UNIVERSITY OF LONDON
IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2007

BEng Honours Degree in Computing Part III
MSc in Computing Science
MEng Honours Degree in Electrical Engineering Part IV
MSc in Computing for Industry
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C317=I3.16=E4.32

GRAPHICS

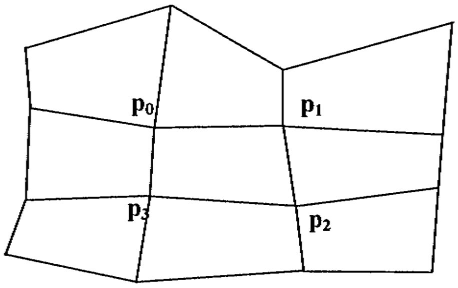
Wednesday 2 May 2007, 14:30
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Shading methods

A scene is defined as a regular array of planar quadrilaterals part of which is shown in the figure.



	X	Y	Z
p_0	1	6	10
p_1	6	6	10
p_2	7	0	13
p_3	0	2	12

The scene is being drawn in monochrome with one light source which is located at the origin. The viewpoint is also at the origin and the view direction is along the positive z axis. Only Lambertian shading is used.

a. Explain, with reference to the central quadrilateral, how Gouraud shading could be used to calculate an intensity value in the range 0-255 for each pixel inside the projected quadrilateral.

b. Explain how Phong shading could be used to calculate an intensity value in the range 0-255 for each pixel inside the projected quadrilateral.

c. What effects can be demonstrated with Phong shading that cannot be created by Gouraud shading?

d. Explain how the technique known as bump mapping could be used to create a special surface appearance.

e Calculate the outer surface normal vector (ie the one that is directed towards the viewpoint) for the central quadrilateral.

f. Assuming that only flat Lambertian shading is being used, the constant for diffuse reflection k_d = *1.0* and the light source intensity is 1.0, calculate the shade value for the central quadrilateral.

The six parts carry, respectively, 20%, 20%, 15%, 15%, 15% and 15% of the marks.

2. Texture Mapping

In a polygon rendering system a scene is to be viewed from the origin, with the viewplane at z=10 in the world coordinate system. A polygon has 3D coordinates [-20,-20,20], [20,-20,20], [-20,0,60] and [20,0,60], and an internal point has coordinate [-15,-5,50].

a What are the projected coordinates of the five points onto the view plane.

b Assuming that a texture defined in the (α,β) space with the restriction that $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ is to be mapped exactly to the polygon, derive a vector expression for an internal point of the polygon in terms of the texture coordinates, the position of the bottom left hand corner and the edge vectors on the viewplane.

c Calculate the value of the texture coordinates of the internal point defined above in both the three dimensional case and the projected two dimensional case.

d Given that the scene is drawn in a window defined by (-100, -100, 100, 100) and that the window is mapped to a raster area of 400 by 400 pixels, calculate the pixel coordinates of the five points defined above, assuming that the pixel origin is at the bottom left hand corner.

e Explain, with the aid of a suitable diagram, how a differential algorithm could be used to determine the (α,β) coordinates of the internal point at the raster level. What numerical result do you get for the internal point defined above?

The five parts carry equal marks.

3. Ray Tracing and Radiosity

a Briefly explain which illumination effects can be achieved with ray tracing and how these effects are achieved. What is the key difference between ray tracing and radiosity?

b A particular recursive ray tracing engine uses two routines: A routine `Colour CastRay(Ray r, int depth)` and a routine `Colour ColourRay(Ray r, Object o, Intersection i, Normal n, int depth)`. Using pseudo code explain how you would implement these two routines.

c A ray originates at point **V** and is parallel with direction vector **d**. A right-angled triangle is given by three points $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ (the right angle is at point $\mathbf{P_1}$). Show in detail how you can calculate the intersection between the ray and the face defined by the triangle.

d In a concrete example, a ray starts at $\mathbf{v} = (9, 9, 0)$ and has a direction vector $\mathbf{d} = (0, 0, 1)$. The points of the triangle are given as $\mathbf{P_1} = (8, 8, 10)$, $\mathbf{P_2} = (12, 8, 10)$, and $\mathbf{P_3} = (8, 8, 10)$. Calculate whether the ray intersects the face defined by the triangle.

The four parts carry, respectively, 20%, 30%, 30% and 20% of the marks.

4. Surface Modelling using the Coon's Patch

Sixteen points on a terrain map are given in the table below. The tabulated values are heights (y co-ordinates) taken over an x-z grid of unit spacing.

		X coordinate			
		0	1	2	3
Z Coordinate	0	10	11	12	13
	1	10	**12**	**14**	16
	2	11	**13**	**15**	19
	3	12	13	14	15

A set of Coon's patchs are to be used to draw the terrain. This question refers to the patch that will be used to interpolate the four points shown in boldface in the above table having co-ordinates:

P(0,0) = (1,12,1), **P**(1,0) = (2,14,1), **P**(0,1) = (1,13,2) and **P**(1,1) = (2,15,2)

Note that the parameters defining the patch μ and ν are both in the range [0..1]. The μ parameter follows the direction of the X axis and the ν that of the Z axis in the above table.

a Determine the eight gradients with respect to the parameters at the four corners of the patch.

b Given that a cubic spline curve patch has coefficients defined by the following vector equation:

$$\begin{bmatrix} \mathbf{a_0} \\ \mathbf{a_1} \\ \mathbf{a_2} \\ \mathbf{a_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P_0} \\ \mathbf{P_0'} \\ \mathbf{P_1} \\ \mathbf{P_1'} \end{bmatrix}$$

determine the equation of the curve P(μ,0) which is one of the bounding curves that bound the Coon's patch

c Given that the Coon's patch has the following equation:

$$\mathbf{P}(\mu,\nu) = \mathbf{P}(\mu,0)(1-\nu) + \mathbf{P}(\mu,1)\nu + \mathbf{P}(0,\nu)(1-\mu) + \mathbf{P}(1,\nu)\mu - \mathbf{P}(0,0)(1-\nu)(1-\mu) - \mathbf{P}(0,1)\nu(1-\mu) - \mathbf{P}(1,0)(1-\nu)\mu - \mathbf{P}(1,1)\nu\mu$$

Using your answer to part b and assuming that the other three bounding curves are defined as follows:

	$\mathbf{a_0}$	$\mathbf{a_1}$	$\mathbf{a_2}$	$\mathbf{a_3}$
P(μ,1)	(1,13,2)	(1,2,0)	(0,-1,0)	(0,1,0)
P(0,ν)	(1,12,1)	(0,1,1)	(0,0.5,0)	(0,-0.5,0)
P(1,ν)	(2,14,1)	(0,1.5,1)	(0,0,0)	0,-0.5,0)

find the co-ordinate of the patch at its centre (μ=ν=0.5)

d Determine the gradient of the surface with respect to the parameter μ at the centre of the patch (μ=ν=0.5).

The four parts carry, respectively, 20%, 20%, 30%, and 30% of the marks