Tutorial 5: Spline Curves and Surfaces

1. A four knot, two dimensional Bezier curve is defined by the following table

	(x, y)
$\mathbf{P}_0$	(0, 0)
$\mathbf{P}_1$	(2, 3)
$\mathbf{P}_2$	(3, -1)
$\mathbf{P}_3$	(0, 0)

a. Use de Casteljau's construction to sketch the curve.

b. Calculate the coefficients $\mathbf{a}_0$, $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ of the corresponding cubic spline patch:

$$\mathbf{P}(\mu) = \mathbf{a}_3\mu^3 + \mathbf{a}_2\mu^2 + \mathbf{a}_1\mu + \mathbf{a}_0$$

c. Differentiate the spline patch equation to find $\mathbf{P}'(\mu)$ and hence show that the gradient at $\mathbf{P}_3$ is the same as the gradient of the line joining $\mathbf{P}_3$ to $\mathbf{P}_2$.

2. A Coons surface patch is to be drawn using the following array of points:

		μ			
		-1	0	1	2
ν	-1	(0, 0, 0)	(0, 10, 5)	(0, 20, 10)	(0, 30, 20)
	0	(10, 0, 5)	(10, 10, 20)	(10, 25, 30)	(15, 35, 40)
	1	(20, 0, 10)	(20, 12, 40)	(20, 30, 50)	(25, 40, 30)
	2	(30, 0, 5)	(35, 15, 30)	(40, 35, 40)	(50, 50, 20)

We are interested in the patch constructed on the centre knots, $\mathbf{P}(0, 0)$, $\mathbf{P}(0, 1)$, $\mathbf{P}(1, 0)$ and $\mathbf{P}(1, 1)$.

a. Find the equations of the four cubic spline patches that bound the Coons Patch:

$$\mathbf{P}(\mu, 0), \mathbf{P}(\mu, 1), \mathbf{P}(0, \nu), \mathbf{P}(1, \nu)$$

These are each parametric cubic splines of the form:

$$\mathbf{P} = \mathbf{a}_3\mu^3 + \mathbf{a}_2\mu^2 + \mathbf{a}_1\mu + \mathbf{a}_0 \quad \text{or} \quad \mathbf{P} = \mathbf{a}_3\nu^3 + \mathbf{a}_2\nu^2 + \mathbf{a}_1\nu + \mathbf{a}_0$$

The parameters for either form can be found using:

$$\begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{P}_i \\ \mathbf{P}'_i \\ \mathbf{P}_{i+1} \\ \mathbf{P}'_{i+1} \end{pmatrix}$$

b. Find the point at the centre of the patch using the equation:

$$\begin{aligned} \mathbf{P}(\mu,\nu) \quad = \quad & \mathbf{P}(\mu,0)(1-\nu) + \mathbf{P}(\mu,1)\nu + \mathbf{P}(0,\nu)(1-\mu) + \mathbf{P}(1,\nu)\mu \\ & - \mathbf{P}(0,0)(1-\mu)(1-\nu) - \mathbf{P}(0,1)(1-\mu)\nu - \mathbf{P}(1,0)\mu(1-\nu) - \mathbf{P}(1,1)\mu\nu \end{aligned}$$

NB: This numerical calculation is rather tedious unless you use a programmable calculator, spreadsheet or software such as MatLab (which is available on the lab machines).