IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2009

BEng Honours Degree in Computing Part III
BEng Honours Degree in Information Systems Engineering Part III
MEng Honours Degree in Information Systems Engineering Part III
MSci Honours Degree in Mathematics and Computer Science Part IV
BSc Honours Degree in Mathematics and Computer Science Part III
MSci Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degrees in Computing Part IV
MSc in Computing Science
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute
This paper is also taken for the relevant examinations for the Associateship of the Royal College of Science

PAPER C317=I3.16=E4.32

GRAPHICS

Wednesday 6 May 2009, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 **Scene Transformations**

A computer animation is being created as part of a film title sequence. In part of it several objects begin to spin about their centres, and then shrink while moving to the same vanishing point. The scene is to be drawn in orthographic projection with the view direction parallel to the z-axis.

a If one of the objects is located at the point $[10, 10, 100]$ and the spin axis passes through the object centre and is parallel to the z-axis, what is the transformation matrix that will spin it clockwise by 10^o per frame step? You can assume that $\sin(10°) = 0.17$ and $\cos(10°) = 0.98$.

b In the second phase of the animation the axis of spin changes to the line joining the centre of each object to the vanishing point $[V_x, V_y, V_Z]$. V_z of the vanishing point is larger than the z coordinate of any object point, and again the spin is clockwise by 10^o per frame step. Derive the component matrices of this transformation for a general object with centre $[O_x, O_y, O_Z]$. You need not multiply the individual matrices together.

c In the final phase of the animation spin continues on the same axis at the same rate, but each object moves towards the vanishing point shrinking in size as it goes. The sequence will last for 100 frames, and during each frame each object will shrink by a factor of 0.9, and will move towards the vanishing point by 1/10 of its current distance from the vanishing point. Derive the component matrices of the transformation that will take each object from frame f to frame $f + 1$. You need not multiply the individual matrices together.

The three parts carry, respectively, 30%, 35%, and 35% of the marks.

2 **Bezier Curves**

A cubic Bezier curve is constructed using the points $P_0 = [0, 0]$, $P_1 = [1, 4]$, $P_2 = [8, 8]$, $P_3 = [4, 2]$

a Draw an accurate sketch showing how the midpoint of the Bezier curve can be constructed using the de Casteljau method.

b Given that the blended form of the Bezier curve is defined by the equation:

$$\mathbf{P}(\mu) = \sum_{i=0}^{N} \mathbf{P}_{\mathrm{i}} W(N, i, \mu)$$

where $W(N, i, \mu)$ is called the Bernstein blending function:

$$W(N, i, \mu) = \binom{N}{i} \mu^i (1 - \mu)^{N-i}$$

$$\binom{N}{i} = \frac{N!}{(N-i)!i!}$$

Calculate the points on the curve for which $\mu = 1/4$ and $\mu = 3/4$.

c Find the gradient of the curve at the points $\mu = 1/4$.

d Use your answers to the previous parts to draw an accurate sketch of the curve.

e Using the blending form of the Bezier curve prove that for any number of points the gradient at the end of the curve is parallel to the line joining the last two points.

f Briefly describe an application where cubic Bezier curves are an effective method of constructing spline curve patches and are preferable to simply using a parametric cubic polynomial.

The six parts carry, respectively, 15%, 15%, 20%, 15%, 20%, and 15% of the marks.

3 Texture mapping

a Briefly explain the following techniques and how they can be used to improve the realism in computer graphics:

i) Texture mapping

ii) Bump mapping

iii) Displacement mapping

iv) Environment mapping

b Describe in detail how you can interpolate texture coordinates taking perspective projection into account.

c Consider an edge defined by two vertices $\mathbf{v}_1 = (-1, 0, 2)^T$ and $\mathbf{v}_2 = (1, 0, 4)^T$. The vertices have 1D texture coordinates $s_1 = 0.2$ and $s_2 = 0.5$. Assume that the edge is viewed from the origin with the image plane at $z = 1$. For the point $\mathbf{p} = (0, 0, 1)^T$ on the image plane compute the texture coordinate s.

d What is meant by aliasing in the context of texture mapping and how can this problem be avoid?

The four parts carry, respectively, 40%, 20%, 20%, and 20% of the marks.

4 Shading and Ray tracing

The Phong illumination model is defined by the following equation:

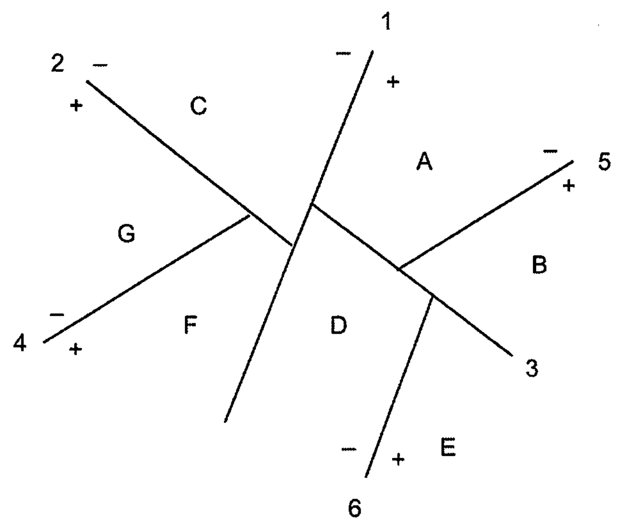
$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)\frac{\Phi_s}{4\pi d^2}$$

a Draw a sketch that explains the geometric significance of the vectors $\mathbf{n}, \mathbf{l}, \mathbf{v}$ and $\mathbf{r}$ appearing in the Phong illumination model as defined by the equation above.

b Explain the Phong illumination model as defined by the equation above in terms of ambient, diffuse and specular illumination.

c Explain how the Phong illumination model defined in the equation above can be extended to compute the illumination in a recursive ray-tracing algorithm taking into account shadows, reflections and transparency.

d Binary Space Partition (BSP) trees are used in ray tracing to speed up the computations of intersections of objects with the ray. Build a BSP tree for the scene shown below. Represent the '+' side of each cutting plane as the right child and the '−' side as the left child.



e Describe how one can model soft shadows using ray tracing.

The five parts carry equal marks.