Using Resolution to Generate Natural Proofs

David Robinson
*Manchester University
Oxford Road, Manchester
robinde0@cs.man.ac.uk

Abstract

I am investigating a method of using resolution theorem provers to automatically generate natural deduction style proofs. These proofs are closer to human reasoning than resolution proofs and so are simpler to understand. I use a splitting rule to track natural deduction style assumptions and a structural transformation to maintain the structure of the input formula.

1 Introduction

With few rules of inference, resolution is well suited to computer automation and many automated theorem provers use the resolution principle. Choices of orderings and selection functions make the modern resolution calculus very flexible. (Schmidt, 2006) and (Schmidt and Hustadt, To appear) explore the use of first-order resolution to develop tableau calculi for modal logics.

In resolution based natural deduction (RND) (Indrzejczak, 2002), Andrzej Indrzejczak introduces a system called RND that mixes some features of resolution with natural deduction. This combination of the two methods gives a calculus that produces proofs that are easily understood and much more straight forward to produce than for natural deduction.

2 RND for Modal Logic

Figure 1 shows the rules of the RND calculus for labelled modal logic K. The assumption set associated with each formula is omitted for clarity. The αE, βE and $\Box$E rules are elimination rules and the αI, βI and $\Box$I rules are introduction rules. The α rules allow manipulation of α formulae. The β rules are used to change β formulae between equivalent forms, for example $\neg\varphi \vee \psi$ into $\varphi \rightarrow \psi$. The α and β introduction and elimination rules are the mirror of each other. The R' rule is the resolution rule. The [Sub] and $\Box$I rules are discharge rules. The $\Box$ rules are the standard natural deduction rules for $\Box$ Fitting (1983).

$$(\alpha E)\ \frac{x{:}\alpha}{x{:}\alpha_1 \quad x{:}\alpha_2} \qquad (\alpha I)\ \frac{x{:}\alpha_1 \quad x{:}\alpha_2}{x{:}\alpha} \qquad (\beta E)\ \frac{x{:}\beta}{x{:}\beta_1 \vee \beta_2} \qquad (\beta I)\ \frac{x{:}\beta_1 \vee \beta_2}{x{:}\beta}$$

$$(R')\ \frac{x{:}\psi \vee \varphi \quad x{:}\theta \vee \neg\varphi}{x{:}\psi \vee \theta} \qquad [Sub]\ \frac{\begin{array}{c}[x{:}\varphi]\\ \vdots \\ x{:}\gamma\end{array}}{x{:}\neg\varphi \vee \gamma} \qquad (\Box E)\ \frac{x{:}\Box\varphi \qquad R(x,y)}{y{:}\varphi} \qquad (\Box I)\ \frac{\begin{array}{c}[R(x,y)]\\ \vdots \\ y{:}\varphi\end{array}}{x{:}\Box\varphi}$$

Figure 1: RND rules for modal logic K

Theorem 1 *The calculus of RND for modal logic K as shown in Figure 1 is sound and complete.*

3 Resolution

The resolution calculus used is ordered resolution with maximal selection (Bachmair and Ganzinger, 2001), together with the splittingless splitting rule (Riazanov and Voronkov, 2001), OH_{Sp}, shown below. The splitting rule introduces new symbols. These q's introduced by applications of the rule can be used to represent RND assumptions and can trace the assumptions through the proof.

Splittingless split:

$$\frac{N \cup \{C \vee D\}}{N \cup \{q \vee C,\ \neg q \vee D\}}$$

if C and D are variable-disjoint.

Figure 2: Splitting through new symbols rule

Theorem 2 *RND assumptions can be simulated by symbols introduced by applications of the splittingless split rule.*

Labelled modal formulae are encoded using a combination of structural transformation, and the standard translation to first-order logic, Π. The standard translation imitates the relational semantics of modal logic. The structural transformation introduces new symbols for subformulae of the formula being encoded. The standard translation and conversion to clause logic allows the application of standard resolution theorem provers to the problem, while the introduced symbols preserve the structure of the original formula. Positive and negative occurrences of subformulae are encoded differently, as shown in Figure 3.

Positive φ	
$\Box\psi$	$Q_\varphi(x) \vee R(x, f(x))$ $Q_\varphi(x) \vee \neg Q_\psi(f(x))$
$\psi_1 \vee \psi_2$	For each ψ_i, $Q_\varphi(x) \vee Q_\theta(x)$, if $\psi_i = \neg\theta$ $Q_\varphi(x) \vee \neg Q_{\psi_i}(x)$, otherwise

Negative φ	
p	$\neg Q_\varphi(x) \vee \neg Q_p(x)$
$\Box\psi$	$\neg Q_\varphi(x) \vee \neg R(x, y) \vee Q_\psi(y)$
$\psi_1 \vee \psi_2$	$\neg Q_\varphi(x) \vee L_1 \vee L_2$ Where, for each i, if $\psi_i = \neg\theta$ then $L_i = \neg Q_\theta(x)$ else $L_i = Q_{\psi_i}(x)$

Figure 3: Positive and Negative Occurrence Encodings

Theorem 3 *Let φ be a formula of modal logic K and Π_{CL}be the encoding given by Figure 3. Then φ is satisfiable in K iff Π_{CL}($\neg\varphi$) is unsatisfiable in first-order logic.*

4 Translating to RND

The clauses produced by OH_{SP} and the encoding of Figure 3 are positive and have the form $q^{\Gamma} \vee Q_\psi(x)$, where q^Γ is a set of literals of the form $q_\theta(x)$. We can define a mapping f that takes an identifiable subset of these positive clauses and returns a tuple $\{\Gamma, x{:}\psi\}$. Each literal in q^Γ is translated to a corresponding formula $lblfmlx\theta$ in Γ. This new set of tuples are RND proof lines where $x{:}\psi$ is the formula derived at the current line, and Γ is the assumption set upon which $x{:}\psi$ depends.

The RND rules being applied at each line can be determined either from the encoding clauses applied in the resolution proof, or by examination of the RND translation.

Theorem 4 *For any modal formula φ, OH_{Sp} together with a suitable encoding and the translation f can be used to automatically generate an RND derivation of φ.*

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