Tutorial 1: 3D space, transformations and animations.

This tutorial uses the following notation:

- Position vectors are denoted by boldface capital letters: **P**, **Q**, **V** etc. Position vectors are the same as Cartesian coordinates, and represent position relative to the origin.
- Direction vectors are indicated by boldface lowercase letters **d**, **n** etc. Direction vectors are independent of any origin.
- Scalars are represented by italics: *a*, *b*, etc.

A plane is an object that is only defined in Cartesian space, however, each plane has a normal vector, whose size is non zero, and whose direction is at right angles to that plane. We can find a normal vector by taking the cross product of any two direction vectors which are parallel to the plane.

Analysis of 3D scenes

1. Given three points:

   $\mathbf{P}_1 = (10, 20, 5)$
   $\mathbf{P}_2 = (15, 10, 10)$
   $\mathbf{P}_3 = (25, 20, 10)$

   find two direction vectors which are parallel to the plane defined by $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$. Hence find a normal vector to the plane.

2. A plane is defined in vector terms by the equation:

   $$\mathbf{n} \cdot (\mathbf{P} - \mathbf{P}_1) = 0$$

   where $\mathbf{P} = (x, y, z)$ is the locus of a point on the plane, and $\mathbf{P}_1$ is any point known to be in the plane.

   For the points given in part 1, expand the vector plane equation to find the Cartesian form of the plane equation, (i.e. $ax + by + cz + d = 0$).

   Verify that you get the same result using either $\mathbf{P}_1$ or $\mathbf{P}_2$.

3. Write a procedure, in any programming language you like, which takes as input three points and returns the coefficients of the Cartesian plane equation (*a*, *b*, *c* and *d*).

4. Starting from any point on a face of a polyhedron, an inner surface normal is a normal vector to the plane of the face whose direction points into the polyhedron.

A tetrahedron is defined by the three points of part 1, and a fourth point $\mathbf{P}_4 = (30, 20, 10)$. Determine whether the normal vector that you calculated in part 1 is an inner surface normal, and if not find the inner surface normal.

5. Two lines intersect at a point $\mathbf{P}_1$, and are in the directions defined by $\mathbf{d}_1$ and $\mathbf{d}_2$. Provided that $\mathbf{d}_1$ and $\mathbf{d}_2$ represent different directions, the two lines define a plane.

   Any point on the plane can be reached by travelling from $\mathbf{P}_1$ in direction $\mathbf{d}_1$ by some distance μ and then in direction $\mathbf{d}_2$ by a distance ν. Using this fact construct the parametric equation of any point on the plane of part 1 in terms of μ, ν, $\mathbf{P}_1$, $\mathbf{P}_2$ and $\mathbf{P}_3$. By taking the dot product with a normal vector to the plane $\mathbf{n}$, show that the parametric plane equation is equivalent to the vector plane equation of part 2.

Animations

6. In a computer graphics animation scene an object is defined as a planar polyhedron. The object centre is located at position P = (0, 0, 10), and the scene is drawn, as normal, in perspective projection with the viewpoint at the origin and the view direction along the z-axis. Calculate the transformation matrix that will shrink the object in size by a factor of 0.8 towards its centre point.

7. Use your matrix of part 1 to check what happens to the points (0, 0, 10) and (0,0,5). Is your result what you expect?

8. In a different animation, the object, defined above is required to rotate clockwise, looking from the origin, while shrinking. In each successive frame it is to rotate by 15° while shrinking to 0.8 of its original size. The rotation axis is to be the z axis, and the shrinkage is, as before, towards the object's centre. Given that $\text{Cos}(15^\circ) = .97$ and $\text{Sin}(15^\circ) = .26$, what is the transformation matrix that will achieve this animation?

9. The scene above is to be drawn in perspective projection with the plane of projection being $z = 2$. Find the combined transformation that will do animation of part 6 followed by the perspective projection. Is your matrix singular?

10. Use your matrix to find the transformation and perspective projection of the points (0, 0, 10) and (0, 0, 5) in homogenous coordinates and then in Cartesian coordinates.

11. The scene is to be viewed from a moving viewpoint specified by its position **C** and a left-handed viewing coordinate system $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$. At one point in the animation the view direction is $\boldsymbol{w} = (-1, 0, 0)^T$, and the viewpoint is given by $\mathbf{C} = (50, 10, -10)$. Given that the view is in the horizontal plane ($\boldsymbol{v} = (0, 1, 0)^T$) find the value of $\boldsymbol{u}$.

12. Hence, or otherwise, find the viewing transformation matrix.