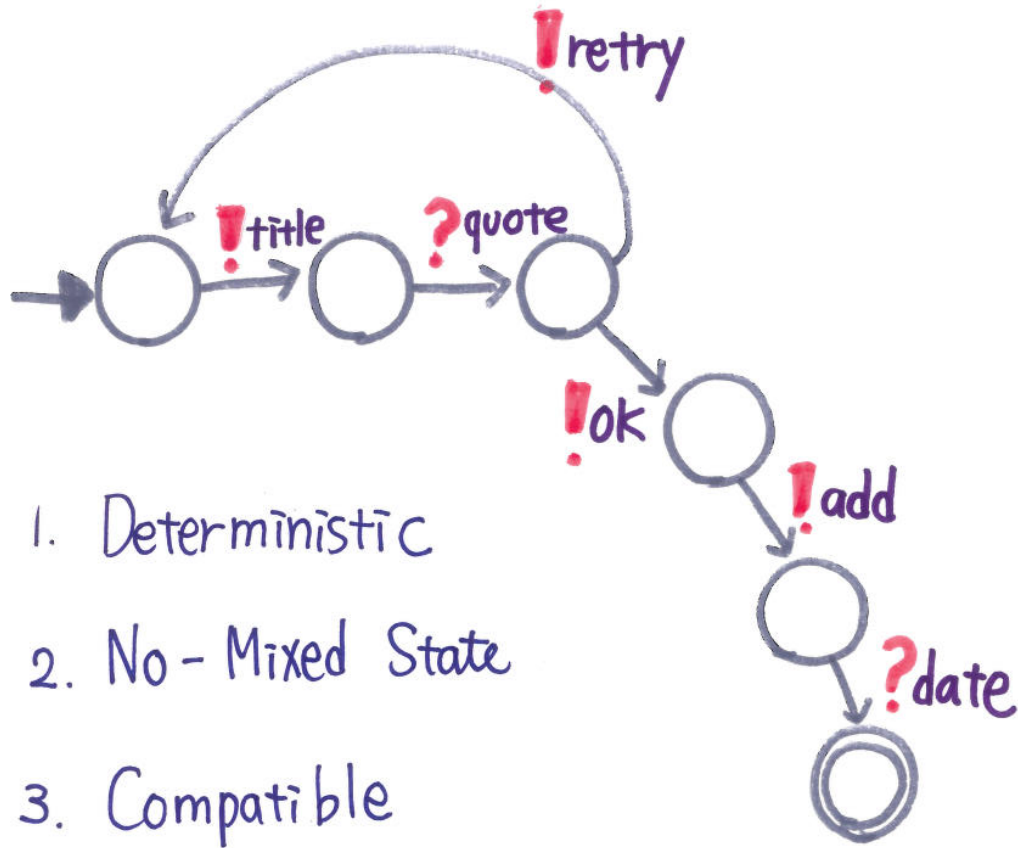
nt! Title ; ? Quote ; ! { ok: ! Add ; ? Date, retry: t }

nt? Title ; ! Quote ; ? { ok: ? Add ; ! Date, retry: t }

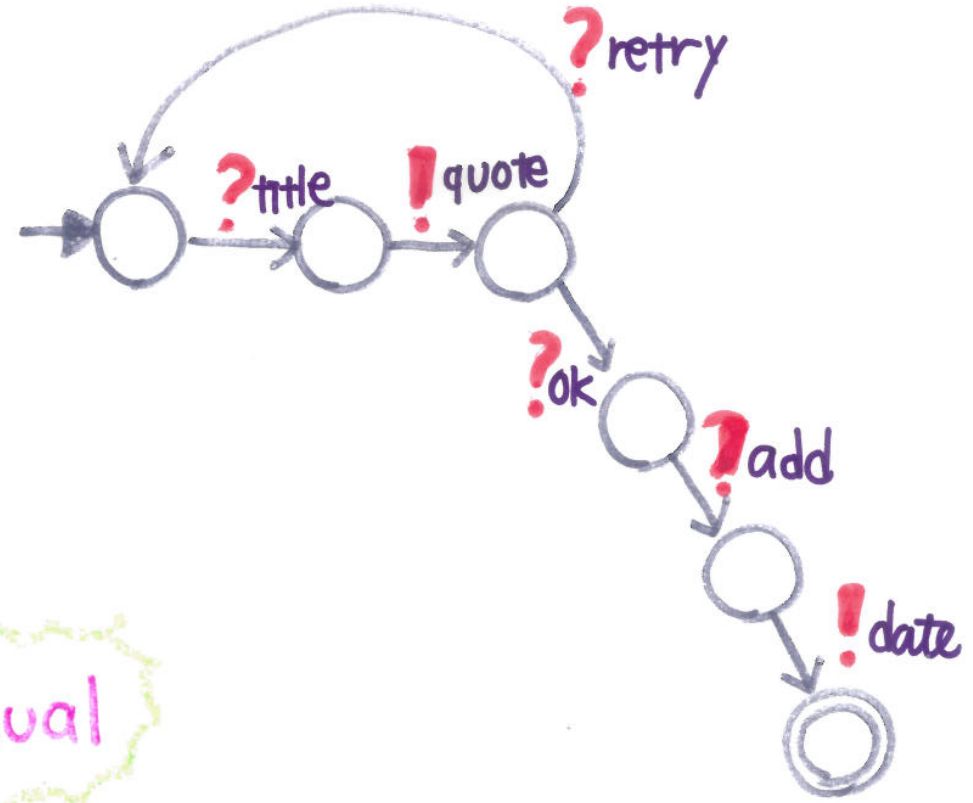
# Communicating Automata [1980s]



dual



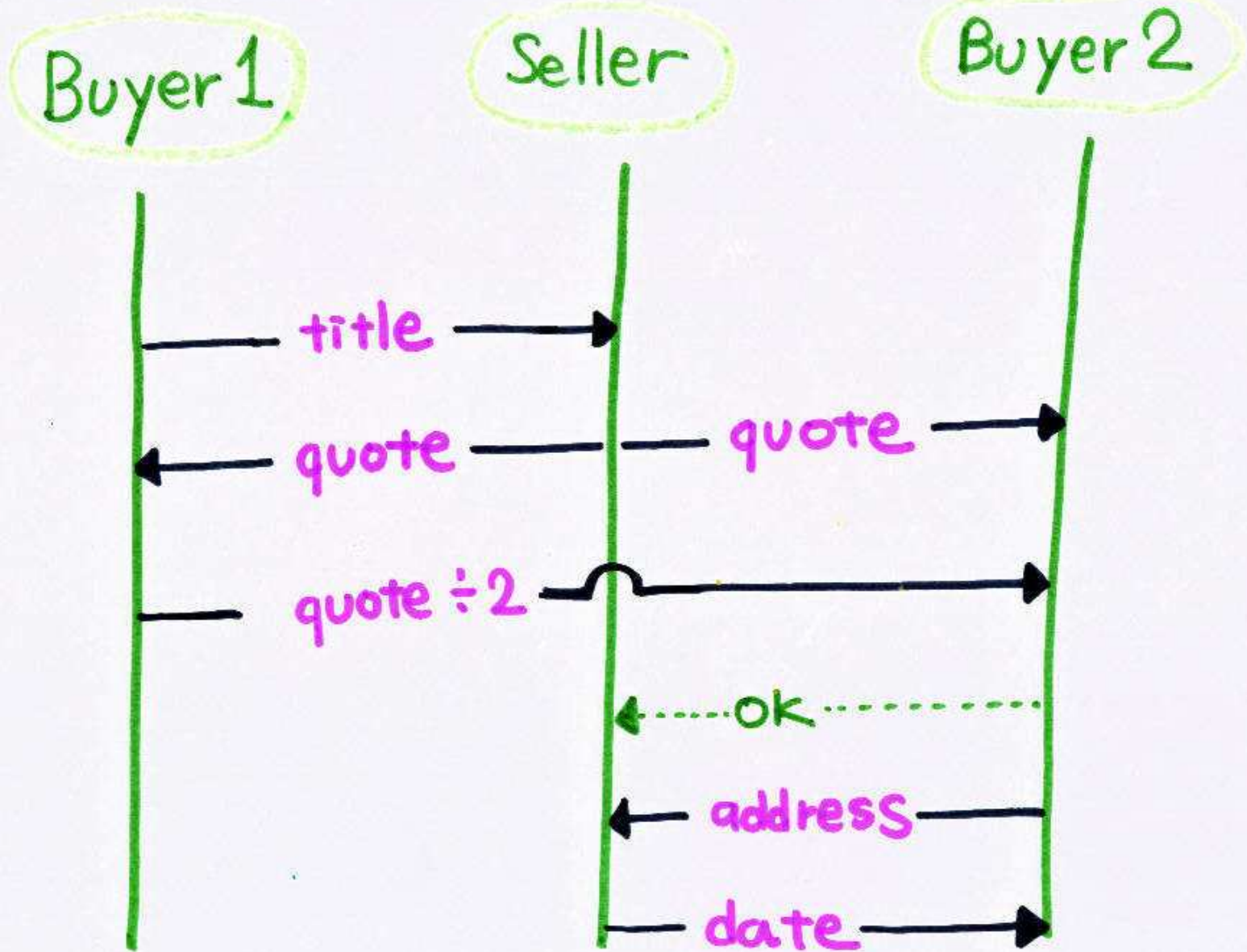
1. Deterministic
2. No - Mixed State
3. Compatible

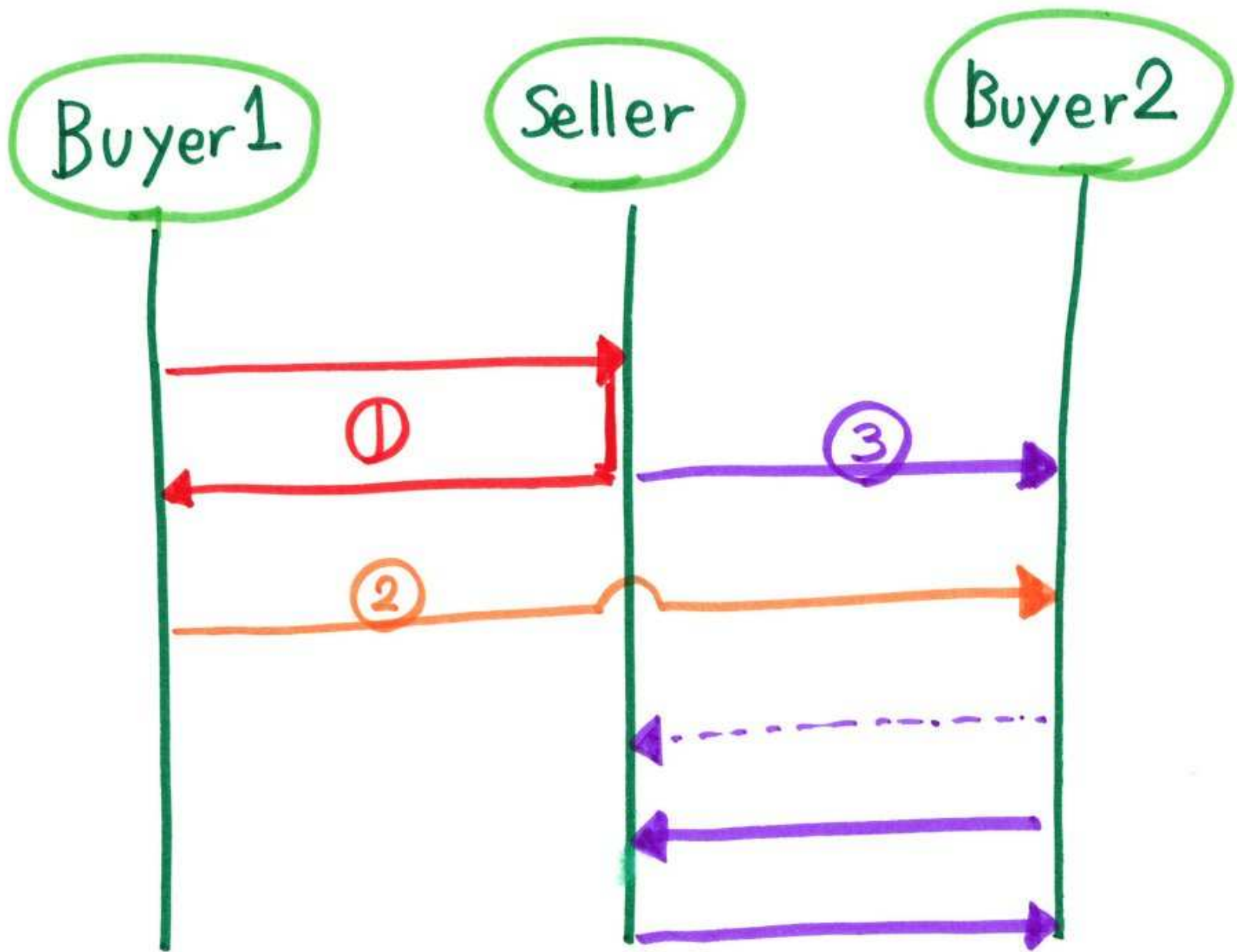


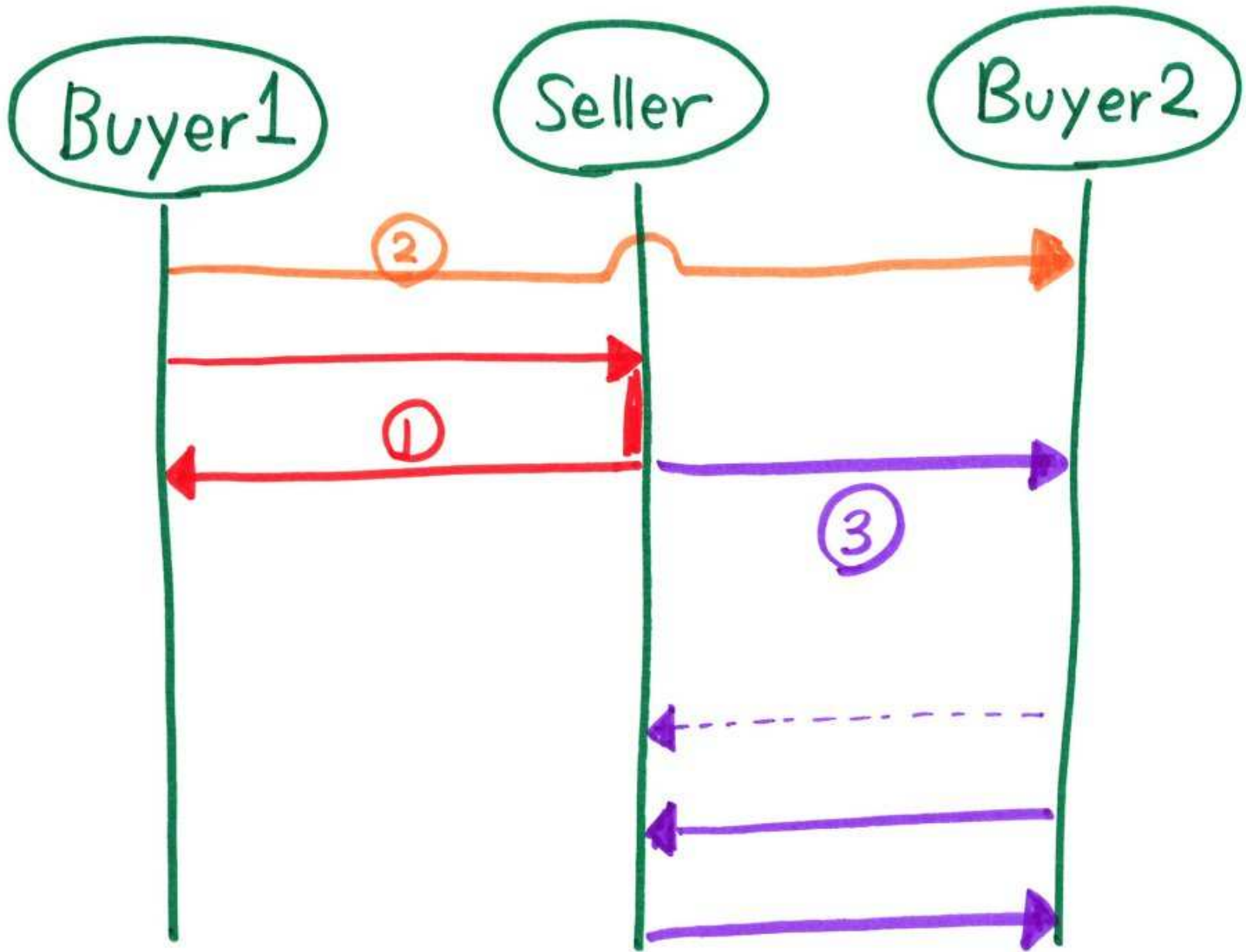
dual

**[Gouda et al 1986]** Two compatible machines without mixed states which are deterministic satisfy deadlock-freedom.

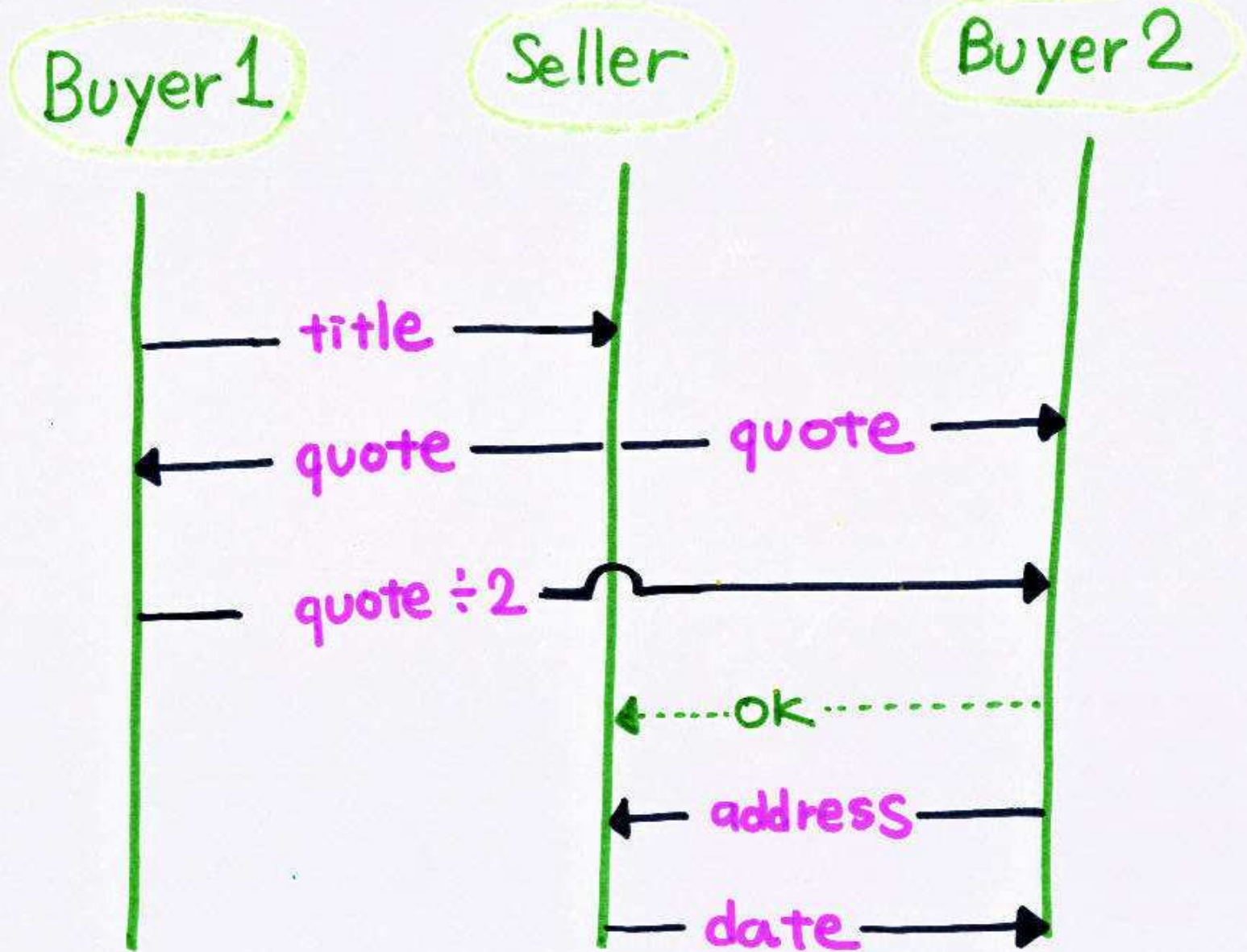
# Multiparty Session Types







# Multiparty Session Types



Alice

Bob

Carol

CA? c ; AB! a

AB? a ; BC! b

BC? b ; CA? c

dual

dual

dual

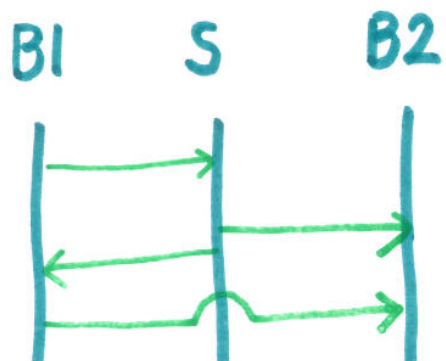
3 dual pairs

If you use  
binary Session  
Types ...

Deadlock!

# Multi party Session Types

[Honda, Yoshida, Carbone 2008]



ⓐ

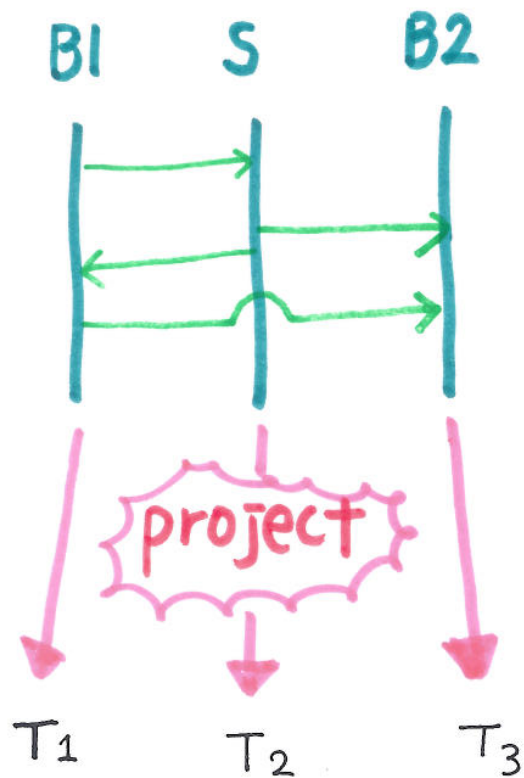
$BI \rightarrow S$  Int.

$S \rightarrow B2$  Char

**STEP 1**

Write Global Type

# Multi party Session Types [Honda, Yoshida, Carbone 2008]



$\textcircled{G}$   $B1 \rightarrow S \text{ Int.}$   
 $S \rightarrow B2 \text{ Char}$

$\textcircled{T}$   $B1? \text{Int. } B2! \text{Char}$

**STEP 1**

Write Global Type

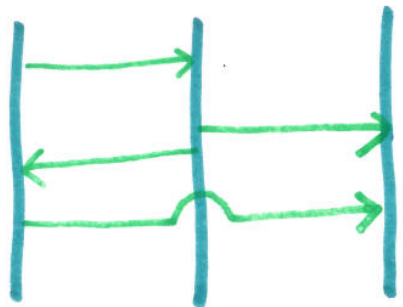
**STEP 2**

Project to Local Types

# Multi party Session Types

[Honda, Yoshida, Carbone 2008]

B1 S B2



(G)

$B1 \rightarrow S \text{ Int.}$

$S \rightarrow B2 \text{ Char}$

STEP 1

Write Global Type

(T)

$B1? \text{Int. } B2! \text{Char}$

STEP 2

Project to Local Type

T<sub>1</sub>

T<sub>2</sub>

T<sub>3</sub>



P<sub>1</sub>

P<sub>2</sub>

P<sub>3</sub>



(P)  $B1?(x). B2! \langle \text{"apple"} \rangle$

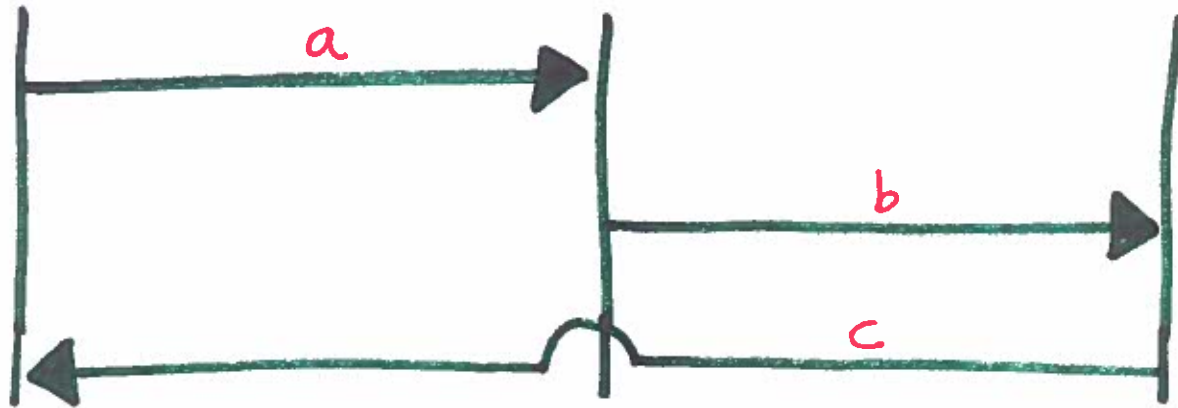
STEP 3

- Static Check
- Generate Code
- Run-time check

Alice

Bob

Carol



Global Type



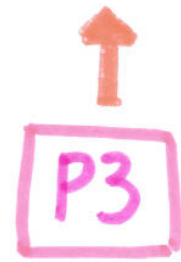
Alice  $AB!a; CA?c$

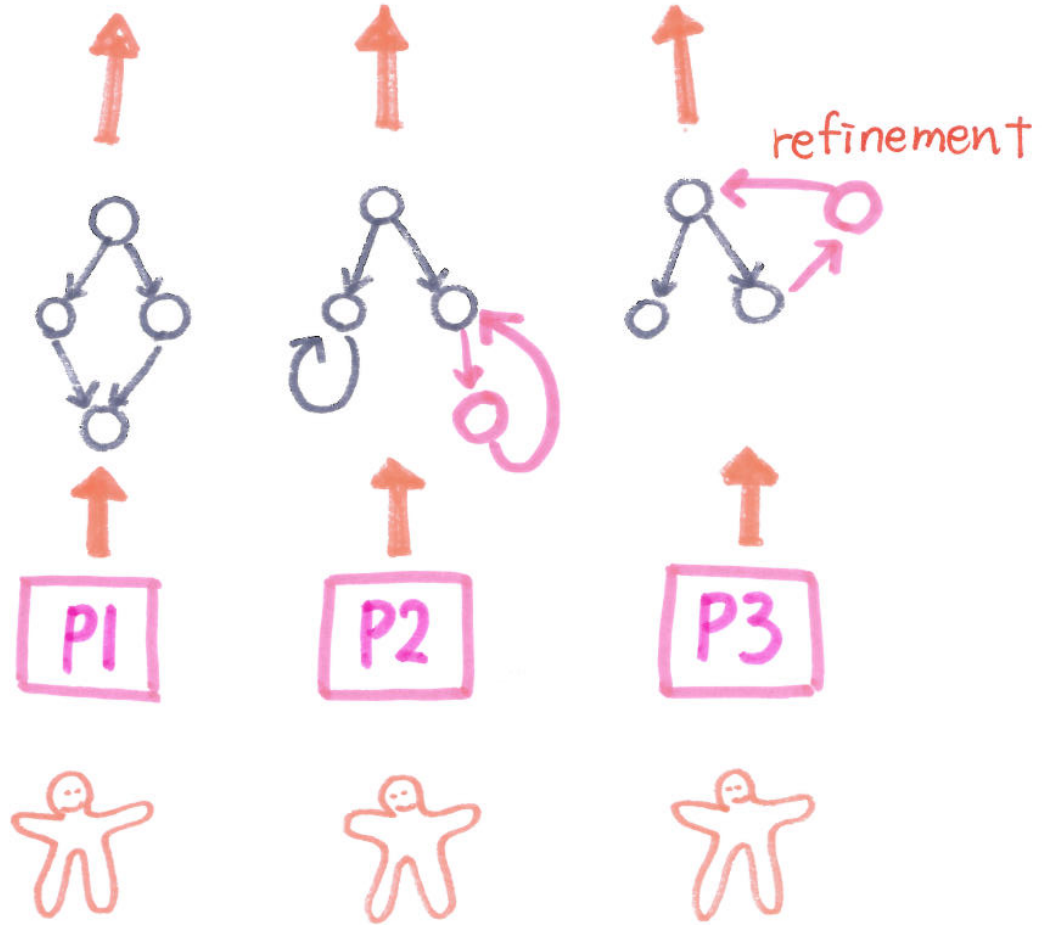
Bob  $AB?a; BC!b$

Carol  $BC?b; CA!c;$

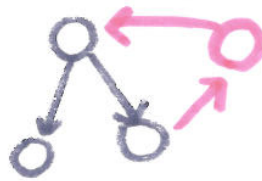
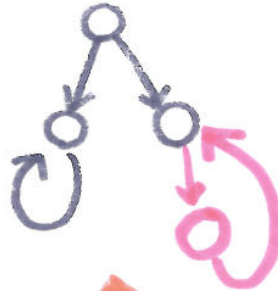
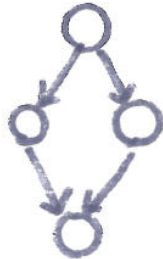
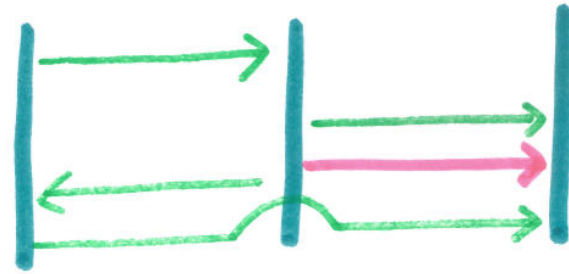
No Deadlock

LOCAL TYPES

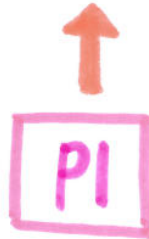




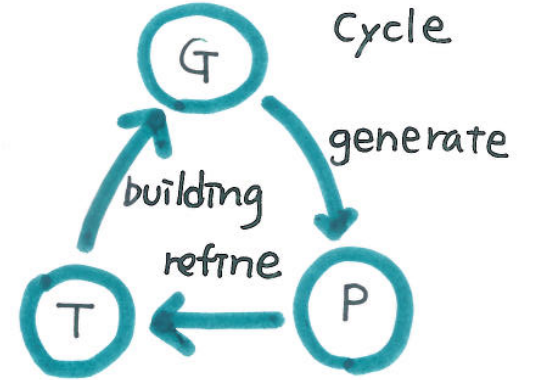
BI S B2



refinement



Software Development Cycle



- Optimisation
- refinement
- inference
- Testing

# Global Types

# Local Types

$G ::= P \rightarrow P' : \{ a_j. G_j \}_{j \in J}$

| mt.  $G$

| t

| end

$T ::= P ! \{ a_j. T_j \}_{j \in J}$

|  $P ? \{ a_j. T_j \}_{j \in J}$

| mt.  $T$

| t

| end

$P, P', \dots$  Participant

$a, b, c$   $\Sigma$  Alphabet

Projection from  $G$  onto  $q$   $G \upharpoonright q$

$$(P \rightarrow P' : \{a_j, G_j\}_{j \in J}) \upharpoonright q$$

$$= \begin{cases} P \upharpoonright \{a_j, G_j \upharpoonright q\}_{j \in J} & q = P \\ P' \upharpoonright \{a_j, G_j \upharpoonright q\}_{j \in J} & q = P' \\ G_i \upharpoonright q = G_j \upharpoonright q & \forall j \in J \text{ otherwise} \end{cases}$$

Projection from  $\Gamma$  onto  $\varrho$       $\Gamma \upharpoonright \varrho$

$$(P \rightarrow P' : \{a_j. \Gamma_j\}_{j \in J}) \upharpoonright \varrho$$

$$= \begin{cases} P ! \{a_j. \Gamma_j \upharpoonright \varrho\}_{j \in J} & \varrho = P \\ P ? \{a_j. \Gamma_j \upharpoonright \varrho\}_{j \in J} & \varrho = P' \\ \Gamma_1 \upharpoonright \varrho = \Gamma_j \upharpoonright \varrho & \forall j \in J \text{ otherwise} \end{cases}$$

**Bad**  $A \rightarrow B : \{ \underline{a}. C \rightarrow D : \underline{c}, \underline{b}. C \rightarrow D : \underline{d} \}$

**Good**  $A \rightarrow B : \{ \underline{a}. C \rightarrow D : \underline{c}, \underline{b}. C \rightarrow D : \underline{c} \}$

# LTS Local Types

$$l ::= pq!a \mid pq?a$$

$$q! \{a_i. T_i\}_{i \in I} \xrightarrow{pq!a_i} T_i$$

$$q? \{a_i. T_i\}_{i \in I} \xrightarrow{qp?a_i} T_i$$

$$\frac{T[mt. T/t] \xrightarrow{l} T'}{mt. T \xrightarrow{l} T'}$$

$(\vec{T}; \vec{w})$

$W_{pq}$   
queue

Send  $T_p \xrightarrow{pq!l} T_p' \Rightarrow$

$$(\dots T_p \dots ; \dots W_{pq} \dots) \xrightarrow{pq!l} (\dots T_p' \dots ; \dots W_{pq} \cdot l \dots)$$

Receive  $T_q \xrightarrow{pq?l} T_q' \Rightarrow$

$$(\dots T_q \dots ; \dots l \cdot W_{pq} \dots) \xrightarrow{pq?l} (\dots T_q' \dots ; \dots W_{pq} \dots)$$

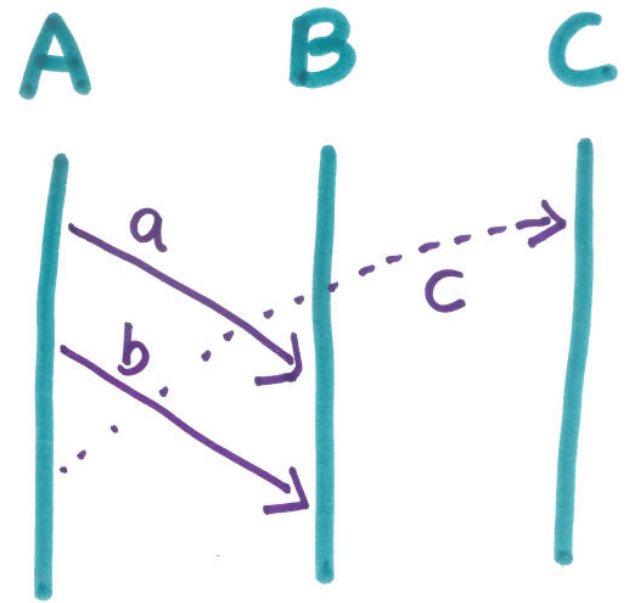
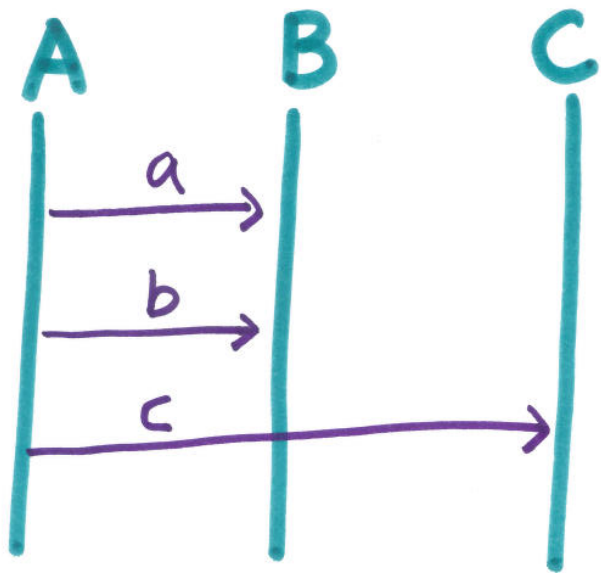
# LTS Global Types

$$p \rightsquigarrow p' : \bar{j} \{a_i. G_i\}_{i \in I}$$

put  $p \rightarrow p' : \{a_i. G_i\}_{i \in I} \xrightarrow{pp'!a_{\bar{j}}} p \rightsquigarrow p' : \bar{j} \{a_i. G_i\}_{i \in I}$

get  $p \rightsquigarrow p' : \bar{j} \{a_i. G_i\}_{i \in I} \xrightarrow{pp'?a_{\bar{j}}} G_{\bar{j}}$

$$\frac{\forall j \in I \quad G_j \xrightarrow{\ell} G'_j \quad p, q \notin \text{sub}(\ell) \quad \frac{G_j \xrightarrow{\ell} G'_j}{q \notin \text{sub}(\ell)} \quad \forall i \in I \setminus \bar{j} \quad G'_i = G_i}{p \rightarrow p' : \{a_i. G_i\}_{i \in I} \xrightarrow{\ell} p \rightarrow p' : \{a_i. G'_i\}_{i \in I} \quad p \rightsquigarrow q : \bar{j} \{a_i. G_i\}_{i \in I} \xrightarrow{\ell} p \rightsquigarrow q : \bar{j} \{a_i. G'_i\}_{i \in I}}$$



$A \rightarrow B:a . A \rightarrow B:b . A \rightarrow C:c$

↓  $AB!a$  OUT

$A \rightsquigarrow B:a . A \rightarrow B:b . A \rightarrow C:c$

↓  $AB?a$  IN

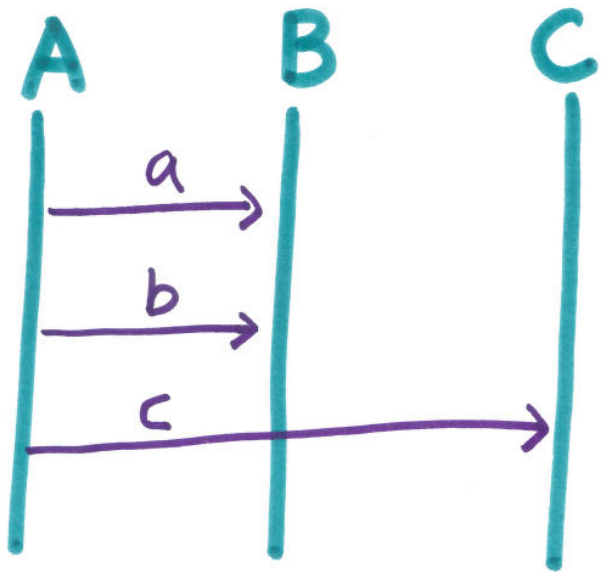
$A \rightarrow B:b \quad A \rightarrow C:c$

↓  $AB!b$  OUT

$A \rightsquigarrow B:b \quad A \rightarrow C:c$

↓  $AB?b$  IN

$A \rightarrow C:c$



$A \rightarrow B:a. A \rightarrow B:b. A \rightarrow C:c$

↓  $AB!a$  OUT

$A \rightsquigarrow B:a. A \rightarrow B:b. A \rightarrow C:c$

↓  $AB?a$  IN

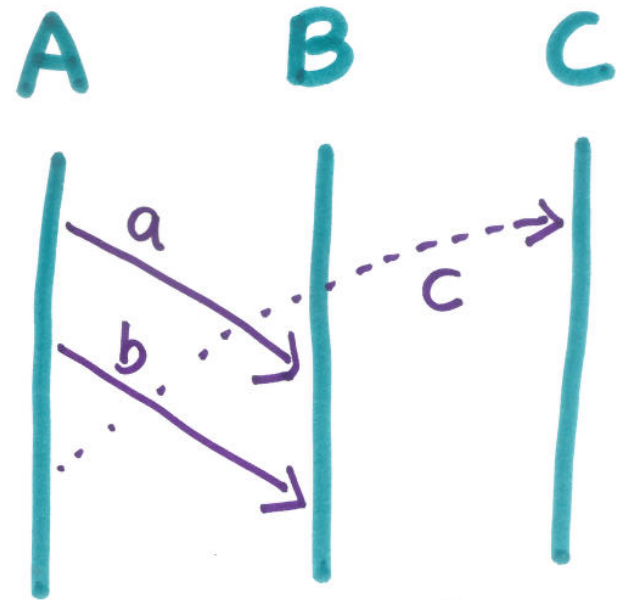
$A \rightarrow B:b. A \rightarrow C:c$

↓  $AB!b$  OUT

$A \rightsquigarrow B:b. A \rightarrow C:c$

↓  $AB?b$  IN

$A \rightarrow C:c$



$A \rightarrow B:a. A \rightarrow B:b. A \rightarrow C:c$

↓  $AB!a$  OUT

↓  $AB!b$  OUT

↓  $AC!c$  OUT

$A \rightsquigarrow B:a. A \rightsquigarrow B:b. A \rightsquigarrow C:c$

↓  $AC?c$  IN

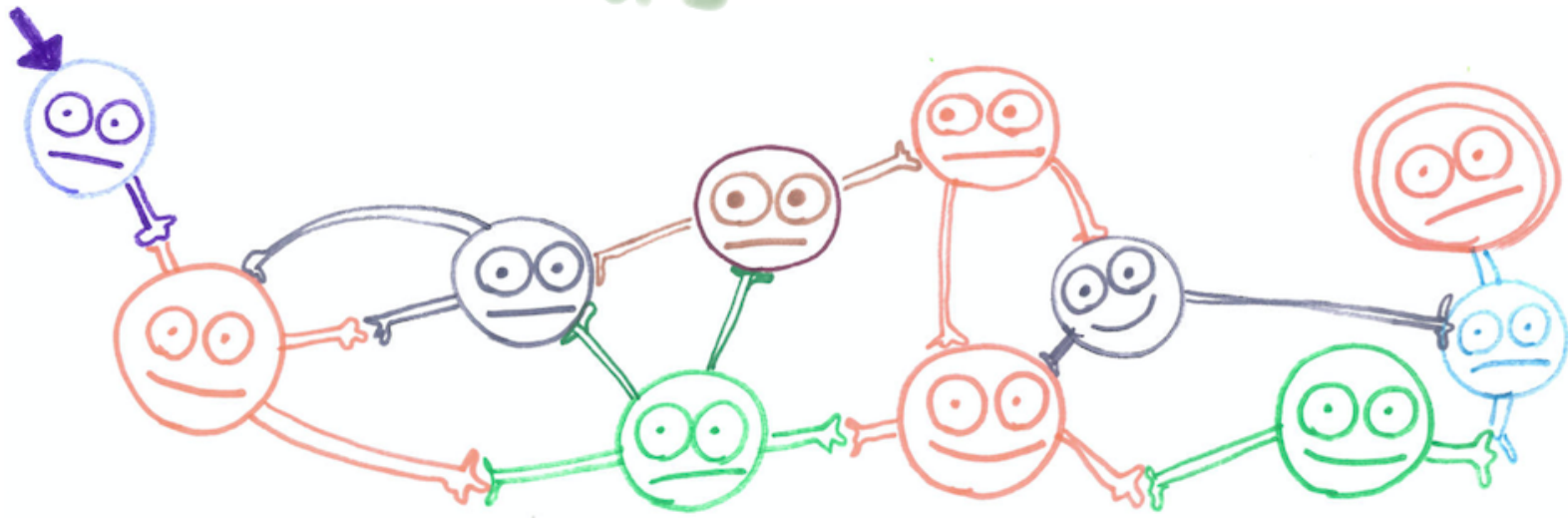
$A \rightsquigarrow B:a. A \rightsquigarrow B:b$

↓  $AB?a$  IN

$A \rightsquigarrow B:b$

# Multiparty Session Types

as



?? Communicating Automata ??

# CFSMs [1980-] ITU notation SDL · MSCS ...

**Def** A CFSM  $M = (Q, C, q_0, \Sigma, \delta)$

$Q$  a finite set of states

$C = \{ pq \in \text{Participant}^2 \mid p \neq q \}$

$q_0$  initial state

$\Sigma$  a finite alphabet of messages

$\delta \subseteq Q \times (C \times \{!, ?\} \times \Sigma) \times Q$  a finite set of transitions

**Def** CS  $S = (M_p)_{p \in \text{Participant}}$

$S \xrightarrow{t} S'$  configuration  $S = (\vec{q}; \vec{w})$   
states queues

Send  
 $(\dots q_p \dots; \dots w_{pq} \dots) \xrightarrow{pq!l} (\dots q'_p; \dots w_{pq} \cdot l \dots)$

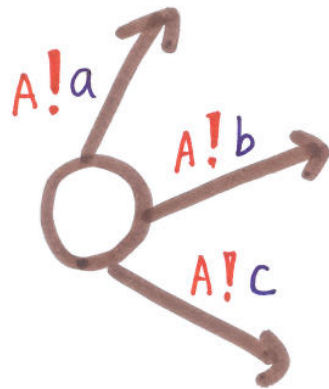
Receive  
 $(\dots q_q \dots; \dots l \cdot w_{pq} \dots) \xrightarrow{pq?l} (\dots q'_q \dots; \dots w_{pq} \dots)$

Deterministic CFSM



# Basic CFSMs

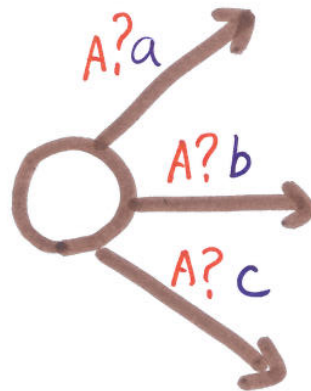
A CFSM is **Basic** if **deterministic**  
**directed**, has **no mixed states**



sending



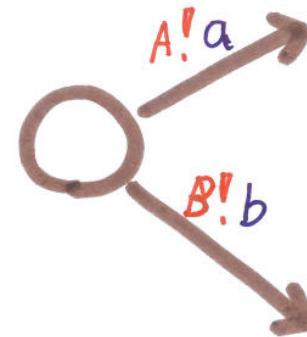
$$T = A! \{a, b, c\}$$



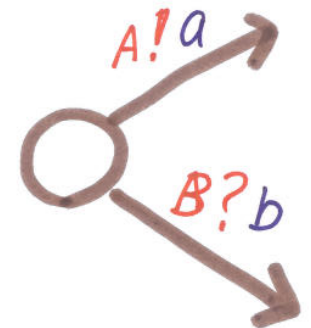
receiving



$$A? \{a, b, c\}$$



non  
directed



mixed



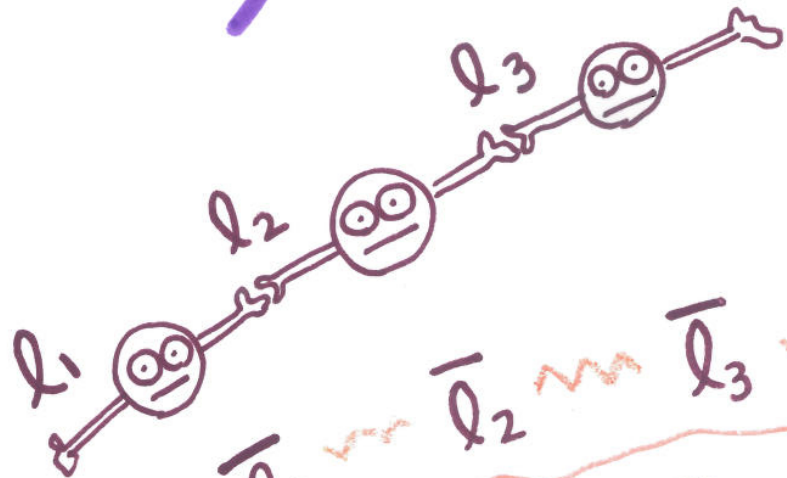
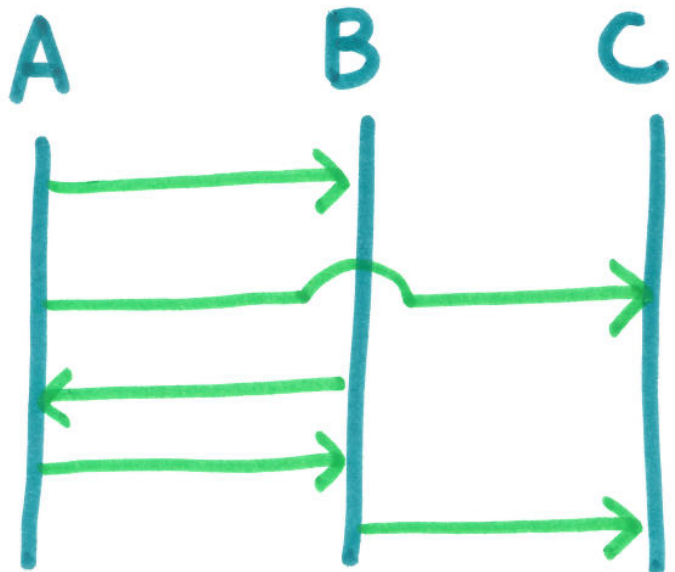
**Theorem 1**  $\text{Tr}(G) \approx \text{Tr}(\{G \upharpoonright P\}_{P \in \text{Participant}})$

Soundness and Completeness between  
a global type and <sup>projected</sup> local types

**Theorem 2**  $\text{Tr}(T) \approx \text{Tr}(A(T))_{T \text{ basic}}$

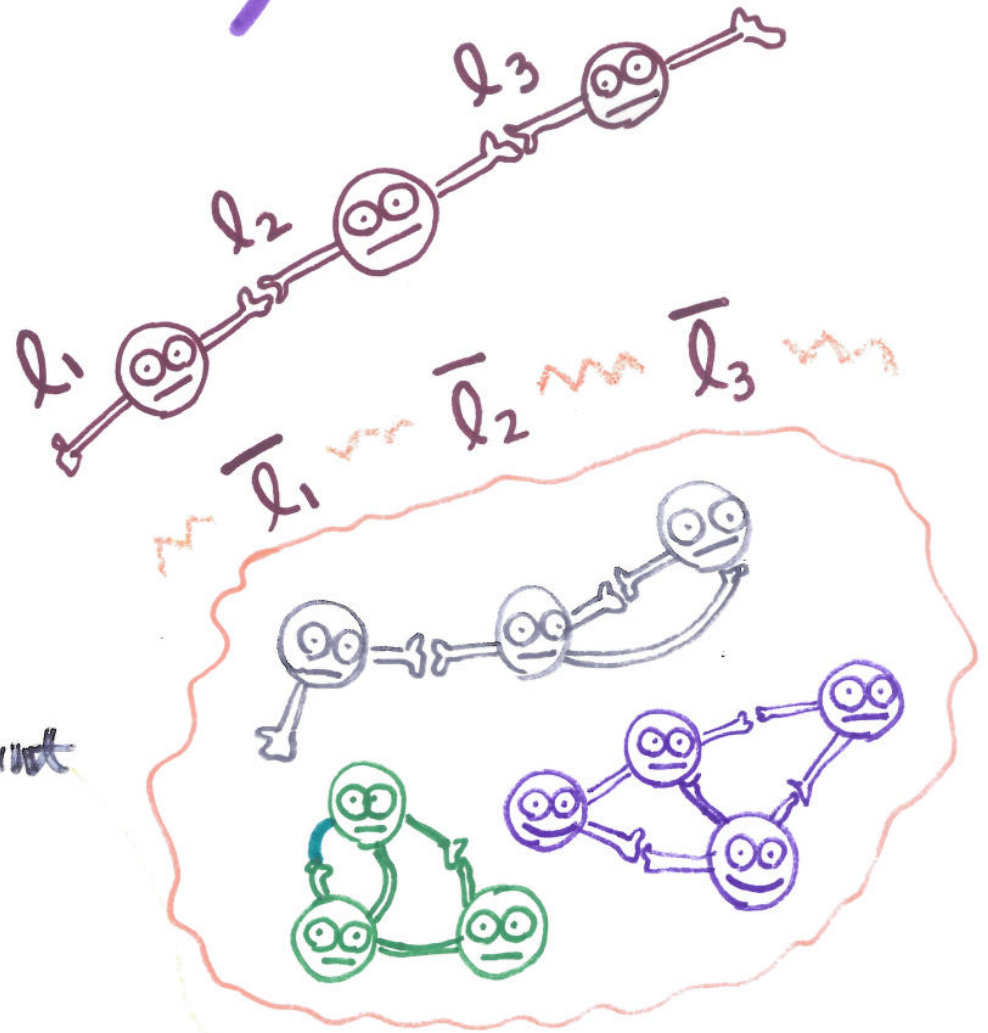
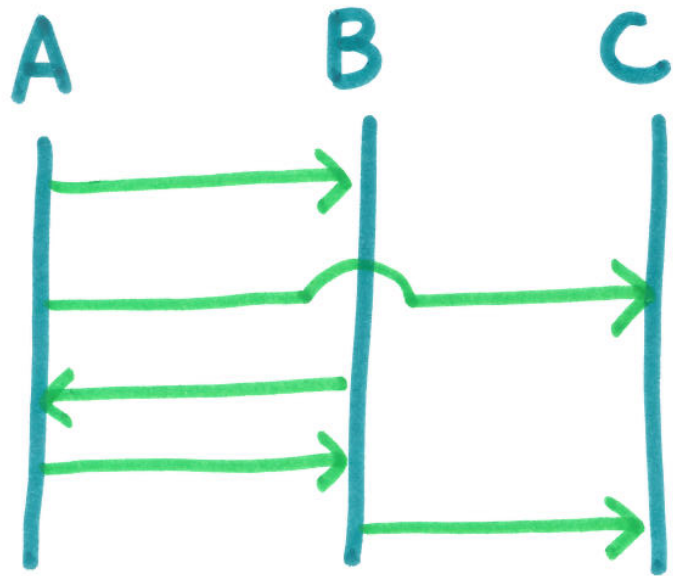
Soundness and Completeness between  
a local type and a basic CFSM of  $T$

# Multiparty Compatibility



*[Faint handwritten notes in purple ink, mostly illegible.]*

# Multiparty Compatibility



Def  $S = (M_p)_{p \in \text{Participant}}$

$\forall s$ .  $s_0 \rightsquigarrow s$   
*1-buffer execution*

if  $M_i$  does action  $l$

then  $(M_{\bar{j}})_{\bar{j} \in P \setminus i}$  do action  $\bar{l}$   
 after some  $\rightsquigarrow$

**Theorem (Safety)** Suppose  $S$  is basic and multiparty compatible.  
Then  $S$  is free from the following errors. Let  $S = (\vec{q}; \vec{w})$

**Deadlock** if  $\vec{w} = \vec{\epsilon}$  and not final, each  $q_p$  is receiving

**Ophan Message** all  $q$  are final, but  $\vec{w} \neq \vec{\epsilon}$


**Unspecified Reception**  $(q_p, p \stackrel{?}{\underline{a}}, q'_p)$  implies  $|w_{pq}| > 1$  and  $w_{pq} \notin \underline{a} \cdot \Sigma^*$

**Proposition Soundness**  $A(\{G \upharpoonright_p\}_{p \in \text{Participant}})$  is  
multiparty compatible. projection

**Proposition** Multiparty Compatibility is decidable.

# Mobility Reading Group

<http://mrg.doc.ic.ac.uk/>



## MobilityReadingGroup

$\pi$ -calculus, Session Types research at Imperial College

[Home](#) [People](#) [Publications](#) [Grants](#) [Talks](#) [Tutorials](#) [Tools](#) [Awards](#) [Kohei Honda](#)

### NEWS

6 Aug 2021

Nobuko Yoshida, with Francisco Ferreira and Adam D. Barwell, conducted an interview with the CONCUR Test-of-Time Award winners, Uwe Nestmann and Benjamin C. Pierce. The full interview can be found [here](#)

24 Mar 2021

Eva passed her viva today, congratulations Dr. Graversen!

### SELECTED PUBLICATIONS

#### 2021

Anson Miu, Francisco Ferreira, Nobuko Yoshida, Fangyi Zhou: [Communication-Safe Web Programming in TypeScript with Routed Multiparty Session Types](#). CC 2021 : 94 - 106.

Silvia Ghilezan, Jovanka Pantovic, Ivan Prokic, Alceste Scalas, Nobuko Yoshida: [Precise Subtyping for Asynchronous Multiparty Sessions](#). POPL 2021 : 16:1 - 16:28.

# Optimising Asynchronous Communication in Rust

Deadlock-Free Message Reordering  
with Multiparty Session Types

Zak Cutner, NY and Martin Vassor

Imperial College  
London

# Introduction

## Rust Language

- Modern systems language focussed on **safety** and **performance**

# Introduction

## Rust Language

- Modern systems language focussed on **safety** and **performance**
- “Most loved language” for past five years on StackOverflow

# Introduction

## Rust Language

- Modern systems language focussed on **safety** and **performance**
- “Most loved language” for past five years on StackOverflow
- Particular emphasis on safe concurrency using **message passing**

# Introduction

## Rust Language

- Modern systems language focussed on **safety** and **performance**
- “Most loved language” for past five years on StackOverflow
- Particular emphasis on safe concurrency using **message passing**
- **Affine** type system is well-suited to session types

# Ring Protocol

## Example

### Global Type

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(\mathit{i32}).\mathit{t} \} \\ \mathit{sub}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(\mathit{i32}).\mathit{t} \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} add(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} add(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ add(i32). t \} \\ sub(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ sub(i32). t \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(i32). t \} \\ \mathit{sub}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(i32). t \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(i32). t \} \\ \mathit{sub}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(i32). t \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

## Example

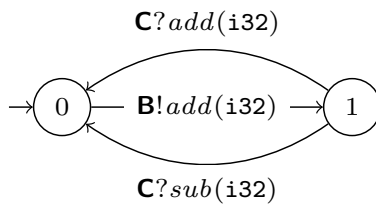
$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(\mathit{i32}).\mathbf{t} \} \\ \mathit{sub}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(\mathit{i32}).\mathbf{t} \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

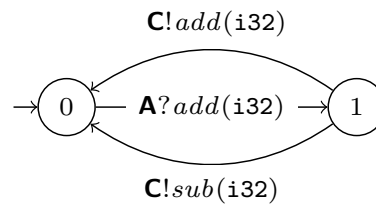
## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(i32). t \} \\ \mathit{sub}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(i32). t \} \end{array} \right\} \end{array} \right\}$$

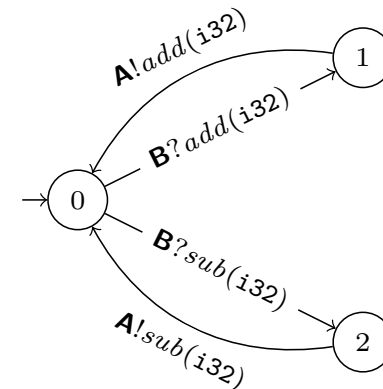
PROJECTION  
↓



PROJECTION  
↓



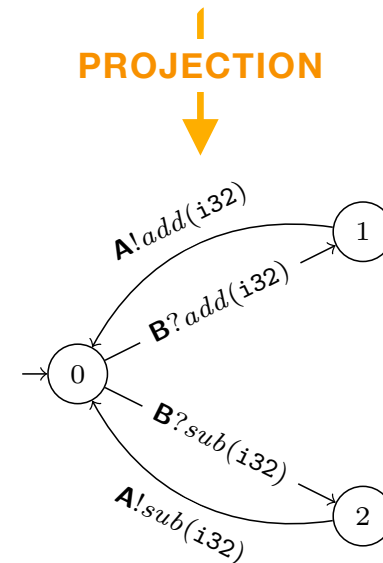
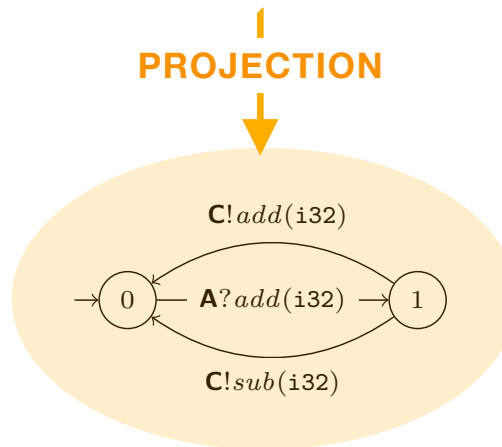
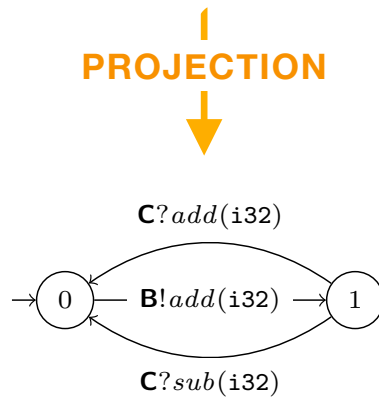
PROJECTION  
↓



# Ring Protocol

## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(i32).t \} \\ \mathit{sub}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(i32).t \} \end{array} \right\} \end{array} \right\}$$



# Challenge

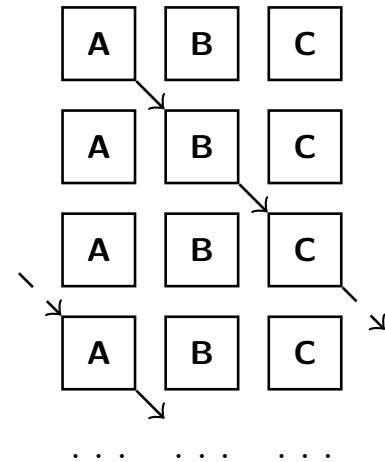
## Asynchronous Orderings

- Global types are inherently **synchronous**

# Challenge

## Asynchronous Orderings

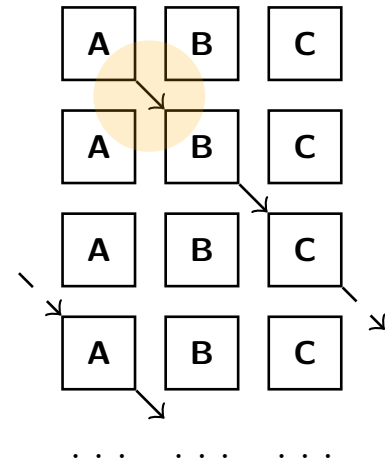
- Global types are inherently *synchronous*
  - Projection provides only one possible ordering



# Challenge

## Asynchronous Orderings

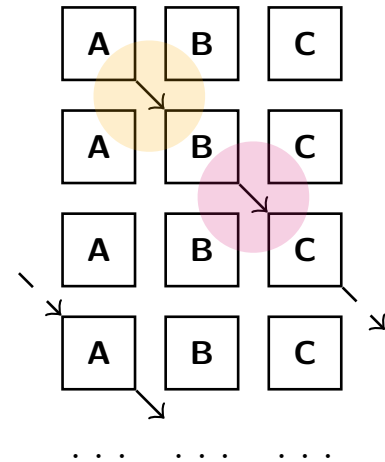
- Global types are inherently *synchronous*
  - Projection provides only one possible ordering



# Challenge

## Asynchronous Orderings

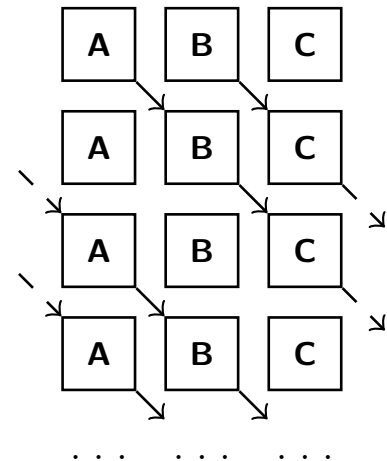
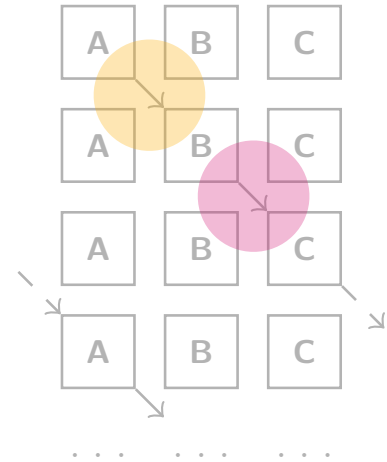
- Global types are inherently *synchronous*
  - Projection provides only one possible ordering



# Challenge

## Asynchronous Orderings

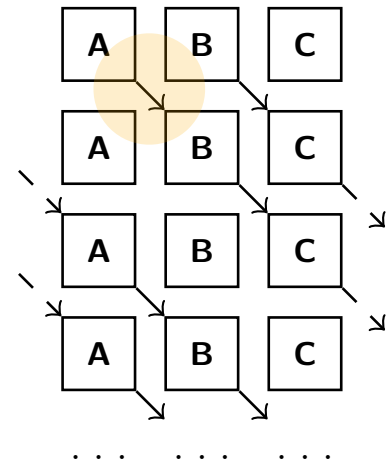
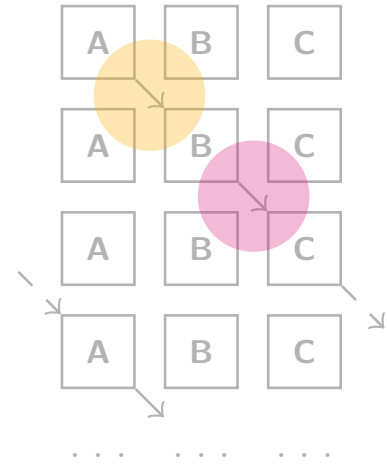
- Global types are inherently **synchronous**
  - Projection provides only one possible ordering
- Interactions can be **reordered** for efficiency while preserving safety



# Challenge

## Asynchronous Orderings

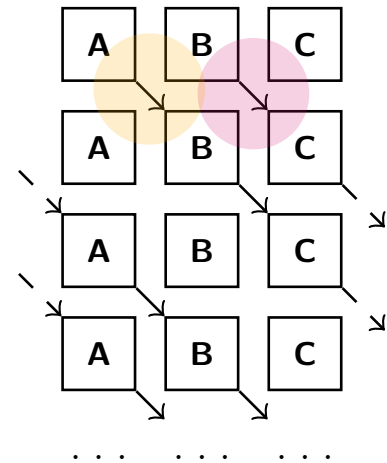
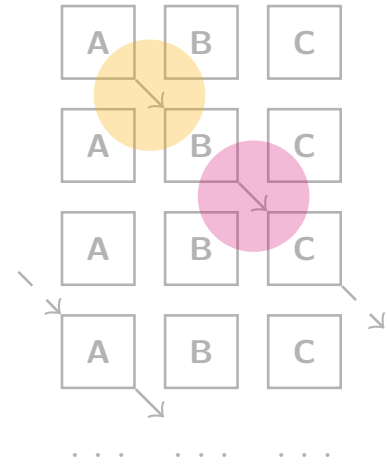
- Global types are inherently **synchronous**
  - Projection provides only one possible ordering
- Interactions can be **reordered** for efficiency while preserving safety



# Challenge

## Asynchronous Orderings

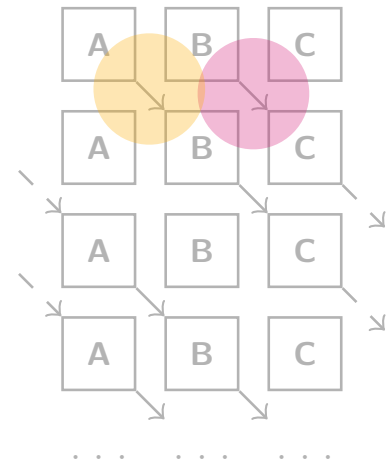
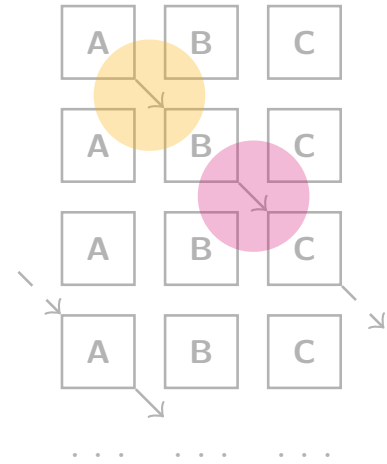
- Global types are inherently **synchronous**
  - Projection provides only one possible ordering
- Interactions can be **reordered** for efficiency while preserving safety



# Challenge

## Asynchronous Orderings

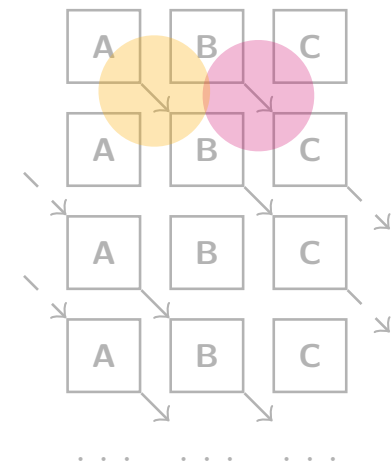
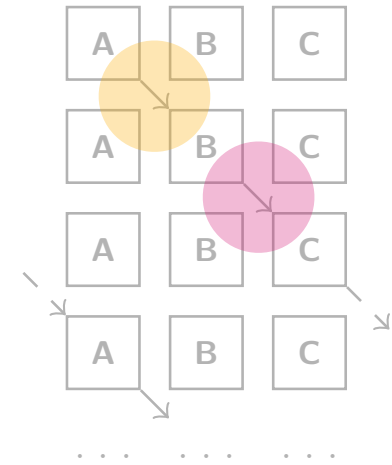
- Global types are inherently **synchronous**
  - Projection provides only one possible ordering
- Interactions can be **reordered** for efficiency while preserving safety
  1. Data **dependencies** must be preserved



# Challenge

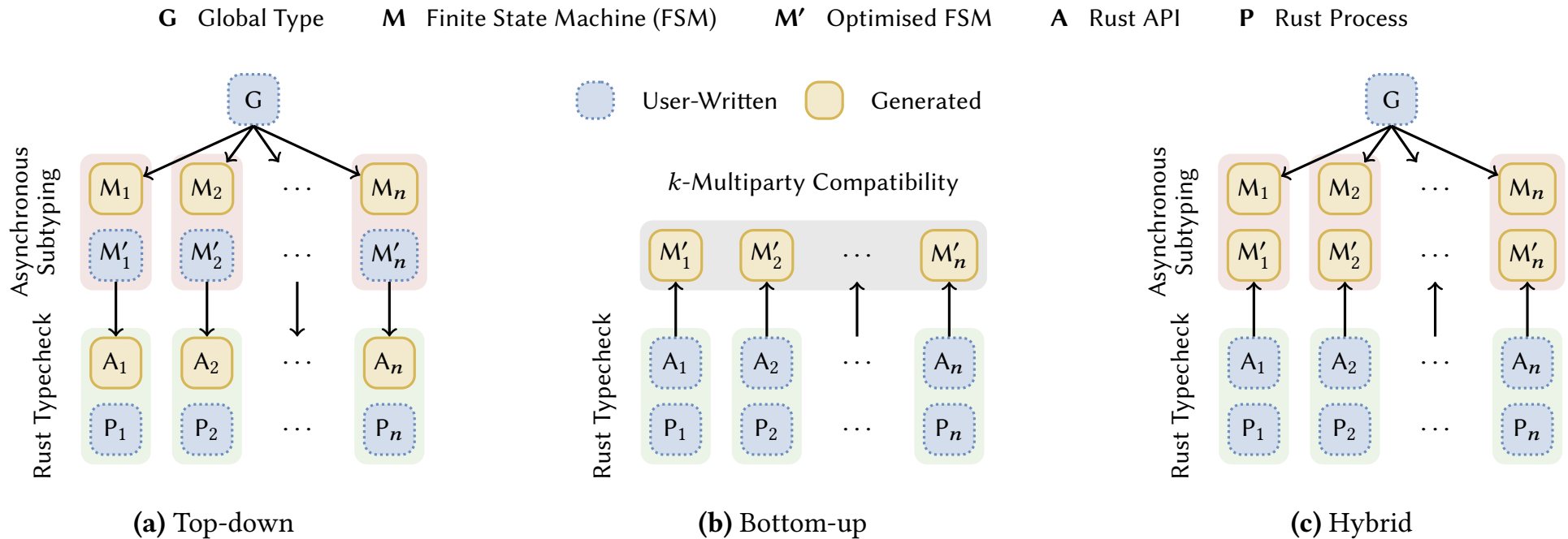
## Asynchronous Orderings

- Global types are inherently **synchronous**
  - Projection provides only one possible ordering
- Interactions can be **reordered** for efficiency while preserving safety
  1. Data **dependencies** must be preserved
  2. **Sound** and **practical** asynchronous reordering rules must be found



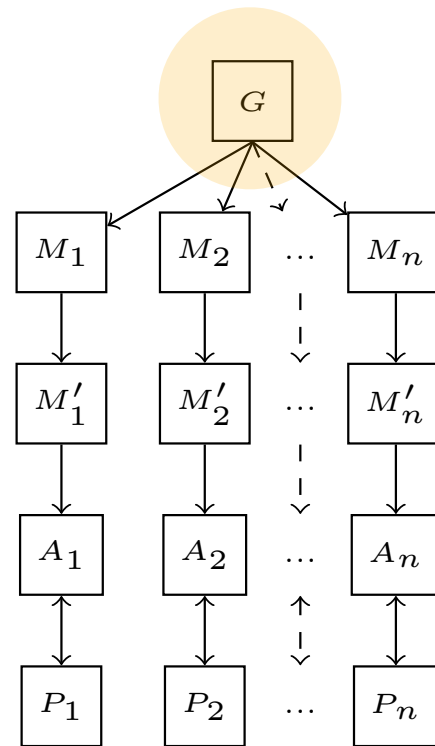
# Rumpsteak Framework

## Three Approaches



# Workflow

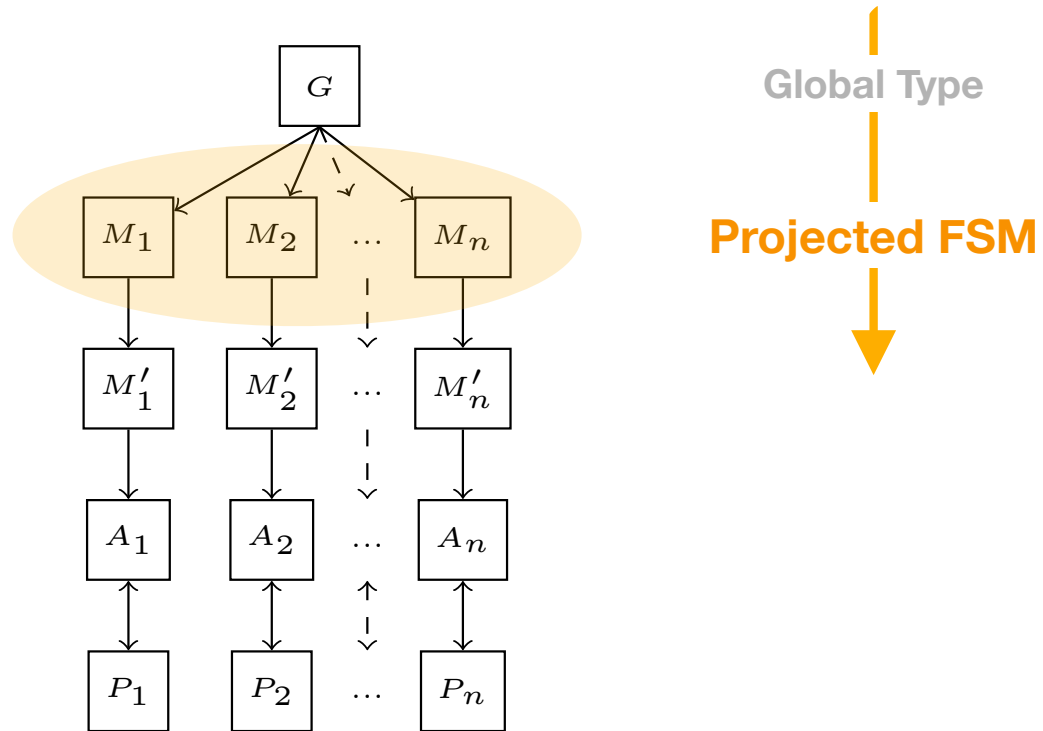
## Top-Down Approach



↓  
Global Type  
↓

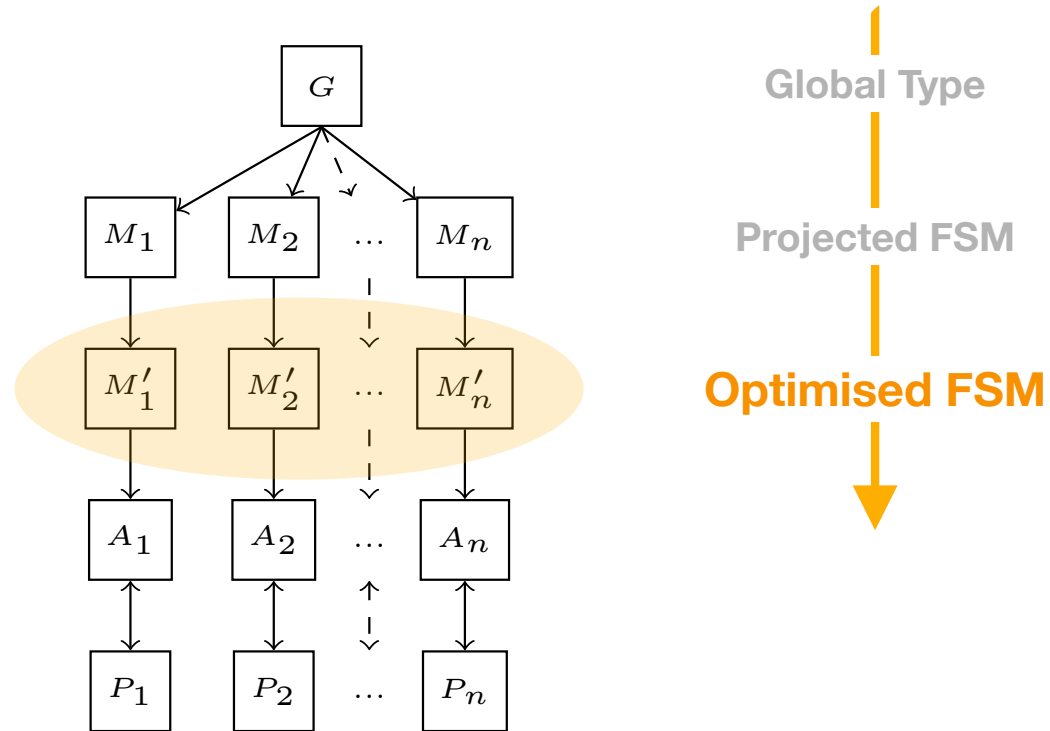
# Workflow

## Top-Down Approach



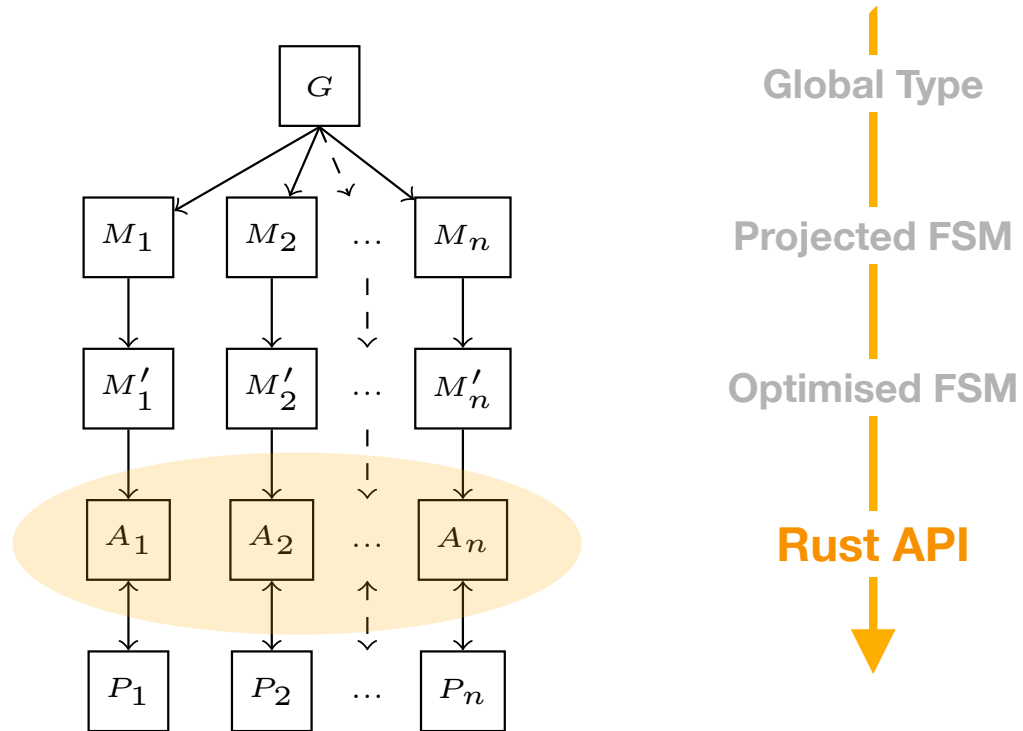
# Workflow

## Top-Down Approach



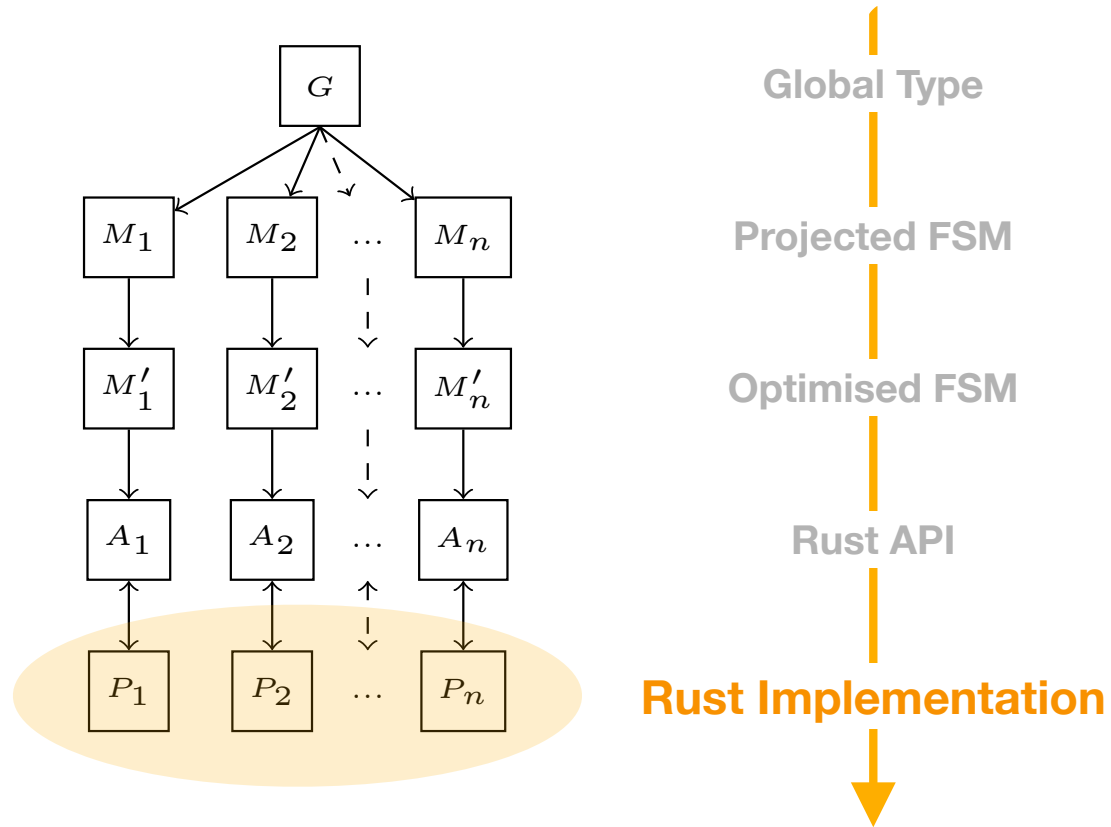
# Workflow

## Top-Down Approach



# Workflow

## Top-Down Approach



# Ring Protocol

## Example

### Global Type

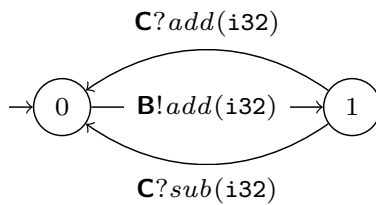
$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(\mathit{i32}).t \} \\ \mathit{sub}(\mathit{i32}).\mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(\mathit{i32}).t \} \end{array} \right\} \end{array} \right\}$$

# Ring Protocol

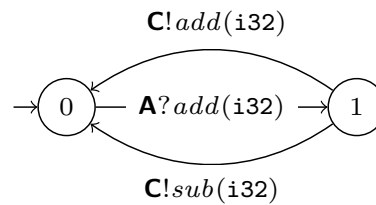
## Example

$$G = \mu t. \mathbf{A} \rightarrow \mathbf{B} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{B} \rightarrow \mathbf{C} : \left\{ \begin{array}{l} \mathit{add}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{add}(i32). t \} \\ \mathit{sub}(i32). \mathbf{C} \rightarrow \mathbf{A} : \{ \mathit{sub}(i32). t \} \end{array} \right\} \end{array} \right\}$$

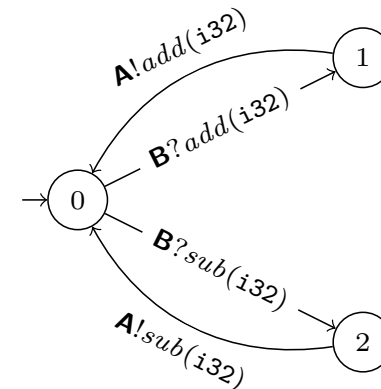
PROJECTION  
↓



PROJECTION  
↓

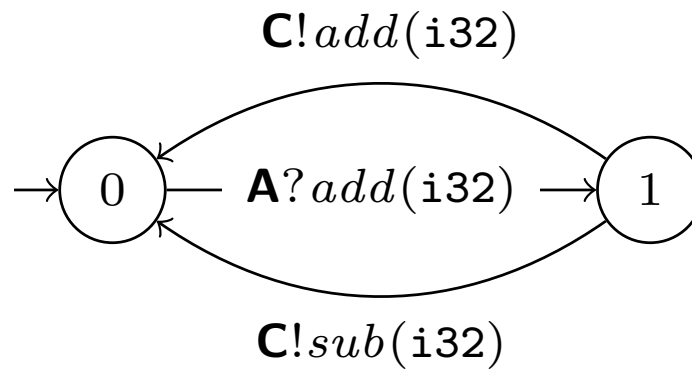


PROJECTION  
↓



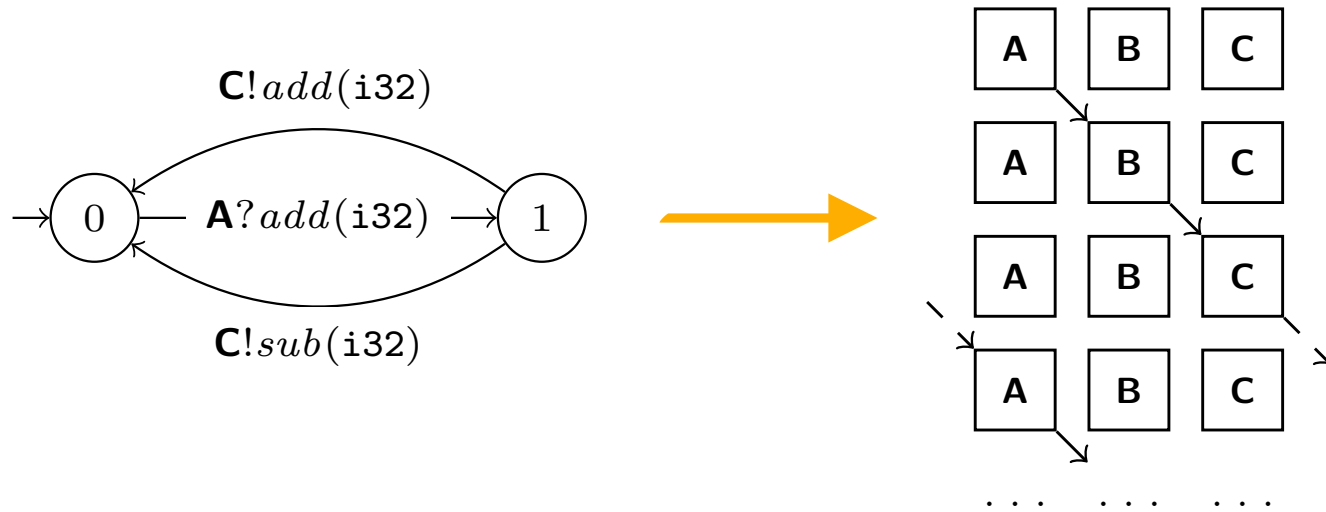
# Ring Protocol

## Example



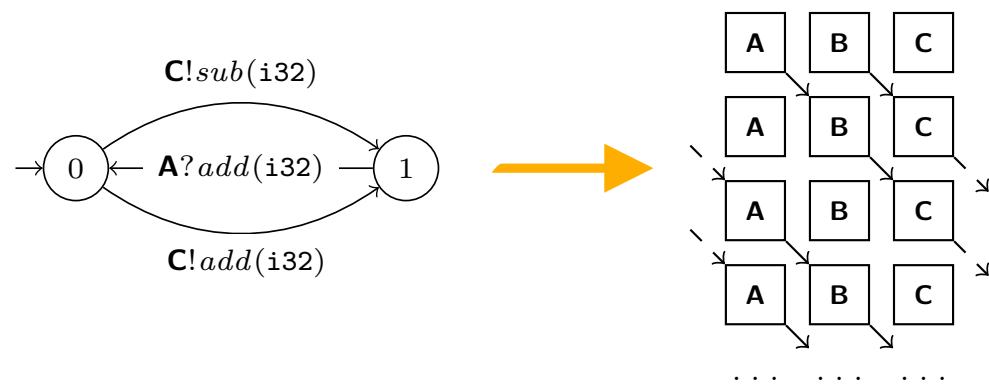
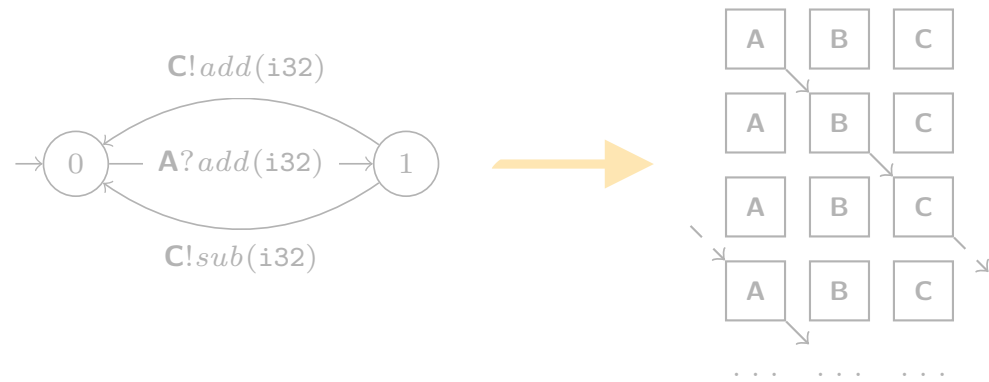
# Ring Protocol

## Example



# Ring Protocol

## Example



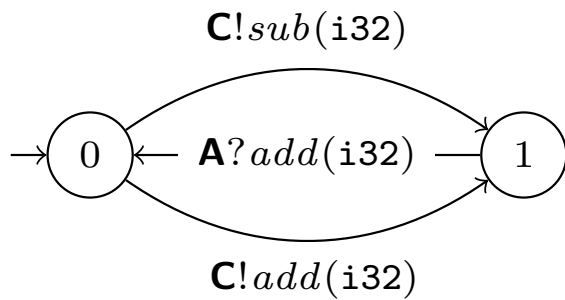
# Scribble

## Protocol Description Language

```
global protocol Ring(role A, role B, role C) {  
  Add(i32) from A to B;  
  choice at B {  
    Add(i32) from B to C;  
    Add(i32) from C to A;  
    do Ring(A, B, C);  
  } or {  
    Sub(i32) from B to C;  
    Sub(i32) from C to A;  
    do Ring(A, B, C);  
  }  
}
```

# Ring Protocol

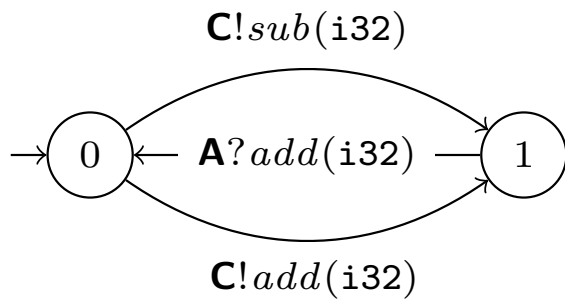
## Rust API



```
#[derive(Role)]  
#[message(Label)]  
struct B(#[route(A)] Receiver, #[route(C)] Sender);
```

# Ring Protocol

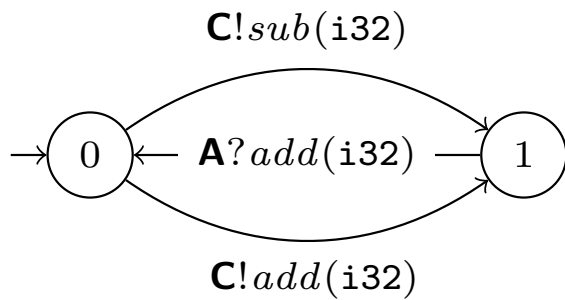
## Rust API



```
#[derive(Role)]  
#[message(Label)]  
struct B(#[route(A)] Receiver, #[route(C)] Sender);
```

# Ring Protocol

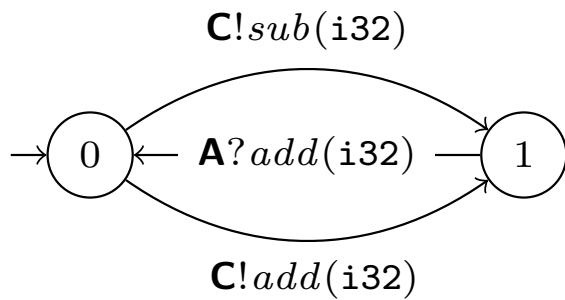
## Rust API



```
#[derive(Role)]  
#[message(Label)]  
struct B(#[route(A)] Receiver, #[route(C)] Sender);
```

# Ring Protocol

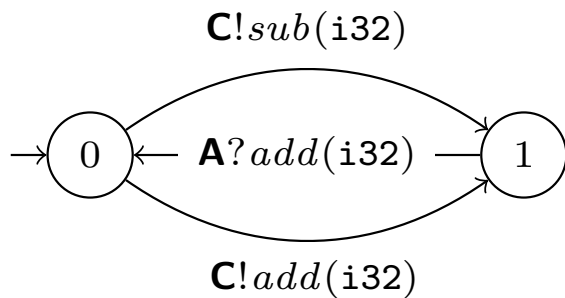
## Rust API



```
#[derive(Role)]  
#[message(Label)]  
struct B(#[route(A)] Receiver, #[route(C)] Sender);  
  
#[derive(Message)]  
enum Label {  
    Add(Add),  
    Sub(Sub),  
}  
  
struct Add(i32);  
struct Sub(i32);
```

# Ring Protocol

## Rust API



```
#[derive(Role)]
#[message(Label)]
struct B(#[route(A)] Receiver, #[route(C)] Sender);

#[derive(Message)]
enum Label {
    Add(Add),
    Sub(Sub),
}

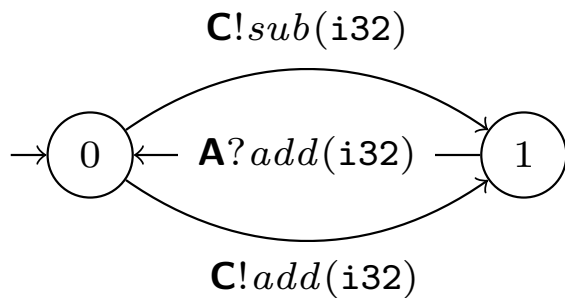
struct Add(i32);
struct Sub(i32);

#[session]
type RingB = Select<C, RingBChoice>;

#[session]
enum RingBChoice {
    Add(Add, Receive<A, Add, RingB>),
    Sub(Sub, Receive<A, Add, RingB>),
}
```

# Ring Protocol

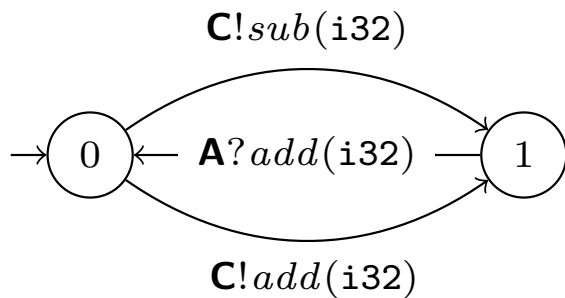
## Implementation



```
async fn ring_b(  
    role: &mut B,  
    mut input: i32,  
) -> Result<Infallible> {  
    try_session(role, |mut s: RingB<'_, _>| async {  
        loop {  
            let x = input * 2;  
  
            s = if x > 0 {  
                let s = s.select(Add(x)).await?;  
                let (Add(y), s) = s.receive().await?;  
                input = y + x;  
                s  
            } else {  
                let s = s.select(Sub(x)).await?;  
                let (Add(y), s) = s.receive().await?;  
                input = y - x;  
                s  
            };  
        }  
    })  
    .await  
}
```

# Ring Protocol

## Implementation

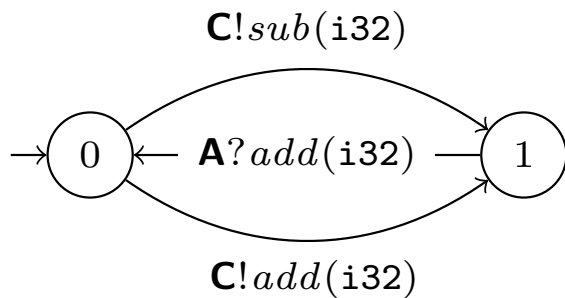


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation

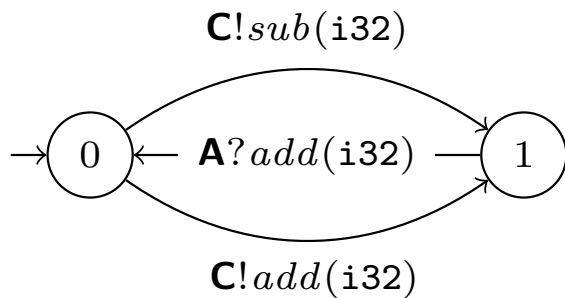


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

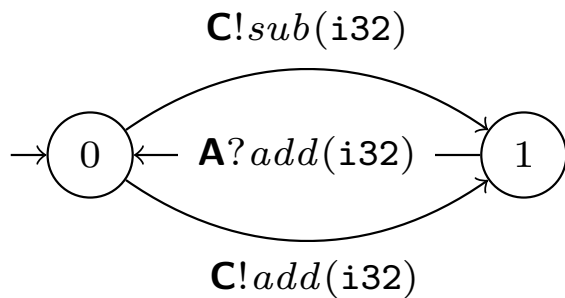
## Implementation



```
async fn ring_b(  
    role: &mut B,  
    mut input: i32,  
) -> Result<Infallible> {  
    try_session(role, |mut s: RingB<'_, _>| async {  
        loop {  
            let x = input * 2;  
  
            s = if x > 0 {  
                let s = s.select(Add(x)).await?;  
                let (Add(y), s) = s.receive().await?;  
                input = y + x;  
                s  
            } else {  
                let s = s.select(Sub(x)).await?;  
                let (Add(y), s) = s.receive().await?;  
                input = y - x;  
                s  
            };  
        }  
    })  
    .await  
}
```

# Ring Protocol

## Implementation

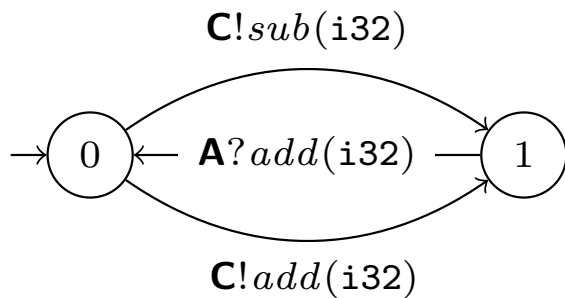


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation

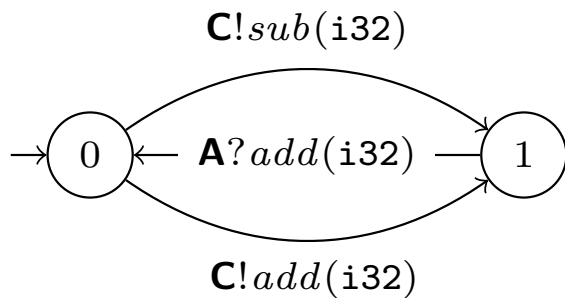


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation

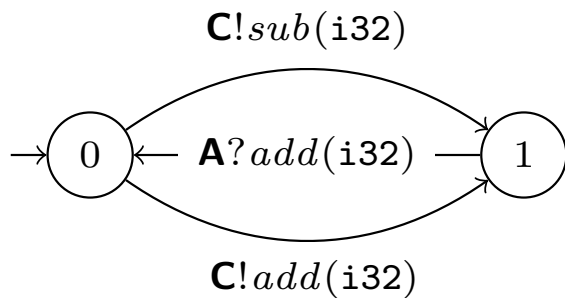


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation

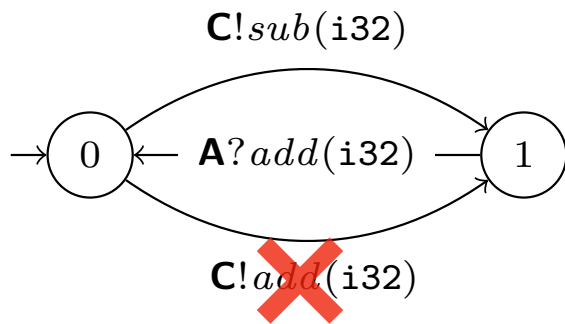


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation

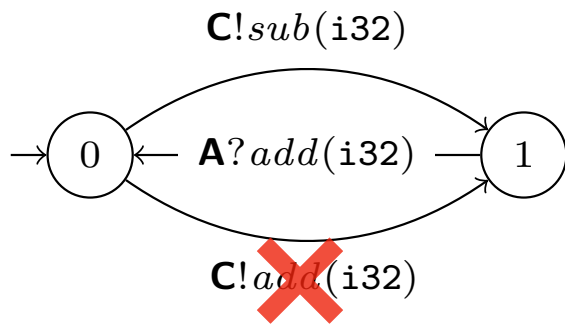


```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
                s
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
                s
            };
        }
    })
    .await
}
```

# Ring Protocol

## Implementation



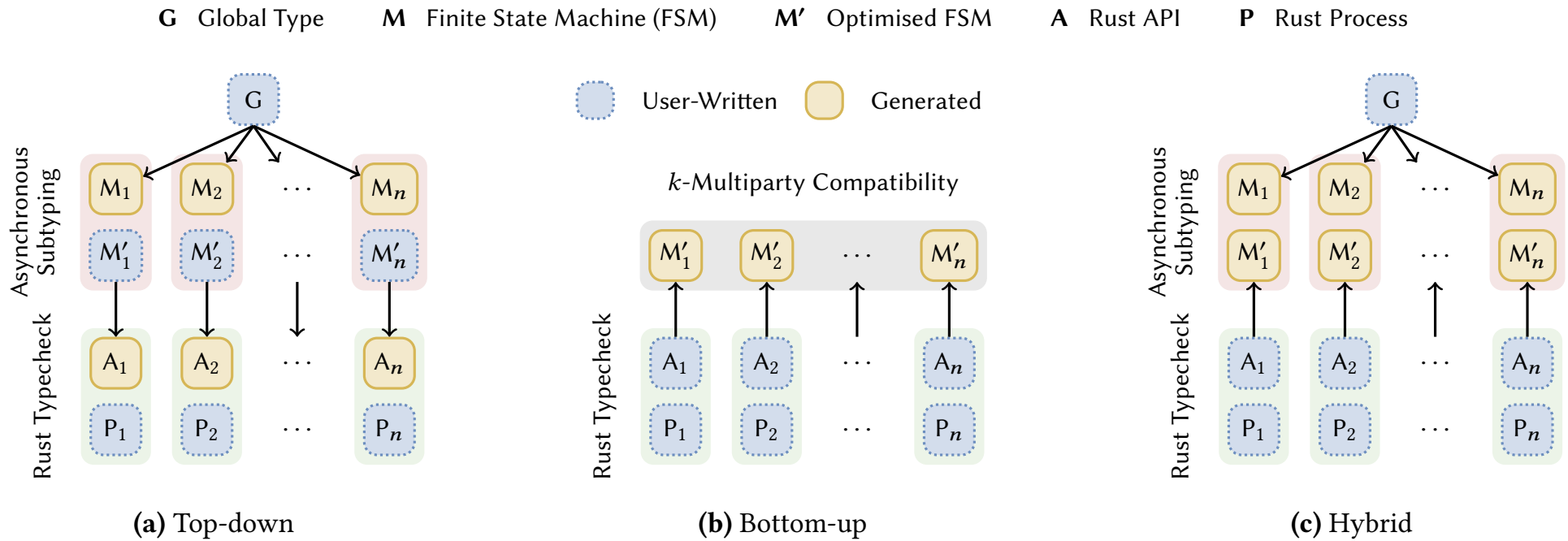
```
async fn ring_b(
    role: &mut B,
    mut input: i32,
) -> Result<Infallible> {
    try_session(role, |mut s: RingB<'_, _>| async {
        loop {
            let x = input * 2;

            s = if x > 0 {
                let s = s.select(Add(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y + x;
            } else {
                let s = s.select(Sub(x)).await?;
                let (Add(y), s) = s.receive().await?;
                input = y - x;
            }
            s
        }
    });
}
.await
}
```

method not found in `rumpsteak::Select<'\_, B, C, RingBChoice<'\_, B>>`

# Rumpsteak Framework

## Three Approaches



# Theories for Communication Optimisation

## Asynchronous Reordering Revisited

How do we check that asynchronous reorderings are **safe**?

# Theories for Communication Optimisation

## Asynchronous Reordering Revisited

How do we check that asynchronous reorderings are **safe**?

1. Asynchronous subtyping relation [Ghilezan et al., 2021]

# Theories for Communication Optimisation

## Asynchronous Reordering Revisited

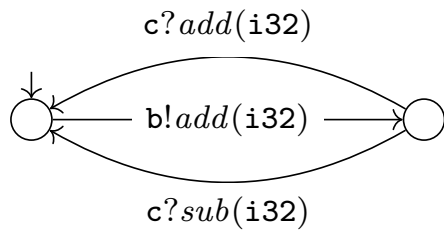
How do we check that asynchronous reorderings are **safe**?

1. Asynchronous subtyping relation [Ghilezan et al., 2021]
2.  $k$ -multiparty compatibility [Lange and Yoshida, 2019]

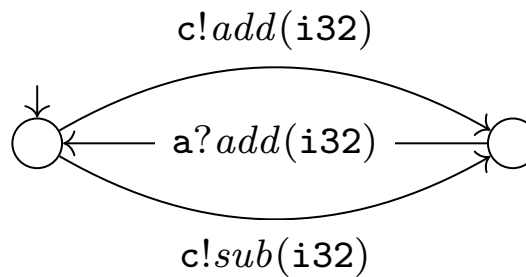
# Safety

## *k*-Multiparty Compatibility

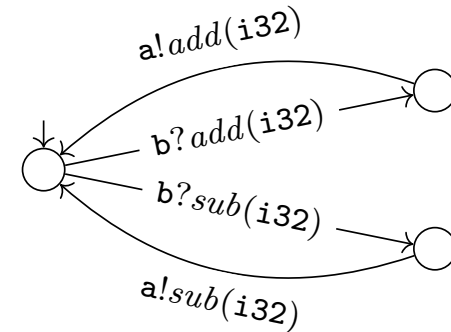
OPTIMISED A



OPTIMISED B



OPTIMISED C

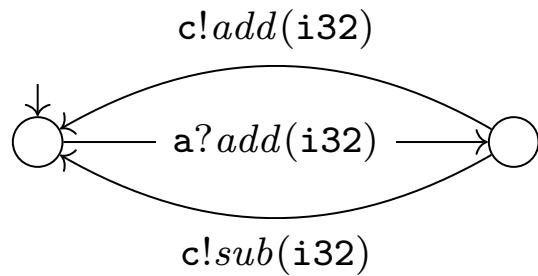


Safe?

# Safety

## Asynchronous Subtyping

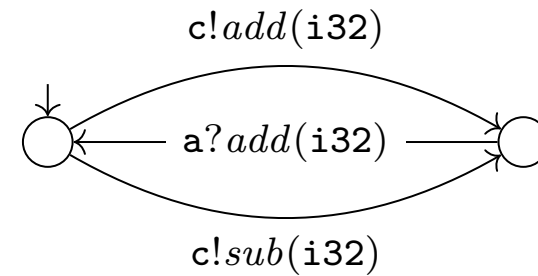
PROJECTED B



Safe?



OPTIMISED B



# Session Decomposition

## Challenge

Our asynchronous subtyping must subsume **synchronous** subtyping

$$\mathbb{T} = \& \mathbf{q} \begin{cases} ?\text{done}(\text{int}) \\ ?\text{fail}(\text{bool}) \end{cases} \leq \mathbf{q} ?\text{done}(\text{int}) \quad \mathbf{r} !\text{cont}(\text{int}) \leq \bigoplus \mathbf{r} \begin{cases} !\text{cont}(\text{int}) \\ !\text{stop}(\text{unit}) \end{cases} = \mathbb{T}'$$

...plus **multiparty asynchronous send/receive reorderings** are **hard to formalise!**

$$\mathbf{r} !\text{cont}(\text{int}).\mathbf{q} ?\text{done}(\text{nat}) \leq \mathbf{q} ?\text{done}(\text{nat}).\mathbf{r} !\text{cont}(\text{int})$$

$$\mathbf{r} ?\text{cont}(\text{int}).\mathbf{q} ?\text{done}(\text{int}) \leq \mathbf{q} ?\text{done}(\text{int}).\mathbf{r} ?\text{cont}(\text{int})$$

$$\mathbf{r} !\text{cont}(\text{int}).\mathbf{q} !\text{done}(\text{int}) \leq \mathbf{q} !\text{done}(\text{int}).\mathbf{r} !\text{cont}(\text{int})$$

# Session Decomposition

## Key Insight and Novelty

Session decomposition — decompose a session into subtrees

- single-input (SI), single-output (SO), single-input-single-output (SISO)

Then, we formalise message reorderings as a refinement for SISO trees only!



$$\llbracket T \rrbracket_{\text{SI}} = \{ \mathbf{q}?\text{done}(\text{int}), \mathbf{q}?\text{fail}(\text{bool}) \}$$

$$\llbracket T' \rrbracket_{\text{SO}} = \{ \mathbf{r}!\text{cont}(\text{int}), \mathbf{r}!\text{stop}(\text{unit}) \}$$

# Asynchronous Subtyping

## SISO Refinement

SISO trees are just paths — i.e. sequences of inputs and outputs!

$$\begin{array}{c}
 \frac{}{\text{end} \lesssim \text{end}} \\
 \\
 \frac{S' \leqslant S \quad W \lesssim W'}{\text{p?}\ell(S).W \lesssim \text{p?}\ell(S').W'} \\
 \\
 \frac{S \leqslant S' \quad W \lesssim W'}{\text{p!}\ell(S).W \lesssim \text{p!}\ell(S').W'} \\
 \\
 \frac{S' \leqslant S \quad W \lesssim \mathcal{A}^{(\mathbf{p})}.W' \quad \text{act}(W) = \text{act}(\mathcal{A}^{(\mathbf{p})}.W')}{\text{p?}\ell(S).W \lesssim \mathcal{A}^{(\mathbf{p})}.\text{p?}\ell(S').W'} \\
 \\
 \frac{S \leqslant S' \quad W \lesssim \mathcal{B}^{(\mathbf{p})}.W' \quad \text{act}(W) = \text{act}(\mathcal{B}^{(\mathbf{p})}.W')}{\text{p!}\ell(S).W \lesssim \mathcal{B}^{(\mathbf{p})}.\text{p!}\ell(S').W'}
 \end{array}$$

$$\mathcal{A}^{(\mathbf{p})} ::= \mathbf{q?}\ell(S) \parallel \mathbf{q?}\ell(S).\mathcal{A}^{(\mathbf{p})} \quad \mathcal{B}^{(\mathbf{p})} ::= \mathbf{r?}\ell(S) \parallel \mathbf{q!}\ell(S) \parallel \mathbf{r?}\ell(S).\mathcal{B}^{(\mathbf{p})} \parallel \mathbf{q!}\ell(S).\mathcal{B}^{(\mathbf{p})} \quad (\mathbf{q} \neq \mathbf{p})$$

$$\frac{\forall U' \in [\mathbf{T}']_{\text{so}} \quad \forall V \in [\mathbf{T}]_{\text{si}} \quad \exists W' \in [\mathbf{U}']_{\text{si}} \quad \exists W \in [\mathbf{V}]_{\text{so}} \quad W' \lesssim W}{\mathbf{T}' \leqslant \mathbf{T}}$$

# Asynchronous Subtyping

## Existing work

- Relation given by [Ghilezan et al., 2021]

# Asynchronous Subtyping

## Existing work

- Relation given by [Ghilezan et al., 2021]
  - Sound 

# Asynchronous Subtyping

## Existing work

- Relation given by [Ghilezan et al., 2021]
  - Sound 
  - Complete 

# Asynchronous Subtyping

## Existing work

- Relation given by [Ghilezan et al., 2021]
  - Sound ✓
  - Complete ✓
  - Decidable [Lange and Yoshida, 2017] ✗

# Asynchronous Subtyping

## Existing work

- Relation given by [Ghilezan et al., 2021]
  - Sound ✓
  - Complete ✓
  - Decidable [Lange and Yoshida, 2017] ✗
- Our aim is a sound and decidable algorithm!

# Algorithm for Asynchronous Subtyping

## Practical, Sound and Terminating

1. **Bound** the number of times we unroll recursions
2. Only unwrap choice **on demand**

# Asynchronous Subtyping

## Session Type Prefix

$\pi, \rho$	$::=$	$\epsilon$	empty prefix
		$p!l(S)$	message send
		$p?l(S)$	message receive
		$\pi_1.\pi_2$	concatenation

# Asynchronous Subtyping

## Reduction Rules

$$\mathcal{A}^{(p)} ::= q?l(S) \mid q?l(S).\mathcal{A}^{(p)} \quad (p \neq q)$$

$$\frac{S' \leq S}{\langle p?l(S).\pi, \mathcal{A}^{(p)}.p?l(S').\pi' \rangle \rightarrow \langle \pi, \mathcal{A}^{(p)}.\pi' \rangle} [\text{RED-}\mathcal{A}]$$

# Asynchronous Subtyping

## Reduction Rules

$$\mathcal{A}^{(p)} ::= q?l(S) \mid q?l(S).\mathcal{A}^{(p)} \quad (p \neq q)$$

$$\frac{S' \leq S}{\langle p?l(S).\pi, \mathcal{A}^{(p)}.p?l(S').\pi' \rangle \rightarrow \langle \pi, \mathcal{A}^{(p)}.\pi' \rangle} [\text{RED-}\mathcal{A}]$$

# Asynchronous Subtyping

## Reduction Rules

$$\mathcal{A}^{(p)} ::= q?l(S) \mid q?l(S).\mathcal{A}^{(p)} \quad (p \neq q)$$

$$\frac{S' \leq S}{\langle p?l(S).\pi, \mathcal{A}^{(p)}.p?l(S').\pi' \rangle \rightarrow \langle \pi, \mathcal{A}^{(p)}.\pi' \rangle} [\text{RED-}\mathcal{A}]$$

# Asynchronous Subtyping

## Reduction Rules

$$\mathcal{A}^{(p)} ::= q?l(S) \mid q?l(S).\mathcal{A}^{(p)} \quad (p \neq q)$$

$$\frac{S' \leq S}{\langle p?l(S).\pi, \mathcal{A}^{(p)}.p?l(S').\pi' \rangle \rightarrow \langle \pi, \mathcal{A}^{(p)}.\pi' \rangle} [\text{RED-}\mathcal{A}]$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?\ell(S).q?m(S'), q?m(S').p?\ell(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?\ell(S).q?m(S'), q?m(S').p?\ell(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$



# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle \quad \checkmark$$

$\uparrow$   
 $\mathcal{A}^{(p)}$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$



$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$



$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle$$



$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle \quad \checkmark$$

$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle \quad \times$$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle \quad \checkmark$$

$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle \quad \times$$

$\uparrow$   
 $\mathcal{A}^{(p)}$

# Asynchronous Subtyping

## Reduction Rules

$$\langle p?l(S).q?m(S'), q?m(S').p?l(S) \rangle \xrightarrow{?} \langle q?m(S'), q?m(S') \rangle \quad \checkmark$$

$$\langle p?l(S).p?m(S'), p?m(S').p?l(S) \rangle \xrightarrow{?} \langle p?m(S'), p?m(S') \rangle \quad \times$$

$$\uparrow \\ \mathcal{A}^{(p)}$$

$$\mathcal{A}^{(p)} ::= q?l(S) \mid q?l(S).\mathcal{A}^{(p)} \quad (p \neq q)$$

# Theorems

## Termination, Soundness & Complexity

**Lemma 3.** *Given finite prefixes  $\pi$  and  $\pi'$ ,  $\langle \pi \parallel \pi' \rangle$  can be reduced only a finite number of times.*

**Theorem 4 (Termination).** *Our subtyping algorithm always eventually terminates.*

**Theorem 5 (Soundness).** *Our subtyping algorithm is sound.*

**Lemma 6.** *Given finite prefixes  $\pi$  and  $\pi'$ , the time complexity of reducing  $\langle \pi \parallel \pi' \rangle$  is  $O(\min(|\pi|, |\pi'|))$ .*

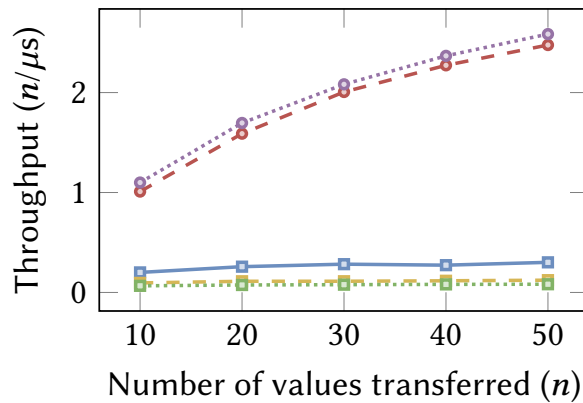
**Theorem 7 (Complexity).** *Consider  $T$  and  $T'$  as (possibly infinite) trees  $\mathcal{T}(T)$  and  $\mathcal{T}(T')$  with asymptotic branching factors  $b$  and  $b'$  respectively. Our algorithm has time complexity  $O(n \min(b, b')^n)$  and space complexity  $O(n \min(b, b'))$  in the worst case to determine if  $T \leq T'$  with bound  $n$ .*

# Evaluation

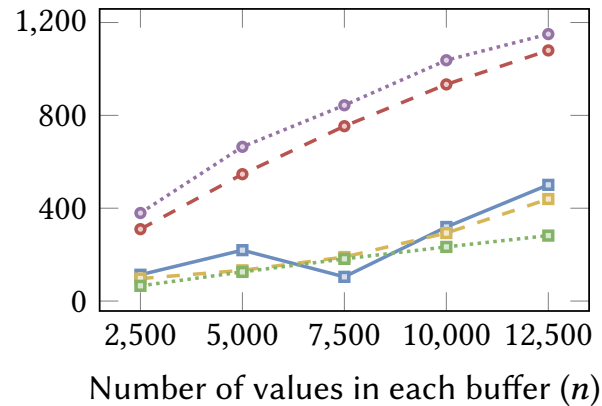
## Rust Framework Benchmarks

—■— SESH    -■- MULTICRUSTY    ...■... FERRITE    —○— RUSTFFT    -○- RUMPSTEAK    ...○... RUMPSTEAK (optimised)

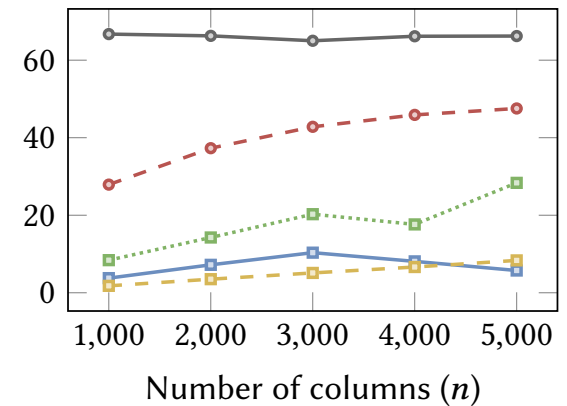
*Stream*



*Double Buffering*



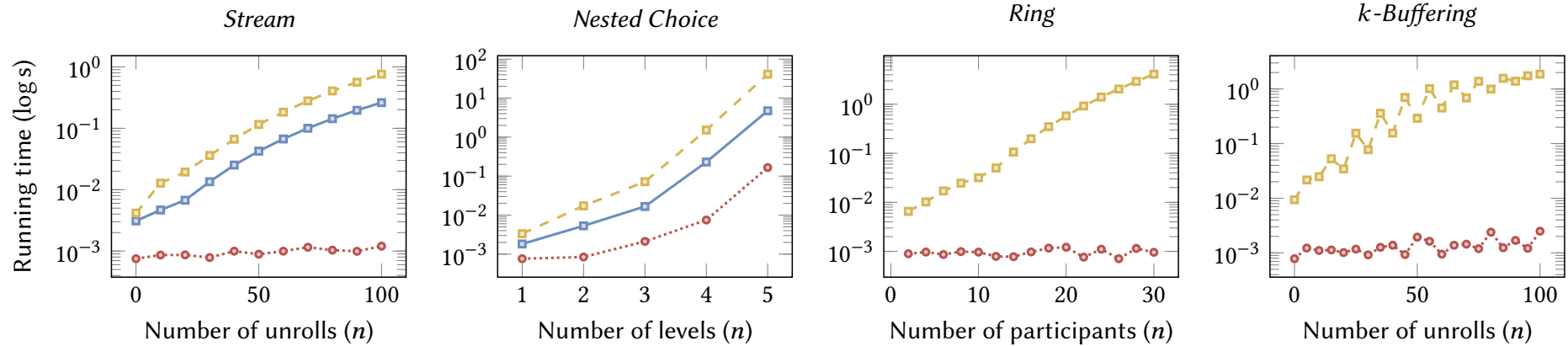
*FFT*



# Evaluation

## Asynchronous Reordering Benchmarks

—■— SOUNDBINARY    -■-  $k$ -MC    ···○··· RUMPSTEAK



# Evaluation

## Expressiveness

Protocol	$n$	AMR	SESH	FERRITE	MULTICRUSTY	RUMPSTEAK	$k$ -MC	SOUNDBINARY
Two Adder	2		✓	✓	✓	✓	✓	✓
Three Adder	3		✗	✗	✓	✓	✓	✗
Stream	2		✓	✓	✓	✓	✓	✓
Optimised Stream	2	✓	✗	✗	✗	✓	✓	✓
Ring	3		✗	✗	✓	✓	✓	✗
Optimised Ring	3	✓	✗	✗	✗	✓	✓	✗
Ring With Choice	3		✗	✗	✓	✓	✓	✗
Optimised Ring With Choice	3	✓	✗	✗	✗	✓	✓	✗
Double Buffering	3		✗	✗	✓	✓	✓	✗
Optimised Double Buffering	3	✓	✗	✗	✗	✓	✓	✗
Alternating Bit	2		✗	✗	✗	✓	✓	✓
Elevator	3	✓	✗	✗	✗	✓	✓	✗
FFT	8		✗	✗	✓	✓	✓	✗
Optimised FFT	8	✓	✗	✗	✗	✓	✓	✗
Authentication	3		✗	✗	✓	✓	✓	✗
Client-Server Log	3		✗	✗	✓	✓	✓	✗
Hospital	2	✓	✗	✗	✗	✗	✗	✓




$n$  Number of participants    AMR Asynchronous message reordering

✓ Expressible    ✗ Expressible using endpoint types (but without deadlock-freedom guarantee)    ✗ Not expressible



# Conclusion

## Multiparty Session Types and Communicating Automata

- Multiparty session types and communicating automata
  - Invited paper in the FCT '21 proceedings
  -  Scribble <https://github.com/scribble>
  -  <https://github.com/nuscr>
- Applications of multiparty session types using communicating automata
  -  <https://github.com/zakcutner/rumpsteak>