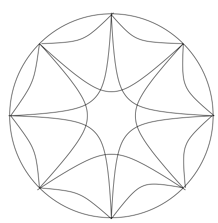
# Complex Systems- Exercises 4

1. Show that the clustering coefficient for a one dimensional lattice with periodic boundary condition (i.e., a circle), as for example in the figure below, can be computed to be

$$C = \frac{3(z-2)}{4(z-1)}$$

which tends to $3/4$ as $z \to \infty$. (Here, $z \ll N$ and $z$ is assumed to be even so that every vertex has $z/2$ connections with its neighbours on one side and $z/2$ connections on the other side.)



2. Find the average distance in a one dimensional lattice of length $\ell$ with $z = 2$ and obtain its asymptotic behaviour as $\ell \to \infty$.

3. We can equivalently define a random graph by its size $N$ and its total number of edges $n$.

(i) What is the total number of possible graphs with this specification?

(ii) Find $z$ and $p$ (as defined in the notes) in terms of $N$ and $n$.

(iii) Starting with the definition of a random network as in the notes, find the expected value $\langle n \rangle$ of the number of edges $n$.

4. Find the expected value and the second moment of the degree of vertices

$$\langle k \rangle = \Sigma_{k=1}^{\infty} k P(k),$$

$$\langle k^2 \rangle = \Sigma_{k=1}^{\infty} k^2 P(k),$$

for the random growing network, where $P(k) = 2^{-k}$. Hence, find $z_2/z_1$ and find the average number of nodes $n$ steps away from a vertex.

**Hint:** Evaluate $\langle k \rangle = 2\langle k \rangle - \langle k \rangle$ and $\langle k^2 \rangle = 2\langle k^2 \rangle - \langle k^2 \rangle$.