Exercise 3

1. Data structures

   (a) Consider the following encoding of the boolean values into the synchronous polyadic pi-calculus:

   $$T\langle l\rangle = l(t,f).\bar{t} \text{ and } F\langle l\rangle = l(t,f).\overline{f}$$

   which represents the true or false value. For the process $R = (\nu\, t,f)\bar{l}\langle t,f\rangle.(t.P \mid f.Q)$, where $t, f \notin fn(P) \cup fn(Q)$, then write the reduction for

   i. $R \mid T$
   ii. $R \mid F$

   (b) Assume we have the following constants,
   $\{Monday, Tuesday, Wednesday, Thursday, Friday, Satuarday, Sunday\}$
   Following the idea of Q(a), write the encoding of each data, such as $Monday(l) = l(mo, tu, we, th, fr, sa, su).\overline{mo}$.

   (c) Consider the following AND operation for the encoding of $\{true, false\}$ in Q(a),

   $$\mathbf{AND}(l,l',l'') = (\nu\, t,f)(\bar{l}\langle t,f\rangle.(t.(\nu\, t',f')(\overline{l'}\langle t',f'\rangle.(t'.T\langle l''\rangle \mid f'.F\langle l''\rangle)) \mid f.F\langle l''\rangle))$$

   i. Write the reduction $\mathbf{AND}(l,l',l'') \mid T\langle l\rangle \mid F\langle l'\rangle$ and $\mathbf{AND}(l,l',l'') \mid T\langle l\rangle \mid T\langle l'\rangle$.
   ii. Similarly, write the OR operation for the encoding of $\{true, false\}$.
   iii. Write the reduction $\mathbf{OR}(l,l',l'') \mid F\langle l\rangle \mid T\langle l'\rangle$ and $\mathbf{OR}(l,l',l'') \mid F\langle l\rangle \mid F\langle l'\rangle$

2. [From exam paper 2009-2010]

   (a) Explain the difference between the monadic asynchronous π-calculus and the polyadic synchronous π-calculus.

   (b) Encode the polyadic synchronous π-calculus into the monadic asynchronous π-calculus.

   (c) Demonstrate that your encoding is correct by showing that there is no mix-up in the following processes (Hint: you can use equational law (tau) and the reduction congruence).

   $$[\![\overline{a}\langle b_1,b_2\rangle.P_1]\!] \mid [\![\overline{a}\langle c_1,c_2\rangle.P_2]\!] \mid [\![a(x_1,x_2).P_2]\!]$$

   (i.e. x_1, x_2 are only replaced by b_1, b_2 or c_1, c_2)

3. Natural numbers can be encoded in polyadic synchronous π-calculus through branching and selection. Consider the following encoding of natural numbers and the successor function:
   $[\![n]\!]_u = !u(x).x \lhd l_n$
   $[\![succ]\!]_u = !u(y,z).(\nu\, b)\overline{y}\langle b\rangle.b \rhd \{l_0 : [\![1]\!]_z \parallel ... \parallel l_i : [\![i+1]\!]_z \parallel ... \parallel l_n : [\![n+1]\!]_z\}$

(a) Try to explain how the two functions work.

(b) Write the reduction $[\![3]\!]_e \mid [\![succ]\!]_f \mid a(y).\overline{e}\langle y\rangle \mid \overline{f}\langle a, c\rangle$.

4. Show the relation of the syntax between the different π-calculi we have studied so far (synchronous, asynchronous, monadic, polyadic). You need to draw a venn diagram to show the relation.