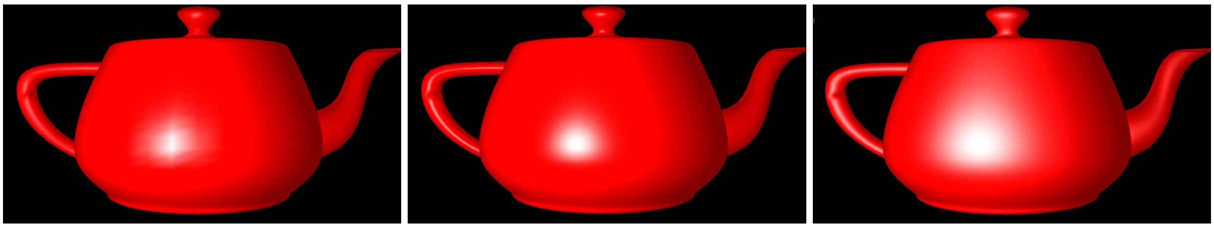
Approximation of Illumination



(a) Gouraud illumination (b) Phong illumination (c) Blinn-Phong illumination

Figure 1: Different variants of the application of the Phong illumination model.

The Phong model

The Phong Illumination Model is a lighting model in 3D computer graphics that is used to calculate the illumination of objects. The model was named after its developer Bùi Tóng Phong and was first introduced in 1975 [4]. An example is shown in Figure 1b.

The Phong model is suitable for the representation of smooth, plastic-like surfaces. The highlight of the surface is described by the term $\cos^n(\theta)$, where the parameter n determines the roughness of the surface.

It is a completely empirical model that is not based on any real physics. This means that it contradicts the principle of energy conservation, which dictates that a surface can reflect at most as many photons as it is hit by. Other lighting models such as the Schlick [5] or Cook-Torrance models [2], are completely or almost physically plausible. In addition, the Phong lighting model is slow to compute compared to the, e.g., the Blinn lighting model, which provides comparable image quality. Despite its shortcomings, the Phong lighting model is still frequently used due to its simplicity.

Assumptions:

- Light sources are point-shaped, i.e. not spatial extent,
- the geometry of the surfaces, except for the surface normals, is ignored,
- diffuse and mirror reflection is only locally modelled,
- ambient reflection is globally modelled.

The Phong lighting model describes the reflection of light as a combination of ambient, ideally diffuse and ideally reflective reflection.

$$I_{\text{out}} = I_{\text{ambient}} + I_{\text{diffuse}} + I_{\text{specular}} \tag{1}$$

Ambient component

The ambient component of the reflected light is independent of the angle of incidence of the light ray from the point source and the angle of view of the observer of the scene. It depends on the ambient light constant for all points on all surfaces and an empirically determined reflection factor (material constant).

$$I_{\text{ambient}} = I_{\text{a}}\, k_{\text{ambient}}, \tag{2}$$

with I_{a} intensity of the ambient light and k_{ambient} as material constant.

Diffuse component

For diffuse reflection, the light is reflected in all directions regardless of the viewer's point of view (Lambertsches' law). However, the luminous intensity of the reflected light from a point source is still dependent on the angle of incidence, as the illumination of the surface changes with the incidence angle. The luminous intensity of the diffuse component is thus dependent on the angle of incidence of the light ray from the point source and on an empirically determined reflection factor (material constant), but is independent of the angle of view of the observer of the scene.

$$I_{\text{diffuse}} = I_{\text{in}}\, k_{\text{diffus}}\, \cos\varphi = I_{\text{in}}\, k_{\text{diffus}}\, (\vec{L} \cdot \vec{N}), \tag{3}$$

with I_{in} the luminous intensity of incident light ray from the point source, k_{diffuse} as empirically determined reflection factor for diffuse component of reflection, and φ the angle between normal vector of surface $\vec{N}$ and unit vector in direction of incident light ray $\vec{L}$.

Reflective (Specular) component

For mirror reflection, the light is reflected in a certain environment of the ideal direction of reflection. The luminous intensity of the reflected light is dependent on the angle of incidence of the light ray from the point source, on an empirically determined reflection factor (material constant) and on the surface texture and the viewing angle of the observer of the scene.

$$I_{\text{specular}} = I_{\text{in}}\, k_{\text{specular}}\, \cos^n\theta = I_{\text{in}}\, k_{\text{specular}}\, (\vec{R} \cdot \vec{V})^n, \tag{4}$$

with I_{in} the luminous intensity of incident light ray from the point source , k_{specular} an empirically determined reflection factor for mirror reflecting component, θ the angle between ideal reflection direction of the incident light ray $\vec{R}$ and viewing direction of the viewer $\vec{V}$, and n as constant exponent to describe the surface texture (rough surface: $n < 32$, smooth surface: $n > 32$, $n = \infty$ would be a perfect mirror).

Furthermore, a normalization factor should be used to ensure that the brightness of large mirror exponents n does not decrease. As a reasonable factor is for example $\frac{n+2}{2\,\pi}$.

Thus, the specular reflection component can be computed as

$$I_{\text{specular}} = I_{\text{in}}\, k_{\text{specular}}\, \frac{n+2}{2\,\pi}\, \cos^n\theta = I_{\text{in}}\, k_{\text{specular}}\, \frac{n+2}{2\,\pi}\, (\vec{R} \cdot \vec{V})^n \tag{5}$$

The complete Phong model can therefore be defined as

$$\begin{aligned} I_{\text{out}} &= I_{\text{a}}\, k_{\text{ambient}} + I_{\text{in}}\, k_{\text{diffus}}\, \cos\varphi + I_{\text{in}}\, k_{\text{specular}}\, \frac{n+2}{2\,\pi}\, \cos^n\theta & (6)\\ &= I_{\text{a}}\, k_{\text{ambient}} + I_{\text{in}}\, (k_{\text{diffus}}\, \cos\varphi + k_{\text{specular}}\, \frac{n+2}{2\,\pi}\, \cos^n\theta) & (7)\\ &= I_{\text{a}}\, k_{\text{ambient}} + I_{\text{in}}\, \left[k_{\text{diffus}}\, (\vec{L} \cdot \vec{N}) + k_{\text{specular}}\, \frac{n+2}{2\,\pi}\, (\vec{R} \cdot \vec{V})^n\right], & (8) \end{aligned}$$

with $k_{\text{diffus}} + k_{\text{specular}} \leq 1$ and $k_{\text{ambient}} \leq 1$.

If several light sources are present, the respective components are first calculated separately for each light source and then summed up.

Blinn variant

In image synthesis, the Blinn illumination model (also called Blinn-Phong model) is a local illumination model for light reflection on surfaces. The Phong lighting model is used as the basis. By using so-called halfway vectors, the necessary calculations are accelerated without noticeably influencing the result. The model was described in 1977 by James F. Blinn [1], who also developed bump mapping (see Texture mapping). An example for Blinn-Phong illumination is shown in Figure 1c.

In practice, the Blinn illumination model is used, for example, in OpenGL, as it avoids the calculation of reflection vectors. Instead, the angle halving H is used:

$$H = \frac{\vec{V} + \vec{L}}{\|\vec{V} + \vec{L}\|}, \tag{9}$$

with $\vec{V}$ the normalized vector from point to viewer and $\vec{L}$ the normalized vector from the point to the point light source.

This can be used to calculate the cosine of the angle θ' between the normal $\vec{N}$ and the angle halving $\vec{H}$: $\cos\theta' = \vec{N} \cdot \vec{H}$. Of course, this is only valid under the condition that $\|\vec{N}\| = 1$. This cosine can now be used instead of $\cos\theta$ instead of Eq. 5 to calculate the specular component in Eq. 1:

$$I_{specular} = I_{in} \cdot k_{specular} \cdot \cos^n \theta' = I_{in} \cdot k_{specular} \cdot \left(\frac{(\vec{V} + \vec{L}) \cdot \vec{N}}{\|(\vec{V} + \vec{L})\| \cdot \|\vec{N}\|} \right)^n, \tag{10}$$

with I_{in} the luminous intensity of incident light ray from the point light source, $k_{specular}$ an empirically determined reflection factor for the mirror component of the reflection, and n a constant exponent to describe the surface properties (called “Shininess” in OpenGL).

In order to achieve comparable results with the Blinn model as with the Phong model, the exponent n must be selected four times as large as n in the Phong model.

Gouraud shading

The Phong model is intended to be applied per pixel. In early days and for low-end hardware this is not feasible in real-time. Instead Gouraud shading [3] can be used, which evaluates the Phong model only at the vertex positions and interpolates illumination similar to colour interpolation across the pixels of a projected polygon. An example for Gouraud illumination and the resulting artefacts of this simplification is shown in Figure 1a.

References

[1] James F. Blinn. Models of light reflection for computer synthesized pictures. *SIGGRAPH Comput. Graph.*, 11(2):192–198, July 1977.

[2] R. L. Cook and K. E. Torrance. A reflectance model for computer graphics. *ACM Trans. Graph.*, 1(1):7–24, January 1982.

[3] H. Gouraud. Continuous shading of curved surfaces. *IEEE Trans. Comput.*, 20(6):623–629, June 1971.

[4] Bui Tuong Phong. Illumination for computer generated pictures. *Commun. ACM*, 18(6):311–317, June 1975.

[5] Christophe Schlick. A customizable reflectance model for everyday rendering. In *In Fourth Eurographics Workshop on Rendering*, pages 73–83, 1993.