Coursework 1

1. What are the free names and free variables of the following processes?

   (a) $(\nu\, c)(\overline{b}\langle c\rangle \mid b(x).P)$

   (b) $(\nu\, b)(b(x).P \mid \overline{b}\langle c\rangle \mid x(y).Q)$

   (c) $(\nu\, a)(!a(x).\overline{c}\langle y\rangle \mid a(x).P \mid b(y).\mathbf{0})$

2. Substitutions:

   (a) Apply $((\nu\, a, b)(!\overline{b}\langle x\rangle \mid \overline{b}\langle a\rangle \mid !(\nu\, c)b(y).P))\{{}^{b}/_{x}, {}^{a}/_{y}\}$

   (b) Is the following application of a substitution correct?

   $$((\nu\, a)(\overline{x}\langle x\rangle \mid a(x).(\nu\, a)\overline{y}\langle a\rangle))\{{}^{a}/_{x}, {}^{b}/_{y}\} \;=\; (\nu\, a)(\overline{a}\langle a\rangle \mid a(x).(\nu\, a)\overline{b}\langle a\rangle)$$

3. Are these processes structurally congruent? Write the justification: if **not**, write the reason; if **yes**, derive $P \equiv Q$ using the rules in the lecture notes.

   (a) $(\nu\, a)Q \mid P \mid !(P \mid (\nu\, a)Q)$ and $(\nu\, a)(P \mid Q) \mid (!((\nu\, a)(P \mid Q)))$

   (b) $(\nu\, a)\overline{c}\langle a\rangle \mid (!(\overline{a}\langle x\rangle \mid b(y).\mathbf{0}))$ and $\overline{a}\langle c\rangle \mid (\nu\, c)(!(\overline{c}\langle x\rangle \mid b(y).\mathbf{0}))$

   (c) $(\nu\, a)(\overline{c}\langle a\rangle \mid a(x).\overline{a}\langle a\rangle)$ and $(\nu\, a)(\overline{c}\langle a\rangle \mid b(x).\overline{b}\langle b\rangle)$

4. Write the reduction step by step using the rules in the lecture notes (you can omit some steps, such as the structural congruence).

   (a) $(\nu\, b)(a(x).\overline{x}\langle b\rangle) \mid !(\overline{a}\langle b\rangle \mid b(x).\mathbf{0})$

   (b) $!a(x).(\overline{b}\langle x\rangle \mid \overline{c}\langle x\rangle) \mid !b(y).(\overline{a}\langle y\rangle \mid c(y).\mathbf{0}) \mid \overline{a}\langle e\rangle$

   (c) $\overline{a}\langle a, b, c\rangle \mid !a(x, y, z).\overline{y}\langle x, z, y\rangle \mid !b(x, z, y).\overline{x}\langle x, y, z\rangle$

   (d) $\overline{a}\langle c\rangle \mid \overline{a}\langle d\rangle \mid \overline{a}\langle e\rangle \mid \overline{a}\langle f\rangle \mid \mathbf{NN}(a)$

   where **New Name Generator** $\mathbf{NN}(a) \stackrel{\text{df}}{=} !a(x).(\nu\, b)\overline{x}\langle b\rangle$.