Exercise 1

1. Which of these processes are syntactically well-formed?

   (a) $\overline{a}\langle b\rangle \mid !a(x).\overline{x}\langle x\rangle$

   (b) $(\nu\, a, b, a)(!!\mathbf{0} \mid x(x).P)$

   (c) $(\nu\, a)(x(a).P)$

   (d) $(\overline{c}\langle a\rangle \mid \overline{a}\langle c\rangle).a(x).P \mid c(x).(\nu\, b)\overline{x}\langle b\rangle$

2. What are the free names and free variables of the following processes?

   (a) $(\nu\, b)(a(x).\overline{x}\langle a\rangle \mid \overline{x}\langle b\rangle)$

   (b) $(\nu\, c)(\overline{b}\langle x\rangle \mid \overline{b}\langle a\rangle \mid (\nu\, b)y(x).\overline{c}\langle y\rangle)$

   (c) $(\nu\, c)(\overline{x}\langle x\rangle \mid \overline{x}\langle y\rangle \mid !x(y).\overline{y}\langle c\rangle)$

   (d) $(\nu\, a)(\overline{b}\langle a\rangle \mid b(x).P)$

3. Substitutions:

   (a) Apply $((\nu\, a)(!\overline{b}\langle x\rangle \mid \overline{b}\langle a\rangle \mid !(\nu\, c)\overline{b}\langle c\rangle))\{{}^{a}/_{x}\}$

   (b) Apply $((\nu\, a)(!\overline{b}\langle x\rangle \mid \overline{b}\langle a\rangle \mid !(\nu\, c)b(x).P))\{{}^{b}/_{x}, {}^{a}/_{y}\}$

   (c) What's a minimal closing substitution for $\overline{a}\langle y\rangle \mid \overline{a}\langle b\rangle \mid y(x).\overline{x}\langle x\rangle$.

   (d) Is the following application of a substitution correct?

   $$((\nu\, a)(\overline{x}\langle x\rangle \mid a(x).(\nu\, a)\overline{y}\langle a\rangle))\{{}^{a}/_{x}, {}^{b}/_{y}\} \;=\; (\nu\, b)(\overline{a}\langle a\rangle \mid b(x).(\nu\, a)\overline{b}\langle a\rangle)$$

4. Are these processes structurally congruent?

   (a) $\overline{a}\langle b\rangle \mid (\nu\, a, b)\overline{b}\langle a\rangle$ and $(\nu\, a, b)(\overline{a}\langle b\rangle \mid \overline{a}\langle b\rangle)$

   (b) $!P \mid !!(Q \mid P)$ and $!P \mid !!(Q \mid P) \mid !(Q \mid P) \mid Q \mid P \mid P$

   (c) $(\nu\, a)b(y).(\overline{a}\langle y\rangle \mid (\nu\, b)\overline{b}\langle x\rangle)$ and $(\nu\, c)b(z).((\nu\, c)(\overline{c}\langle x\rangle) \mid \overline{c}\langle z\rangle)$

5. Reduce these processes

   (a) $(\nu\, b)(a(x).\overline{x}\langle b\rangle) \mid (\nu\, b)(\overline{a}\langle b\rangle \mid b(x))$

   (b) $!(\nu\, a)(\overline{a} \mid !(!b.\overline{c} \mid !a.\overline{b}))$

   (c) $\overline{a}\langle b, a\rangle \mid !a(x, y).\overline{x}\langle x, y\rangle \mid !b(x, y).\overline{x}\langle y, x\rangle$
   where we assume the polyadic asynchronous pi-calculus
   $\overline{a}\langle c_1, .., c_n\rangle \mid a(x_1, .., x_n).P \longrightarrow P\{{}^{c_1, .., c_n}/_{x_1, .., x_n}\}$