Robotics
Practical 3: Probabilistic Motion

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1 Introduction

This week we will revisit robot motion and sensing but now with a probabilistic viewpoint, understanding how to model and reason about uncertainty. This practical lays the groundwork for next week's session where these components will be brought together into a full algorithm for Monte Carlo Localisation, a probabilistic localisation filter.

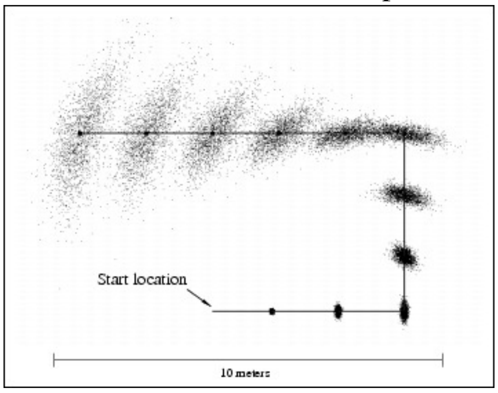
This practical will be ASSESSED. There are **30 marks** to be gained for completing the objectives defined for today's practical, out of a total of 100 for the coursework mark for Robotics over the whole term. Assessment will take place via a short demonstration and discussion of your robots and other results to me or one of the lab assistants AT THE START OF THE PRACTICAL SESSION ON THURSDAY 12th FEBRUARY. No submission of reports, code or other materials is required. We will assign marks based on our judgement of whether each objective has been successfully achieved, and give you feedback and tell you your marks on the spot.

2 Objectives

2.1 Representing and Displaying Uncertain Motion with a Particle Set (16 Marks)

In the first practical we saw clearly that even after careful calibration, a robot's will never do the same thing every time, because of many small factors which are very difficult to model. If it makes an estimate of its motion based on odometry, therefore, this measurement will always have uncertainty relative to its true motion.

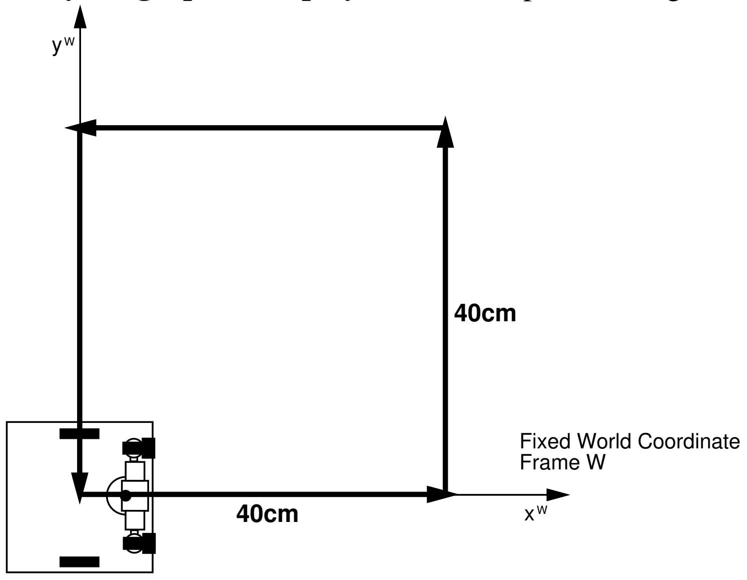
In lectures I explained that in probablistic robotics, one way to represent this uncertain estimate and how it changes over time is via a particle distribution. The figure below shows an example particle distribution evolving over time to represent the growing uncertainty in the position of a robot navigating using only odometry, though with a different scale and motion pattern from what we will be using today.



Your key aim this week is to write a program which represents the uncertain motion of your moving robot in a similar way via a particle distribution, and displays the results in the form of a cloud of particles drawn in real-time using our Pi web interface (see below).

You will need to demonstrate this program to us next week. Program your robot to follow a **full 40cm square trajectory as in Practical 1**, though **make your robot move in 10cm steps, stopping after each movement or turn**. We want to see the particle distribution on screen, updating in real-time after each movement. The particles should move after every robot movement step, and spread out gradually to represent its growing uncertainty during this motion. No outward looking sensors are to be used in this part so the uncertainty should always get bigger after each step.

So, in your demonstration we should see, happening at the same time, your actual robot driving on the floor to complete a square motion in 10cm steps, and on screen a real-time particle display which updates after each movement step. **Please pay attention to matching up the coordinate frames of your robot's real motion, and your graphics display.** Follow the picture we gave in the week 2 practical:



The x coordinate is forward, y left, at the start of the motion, and your robot should make left turns.

Note that you should clear the screen each time you redisplay the particles, so that we just see the new state of the particles rather than showing the whole history as in the figure above.

In preparation for next week your robot should run on the lab **carpet** rather than paper, and you may need to alter your tuning and calibration parameters slightly to get accurate motion on carpet. You don't need a pencil this week. (We will not be judging the accuracy of your robot this week, and I don't want you to spend a long time on calibration, but get it approximately right and if your robot is moving fairly accurately you will be ready for next week.)

At the start of motion, the set of particles should all be initialised to the origin $(x = 0, y = 0, \theta = 0)$. Each time the robot stops, you should update the position of the particles using the equations below:

- After a straight-line period of motion of distance D:

$$\begin{pmatrix} x_{new} \\ y_{new} \\ \theta_{new} \end{pmatrix} = \begin{pmatrix} x + (D + e)\cos\theta \\ y + (D + e)\sin\theta \\ \theta + f \end{pmatrix}$$

- After a pure rotation of angle angle α:

$$\begin{pmatrix} x_{new} \\ y_{new} \\ \theta_{new} \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta + \alpha + g \end{pmatrix}$$

Here e, f and g are noise terms, which are generated for each particle separately by sampling random numbers with zero mean and an appropriate standard deviation from a Gaussian distribution. Adding a small *different* random amount onto the position of each particle like this will cause spreading like you see in the diagram.

The best way to adjust the standard deviations of the Gaussians for each of e, f and g which you sample is by running the whole program for the square trajectory and observing the total amount of particle spreading which occurs. e, f and g represent different things, so each should have a different standard deviation, though all shouldh have zero mean as long as your robot is well calibrated. In principle, the amount your particles spread out should agree with the amount of uncertainty you think your robot has in its motion. So if your robot can complete the square trajectory and get back to its starting point with around a 5cm standard deviation total error, look for this amount of spread in the particles at the end of the motion by looking at your graphics display (you can judge by eye the amount of spreading relative to the size of the 40cm trajectory and 10cm steps).

Note that it usually makes sense to err on the side of too much uncertainty in the way that the particles spread out because when the robot is out in the real world and especially running on carpet it may be less precise than during the experiment we did on paper two weeks ago. We would expect that the particles spreading out to around 5cm standard deviation over the whole square motion would be reasonable for most robots. The following sections have further details.

2.1.1 Web Interface and Displaying Output

You may have already been using the web interface with your Raspberry Pi Robots which makes it easy to interact with programs running on the Pi from any device on the college network (PC, tablet, etc.). Instructions for how to install and use this were given in the setup guide you should have completed in the first week: `http://www.doc.ic.ac.uk/~ajd/Robotics/RoboticsResources/wifi_setup.txt`.

Remember that if you have the webserver correctly installed, you should be able to click straight through to it via this link (using your Pi's MAC address): `http://www.doc.ic.ac.uk/~ajd/robotics/`.

You can copy programs to the Pi through this interface, run them and see text output in the browser.

This week we will be using the other functionality it provides, which is to enables real-time 2D graphics output from your Pis via special formatted Python print statements. You can output simple 2D graphics from a Python program to the web interface which can be used to visualise the state of a particle filter by drawing lines and points. The Python program does not need to call any libraries to achieve this drawing. It simply has to print text in the correct format. While most print statements will lead to the raw text being displayed in the browser, if you print lines with the drawLine or drawParticles keywords they are picked up by the web server and interpreted as graphics commands.

So in Python:

- Drawing lines: `print ("drawLine:" + str(line))`, where `line` is a 4-tuple `(x0, y0, x1, y1)`.

- Drawing particles: `print ("drawParticles:" + str(particles))`, where `particles` should be a list of 3-tuples `(x, y, theta)`.

These drawing commands will use a standard graphics coordinate system to draw in a window within the web display. You define a suitable transformation between (x, y) robot coordinates and screen graphics coordinates to suitably scale and orient your display so that it is clear easy to see what is going on with the particles, and so that the display has the same orientation as the picture above.

Note that a known issue with our graphics functionality through the web browser is that it does not cope perfectly with very rapid drawing calls, or calls to draw a very high number of particles or lines. You should be fine with 100 particles, displayed at a rate up to several times a second.

2.1.2 The Particle Set

The set of particles, each of which has values for $\mathbf{x}_i = (x_i, y_i, \theta_i)$ and weight w_i, can be stored in pre-allocated Python arrays of length `NUMBER_OF_PARTICLES`. A suitable initial value for `NUMBER_OF_PARTICLES` this week is 100. In this week's practical, the weights w_i of the particles will not be important and should just be set to 1/ `NUMBER_OF_PARTICLES`. The weights become important next week when we incorporate sonar measurements.

The web interface provides a simple way to display a set of points on screen to represent the positions of a particle set. You should experiment to find a suitable scale so that the particle set stays within the bound of the interface window at all times.

2.1.3 Generating Random Numbers

Random numbers for use in the particle movement can be easily generated in Python using the `random` module. See `http://docs.python.org/3/library/random.html` for full documentation. In particular, this week you will need to generate random numbers sampled from a Gaussian distribution. The function `random.gauss(mu, sigma)` generates a random number sampled from a Gaussian distribution with mean `mu` and standard deviation `sigma`.

2.2 Waypoint Navigation (8 Marks)

The proof of good localisation is if it can be used to achieve accurate navigation. You can make a point estimate of the current position and orientation of the robot by taking the mean of all of the particles:

$$\bar{\mathbf{x}} = \sum_{i=1}^{N} w_i \mathbf{x}_i \; .$$

Based on this estimate you can then control the robot to move towards a target location. Note that this week, this mean estimate is just calculated from odometry so is not very interesting — as the particles spread out as uncertainty increases, their mean will not change — but with this machinery in place you will be set up for next week when the particle distribution will also be affected by sonar measurements.

Provide your robot with the capability to perform position-based navigation via simple path planning through a set of waypoints. A waypoint is an (W_x, W_y) position relative to the world coordinate frame W through which the should pass. Write a function `navigateToWaypoint(float X, float Y)` which, given the robot's current estimated location (x, y, θ), drives it to the waypoint at (W_x, W_x) specified in metre units. Most straightforwardly this will be achieved by first a turn on the spot through the appropriate angle and then a straight forward motion of the right distance. Refer back to the lecture notes from week 2 on position-based path planning, and please use the same 2D coordinate frame convention defined in the figures in that lecture such that the robot starts at $(x, y, \theta) = (0, 0, 0)$ with its forward direction aligned with the x-axis, the y axis points left and positive θ representing a left turn.

Prove the operation of your path planning function to use by demonstrating a short interactive program where a user is asked to enter (W_x, W_y) coordinates in metres which the robot will then move to before asking for more coordinates. We ask you to demo this in the assessment and give you some coordinates for the robot to drive to, so make sure your program is well tested. (Again, bear in mind that at this stage your robot is navigating only based on odometry so there will be inevitable drift in its position over long motions; but with this machinery in place we will be ready to implement a full MCL localisation and navigation system next week.)

2.3 Sonar Investigation (6 Marks)

The sonar is the crucial outward-looking sensor which we will use to make contact with the mapped world in Monte Carlo Localisation next week.

This week we will make some initial quantitive investigation of its properties. We will attempt to calibrate the sonar by comparing the values it returns with *ground truth* obtained from measurements with a ruler or tape measure (there are some around the lab and I should have some spares). Set up your sonar sensor with a simple program to read continuously and report depth values to the screen (look at the example program `NXT-Ultrasonic_Sensor.py` on your Raspberry Pis to see how to do this).

2.3.1 Sonar Calibration Questions to Answer

On paper, prepare brief answers to these questions, showing any measurements that you recorded to support your answers. We will ask you about all of these at the assessment:

1. When placed facing and perpendicular to a smooth surface such as a wall, what are the minimum and maximum depths that the sensor can reliably measure?
2. Move the sonar so that it faces the wall at a non-orthogonal incidence angle. What is the maximum angular deviation from perpendicular to the wall at which it will still give sensible readings?
3. Do your sonar depth measurements have any systematic (non-zero mean) errors? To test this, set up the sensor at a range of hand-measured depths (20cm, 40cm, 60cm, 80cm, 100cm) from a wall and record depth readings. Are they consistently above or below what they should be?
4. What is the the accuracy of the sonar sensor and does it depend on depth? At each of two chosen hand-measured depths (40cm and 100cm), make 10 separate depth measurements (each time picking up and replacing the sensor) and record the values. Do you observe the same level of scatter in each case?
5. In a range of general conditions for robot navigation, what fraction of the time do you think your sonar gives garbage readings very far from ground truth?