

## Coursework 2

1. (**Structure Congruence**) Are these processes structurally congruent? If so, prove them. In every step of the proof, mention explicitly the name of the rule used. If not, please explain the reason.

$$a(x).!(\nu b)\bar{x}\langle b \rangle \equiv a(y).(\nu b)(\bar{y}\langle b \rangle \mid !(\nu b)\bar{y}\langle b \rangle)$$

2. (**Barb**) Find all the weak barbs of the processes below:

- (a)  $a(x).\bar{x}\langle b \rangle \mid b(y).\bar{c}\langle y \rangle \mid \bar{a}\langle b \rangle \mid b(x).\mathbf{0} \mid (\nu e)(!\bar{a}\langle e \rangle \mid e(x).e(x).\bar{f}\langle x \rangle)$   
 (b)  $(\nu b)(!a(x).\bar{x}\langle b \rangle \mid b(y).\bar{c}\langle y \rangle) \mid \bar{a}\langle b \rangle \mid b(x).\mathbf{0} \mid !((\nu e)(\bar{a}\langle e \rangle \mid e(x).e(x).\bar{f}\langle x \rangle))$

3. (**Name Generator**) Consider the monadic asynchronous  $\pi$ -calculus and the following processes:

$$\begin{aligned} \mathbf{NN}_2\langle a \rangle &= !a(x).(\nu b)\bar{x}\langle b \rangle \\ \mathbf{NN}_3\langle a \rangle &= !a(x).(!(\nu b)\bar{x}\langle b \rangle \mid !(\nu b)\bar{x}\langle b \rangle \mid !(\nu b)\bar{x}\langle b \rangle \mid !(\nu b)\bar{x}\langle b \rangle \mid !(\nu b)\bar{x}\langle b \rangle) \\ \mathbf{NN}_4\langle a \rangle &= (\nu b_1, b_2, b_3, b_4, b_5)!a(x).(\bar{x}\langle b_1 \rangle \mid \bar{x}\langle b_2 \rangle \mid \bar{x}\langle b_3 \rangle \mid \bar{x}\langle b_4 \rangle \mid \bar{x}\langle b_5 \rangle) \\ \mathbf{NN}_5\langle a \rangle &= (\nu b_1)!(\nu b_2)a(x).(\bar{x}\langle b_1 \rangle \mid \bar{x}\langle b_2 \rangle) \mid (\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle \\ \mathbf{NN}_6\langle a \rangle &= !a(x).(\nu c_1, c_2, c_3, c_4, b_1, b_2, b_3, b_4, b_5) \\ &\quad (\mathbf{FW}\langle c_1, x \rangle \mid \mathbf{FW}\langle c_2, x \rangle \mid \mathbf{D}\langle c_3, x, c_4 \rangle \mid \mathbf{K}\langle c_4 \rangle \mid \bar{c}_1\langle b_1 \rangle \mid \bar{c}_2\langle b_2 \rangle \mid \bar{c}_3\langle b_3 \rangle \mid \bar{x}\langle b_4 \rangle \mid \bar{x}\langle b_5 \rangle) \\ \mathbf{NN}_7\langle a \rangle &= !a(x).(\nu c_1, c_2, c_3, c_4, b_1, b_2, b_3, b_4, b_5) \\ &\quad (!\mathbf{FW}\langle c_1, x \rangle \mid !\mathbf{FW}\langle c_2, x \rangle \mid \mathbf{D}\langle c_3, x, c_4 \rangle \mid \mathbf{K}\langle c_4 \rangle \mid \bar{c}_1\langle b_1 \rangle \mid \bar{c}_2\langle b_2 \rangle \mid \bar{c}_3\langle b_3 \rangle \mid \bar{c}_1\langle b_4 \rangle \mid \bar{c}_2\langle b_5 \rangle) \\ \mathbf{NN}_8\langle a \rangle &= !a(x).((\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle \mid (\nu b)\bar{x}\langle b \rangle) \end{aligned}$$

- (a) Which processes are equivalent ( $\cong$ ) to  $\mathbf{NN}_8\langle a \rangle$ ? You do not have to prove the equivalence.
- (b) For all  $\mathbf{NN}_i\langle a \rangle$  such that  $\mathbf{NN}_8\langle a \rangle \not\cong \mathbf{NN}_i\langle a \rangle$ , give a context able to tell the two processes apart: find  $C$  such that  $(C[\mathbf{NN}_i\langle a \rangle] \Downarrow_c$  and  $C[\mathbf{NN}_8\langle a \rangle] \not\Downarrow_c$ ) or  $(C[\mathbf{NN}_i\langle a \rangle] \not\Downarrow_c$  and  $C[\mathbf{NN}_8\langle a \rangle] \Downarrow_c$ ).
4. (**Equators**) Consider the monadic asynchronous  $\pi$ -calculus and

- (a) Using equational reasoning and the equivalence laws given on the slides and the following (EQ3),

$$\mathbf{EQ}\langle a, b \rangle \mid \mathbf{EQ}\langle b, c \rangle \cong \mathbf{EQ}\langle a, b \rangle \mid \mathbf{EQ}\langle a, c \rangle \quad (\text{EQ3})$$

show that  $P \cong Q$ , where:

$$\begin{aligned} P &\stackrel{\text{df}}{=} (\nu d)(\bar{a}\langle c \rangle \mid \mathbf{EQ}\langle c, b \rangle \mid \mathbf{EQ}\langle b, a \rangle \mid d(x).\mathbf{0}) \\ Q &\stackrel{\text{df}}{=} !(\mathbf{FW}\langle a, b \rangle \mid \mathbf{FW}\langle b, a \rangle) \mid \bar{b}\langle a \rangle \mid !\mathbf{FW}\langle a, c \rangle \mid !(\nu e)(\bar{e}\langle c \rangle \mid e(x).\mathbf{FW}\langle x, a \rangle) \end{aligned}$$

In every step of the proof, mention explicitly the name of the main law used.

- (b) Show that  $Q \not\cong R$  where  $R$  is defined in point 4a and

$$R \stackrel{\text{df}}{=} !\mathbf{FW}\langle b, c \rangle \mid \mathbf{EQ}\langle a, b \rangle \mid \bar{a}\langle c \rangle$$

by giving a context able to tell the two processes apart: find  $C$  such that  $C[Q] \Downarrow_f$  and  $C[R] \not\Downarrow_f$ .