

$\rho \in \mathbf{Env} = \mathbf{Var} \mapsto \mathbf{Value}$ **Environments**
 $v \in \mathbf{Value} = \mathbf{Constant} \cup \mathbf{Closure}$ **Values**
 $\mathbf{Closure} ::= [(\mathbf{fn } x \Rightarrow e_0), \rho]$ **Closures**

$\mathbf{eval}(\rho, e) = v$ iff “ e evaluates to v under ρ ”

- $\mathbf{eval}(\rho, e) = v$ can also be read as an specification for building an interpreter for the `Fun` language.
- We will use this specification just as a aid to help us understand the **0-CFA** specification.

Rules

$\mathbf{eval}(\rho, c^\ell) = c$

$\mathbf{eval}(\rho, x^\ell) = \rho(x)$

$\mathbf{eval}(\rho, (t_1^{\ell_1} \mathbf{op } t_2^{\ell_2})^\ell) = \mathbf{apply}(\mathbf{op}, \mathbf{eval}(\rho, t_1^{\ell_1}), \mathbf{eval}(\rho, t_2^{\ell_2}))$

where $\mathbf{apply} : \mathbf{Op} \times \mathbf{Constant} \times \mathbf{Constant} \rightarrow \mathbf{Constant}$

$\mathbf{eval}(\rho, (\mathbf{if } t_0^{\ell_0} \mathbf{ then } t_1^{\ell_1} \mathbf{ else } t_2^{\ell_2})^\ell) = v$

where $v = \begin{cases} \mathbf{eval}(\rho, t_1^{\ell_1}) & \mathbf{eval}(\rho, t_0^{\ell_0}) = \mathbf{true} \\ \mathbf{eval}(\rho, t_2^{\ell_2}) & \mathbf{eval}(\rho, t_0^{\ell_0}) = \mathbf{false} \end{cases}$

$\text{eval}(\rho, (\mathbf{fn} \ x \ \Rightarrow \ e_0)^\ell) = [(\mathbf{fn} \ x \ \Rightarrow \ e_0), \rho]$ **closure creation**

$\text{eval}(\rho, (\mathbf{let} \ x = t_1^{\ell_1} \ \mathbf{in} \ t_2^{\ell_2})^\ell) = \text{eval}(\rho[x \mapsto v_1], t_2^{\ell_2})$
where $v_1 = \text{eval}(\rho, t_1^{\ell_1})$

$\text{eval}(\rho, (t_1^{\ell_1} \ t_2^{\ell_2})^\ell) = \text{eval}(\rho_0[x \mapsto v_2], e_0)$ **function application**
where $\text{eval}(\rho, t_1^{\ell_1}) = [(\mathbf{fn} \ x \ \Rightarrow \ e_0), \rho_0] \wedge$
 $\text{eval}(\rho, t_2^{\ell_2}) = v_2$