

Exercise 5

1. Find all the weak barbs of the processes below:

- (a) $(\nu b)b(x).\bar{x}\langle a \rangle \mid \bar{b}\langle c \rangle \mid \bar{a}\langle b \rangle$
- (b) $(\nu b)(b(x).\bar{x}\langle a \rangle \mid \bar{b}\langle c \rangle \mid \bar{a}\langle b \rangle)$
- (c) $(\nu b, c)(b(x).\bar{x}\langle a \rangle \mid \bar{b}\langle c \rangle \mid \bar{a}\langle b \rangle)$
- (d) $(\nu a)(\bar{a} \mid !(a.\bar{b} \mid b.\bar{c}) \mid c.\bar{a})$
- (e) $!(\nu a)(\bar{a}\langle a, b, c \rangle \mid (\nu b)a(x, y, z).\bar{y}\langle a, b \rangle) \mid \bar{d}\langle a \rangle$

2. Find a separation context which proves:

- (a) $\bar{a} \mid \bar{b} \not\cong \bar{a}$
- (b) $\bar{a} \mid \dots \mid \bar{a} \not\cong \bar{a}$
- (c) $a.\bar{c} \not\cong b.\bar{c}$
- (d) $\bar{c} \mid \bar{a}\langle b \rangle \not\cong \bar{c} \mid \bar{a}\langle d \rangle$
- (e) $a.\bar{b} \not\cong b.\bar{a}$

3. Show the inequality $P \not\cong Q$ where

- (a) $P \stackrel{\text{df}}{=} (\nu a)(\bar{a}\langle b \rangle \mid a(x).\bar{x}\langle c \rangle) \mid x(z).\bar{z}\langle y \rangle \mid y(z).\bar{z}\langle x \rangle$
 $Q \stackrel{\text{df}}{=} \bar{x}\langle y \rangle \mid y(z).\bar{z}\langle x \rangle \mid x(z).\bar{z}\langle y \rangle$
- (b) $P \stackrel{\text{df}}{=} (\nu b)(\bar{b}\langle y \rangle \mid b(x).\bar{a}\langle x \rangle) \mid \bar{a}\langle y \rangle \mid b(x).\bar{x}\langle c \rangle \mid \bar{z}\langle c \rangle$
 $Q \stackrel{\text{df}}{=} \bar{a}\langle y \rangle \mid b(x).\bar{x}\langle c \rangle \mid \bar{z}\langle c \rangle$

(Note: for the open processes, you have to find a closing substitution σ such that $P\sigma \not\cong Q\sigma$.)

4. Show that the following equivalences hold, using the laws seen during the lectures:

- (a) $!(\nu a)a(x).P \mid Q \mid (\nu a)\bar{a}\langle c \rangle \cong !Q$
- (b) $\bar{c} \mid !(a.\bar{b} \mid c.\bar{a} \mid b.\bar{c}) \mid c.\bar{a} \mid (\nu d)d(x).\bar{x}\langle a \rangle \cong !(a.\bar{b} \mid b.\bar{c} \mid c.\bar{a}) \mid \bar{a}$
- (c) $(\nu a, b)(\bar{a}\langle c \rangle \mid !(\bar{c}\langle b \rangle \mid P)) \cong (\nu d)!\bar{c}\langle d \rangle \mid !(P \mid P), a, b \notin \text{fn}(P)$
- (d) $!(a(x, y).\bar{a}\langle x, y \rangle \mid P) \mid !P \cong !P$
- (e) $(\nu a, b, c, e)(\mathbf{FW}\langle a, b \rangle \mid \mathbf{FW}\langle b, c \rangle \mid \mathbf{FW}\langle c, d \rangle \mid \bar{a}\langle e \rangle) \cong (\nu b)\bar{d}\langle b \rangle$
- (f) $\bar{c}\langle a \rangle \mid \mathbf{EQ}\langle a, b \rangle \mid \mathbf{EQ}\langle b, c \rangle \cong \bar{a}\langle c \rangle \mid \mathbf{EQ}\langle b, a \rangle \mid \mathbf{EQ}\langle c, b \rangle$