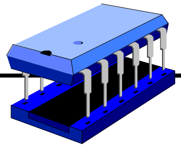

Computer Systems - Architecture

Lecture 4 - Boolean Logic

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(Heavily based on notes by Andrew Davison and Ian Harries)

Learning Outcomes

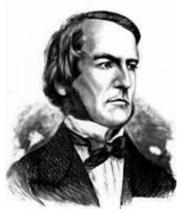
- At the end of this lecture you should:
  - understand how logic relates to computing problems
  - be able to represent Boolean logic problems as:
    - Truth tables
    - Logic circuits
    - Boolean algebra
  - be able to produce circuits for the half adder and full adder
  - have a feeling for how electronic circuits can be joined together to create number manipulators (simple computers???)

What is Logic?

- Dictionary definitions (dictionary.com - reduced!)
  - reason or sound judgement
  - a system of principles of reasoning
  - the science that investigates the principles governing correct or reliable inference
- Branch of philosophy
  - Principles of inference
  - Ancient civilizations (India – the Rigveda, China Gongsun Long – 325 BC)
  - Greek – Aristotle (Syllogistic logic)
  - Modern: John Stuart Mill “The science of reasoning”, Frege, Russell, Gödel...
- You use logic all the time in your everyday life

Boolean Logic

- Named after George Boole
- Provides a system of logical operations
- Rules for combining operations
- Describes their application to binary numbers



0 or 1?

TRUE or FALSE?

YES OR NO?

Simple Example

- "If it is raining then I will take an umbrella"
  - "It is raining" – can be TRUE or FALSE
  - "I will take an umbrella" - can be TRUE OR FALSE
- The truth of "take an umbrella" *depends on* the truth of "raining"
- Can be represented in the form of a truth table:

Raining	Umbrella
False	False
True	True

Can be used to make a decision (i.e. inference):

(take umbrella) = (raining)

Example 2

"If it is raining or the weather forecast is bad then I take an umbrella"

Truth table:

Raining	Bad forecast	umbrella
False	False	False
False	True	True
True	False	True
True	True	True

(take umbrella) = ((raining) OR (bad forecast))

Example 3

"If it is raining and I have no car then I will take an umbrella"

Truth table:

Raining	No Car	Umbrella
False	False	False
False	True	False
True	False	False
True	True	True

(take umbrella) = ((raining) AND (NOT car))

Digital Logic

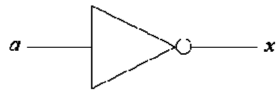
- Computers make decisions using logic
- Basic logic operations
  - NOT
  - AND
  - OR
  - Exclusive OR (XOR)
- Also
  - NOT AND (NAND)
  - NOT OR (NOR)

Digital Logic

- Computers operate electronically using **Logic Gates**
  - One or more inputs
  - One output
  - Input and output are binary digits (0 or 1)
  - 0 = FALSE
  - 1 = TRUE
- Electronic circuits are easily connected together to perform more complex functions, from these basic "building blocks" of computers

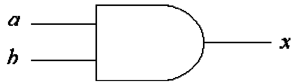
Logic Gates

NOT gate:



a	x
0	1
1	0

AND gate:



a	b	x
0	0	0
0	1	0
1	0	0
1	1	1

What is the truth table for the NOR gate?

A

a	b	x
0	0	1
0	1	1
1	0	1
1	1	0

B

a	b	x
0	0	1
0	1	0
1	0	0
1	1	0

C

a	b	x
0	0	0
0	1	1
1	0	1
1	1	0

D

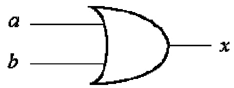
a	b	x
0	0	0
0	1	1
1	0	1
1	1	1

E

a	b	x
0	0	0
0	1	0
1	0	1
1	1	1

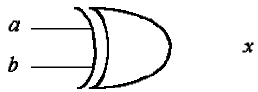
Logic Gates

OR gate:



a	b	x
0	0	0
0	1	1
1	0	1
1	1	1

XOR gate:



a	b	x
0	0	0
0	1	1
1	0	1
1	1	0

How many basic logic gates are required

- NOT
- AND
- OR
- XOR – can be made up of NOT, AND and OR gates
- Draw the circuit……
- What is the corresponding logic equation?

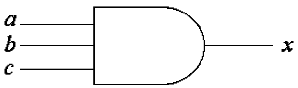
Answer

- A a (XOR) b = (NOT a) AND (NOT b) OR (a OR b)
- B a (XOR) b = a OR ((b AND NOT a) AND b)
- C a (XOR) b = NOT((NOT a AND NOT b) OR (a AND b))
- D a (XOR) b = (a AND NOT b) OR (NOT a AND b)
- E a (XOR) b = (a OR b) AND NOT (a AND b)

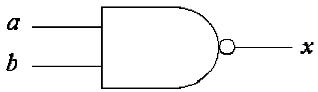
IN FACT C, D or E are correct answers, but D,E more concise

Summary

- Basic logic gates AND, OR and NOT are enough
- Useful to have others - XOR, NAND etc.
- Can have more complex gates (e.g, multiple inputs)

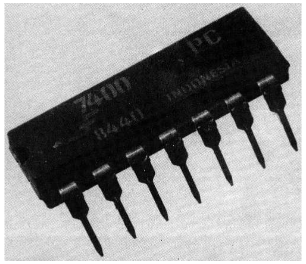


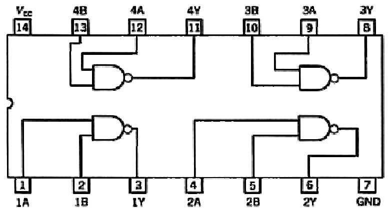
3-input AND gate



NAND gate
(AND followed by NOT)

Practical digital circuits





7400 dual in-line package

Boolean algebra - notation

Infuriatingly, there is no single agreed representation for the logical operators in Boolean algebra:

A AND B º A • B º A Ù B º A & B º AB
A OR B º A + B º A Ú B º A | B
A XOR B º A Å B
NOT A º A¢ º Ø A º $\bar{A}$ (with bar) º A'

We will use A • B, A + B, A'

Boolean algebra - rules

There are many rules that can be used for algebraic manipulation

Negation:
(A')' = A
A•A' = 0
A+A' = 1

Associative:
(A•B)•C = A•(B•C)
(A+B)+C = A+(B+C)

Commutative:
A•B = B•A
A+B = B+A

Distributive:
A•(B+C) = A•B + A•C
A+(B•C) = (A+B) • (A+C)
Note the precedence

Boolean algebra - rules

Single variables:
A•A = A
A+A = A

Simplification rules with 1 and 0:
A•0 = 0
A•1 = A
A+0 = A
A+1 = 1

Boolean Algebra – de Morgan's rules

(A + B)' = A' • B'
(A • B)' = A' + B'
as before, A and B can be any Boolean expression

Can generalise to n Boolean variables:
(A + B + C + D + ...)' = A' • B' • C' • D' • ...
(A • B • C • D • ...)' = A ' + B' + C' + D' + ...

Boolean algebra – rules for XOR

A Å B = A • B' + A' • B

Complement Law for XOR:

(AÅB)' = AÅB' = A'ÅB

Exclusive NOR (NOT XOR) sometimes called an equivalence gate – why?

Addition using logic gates

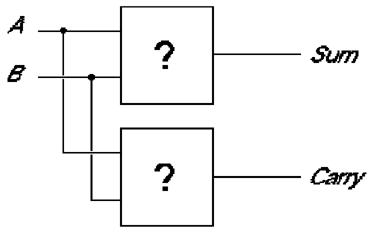
- Suppose we want to add two one-bit inputs?
- What is the truth table for

  The sum?

  Any bit that might carry over?

Addition using logic circuits

In the following circuit:



Upper box is: A. AND B. OR C. NOT D. XOR E. NOR

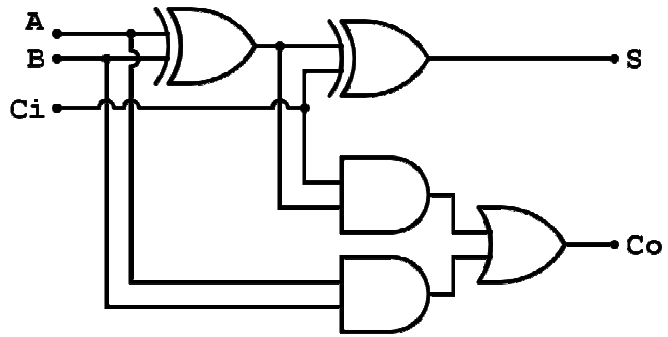
Lower box is: A. AND B. OR C. NOT D. XOR E. NOR

The half-adder

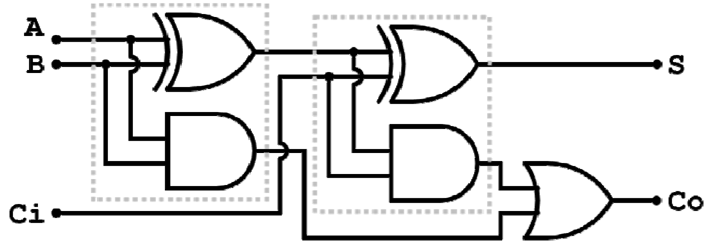
The full adder

- For a full adder, we need another input, the carry bit from the next less significant bit.
- What is the logic circuit for the full adder?

The full-adder



The full adder - redrawn



Effectively two half-adders plus an OR gate

Learning Outcomes

- You should now:
  - understand how logic relates to computing problems
  - be able to represent Boolean logic problems as:
    - Truth tables
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    - Boolean algebra
  - be able to produce circuits for the half adder and full adder
  - have a feeling for how electronic circuits can be joined together to create number manipulators (simple computers???)

This lecture - feedback

- The pace of the lecture was:

A. much too fast B. too fast C. about right D. too slow E. much too slow

- The learning objectives were met:

A. Fully B. Mostly C. Partially D. Slightly E. Not at all