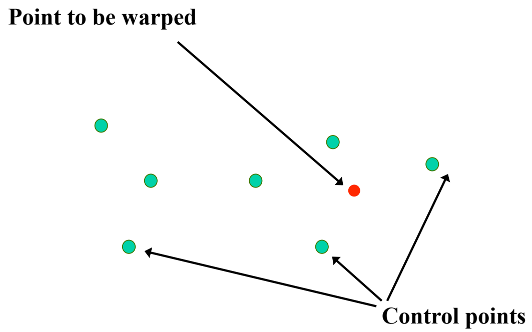
Interactive Computer Graphics

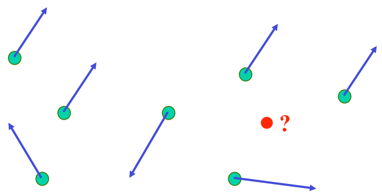
- Lecture 16: Warping and Morphing (cont.)

Non-rigid transformation



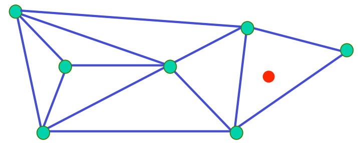
Non-rigid transformation

- For each control point we have a displacement vector
- How do we interpolate the displacement at a pixel?



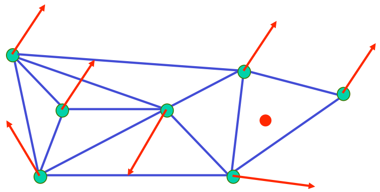
Non-rigid transformation: Piecewise affine

- Partition the convex hull of the control points into a set of triangles



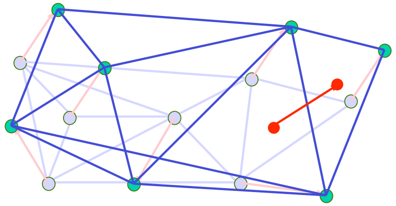
Non-rigid transformation: Piecewise affine

- Partition the convex hull of the control points into a set of triangles



Non-rigid transformation: Piecewise affine

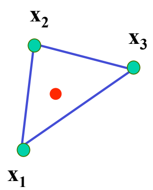
- Partition the convex hull of the control points into a set of triangles



Non-rigid transformation: Piecewise affine

- Find triangle which contains point **p** and express in terms of the vertices of the triangle:

$$\mathbf{p} = \mathbf{x}_1 + \alpha(\mathbf{x}_2 - \mathbf{x}_1) + \beta(\mathbf{x}_3 - \mathbf{x}_1)$$



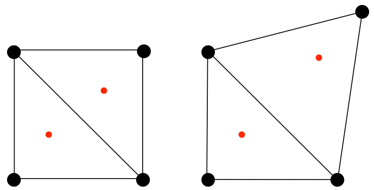
Non-rigid transformation: Piecewise affine

- Or $\mathbf{p} = \gamma\mathbf{x}_1 + \alpha\mathbf{x}_2 + \beta\mathbf{x}_3$ with $\gamma = 1 - (\alpha + \beta)$

- Under the affine transformation this point simply maps to

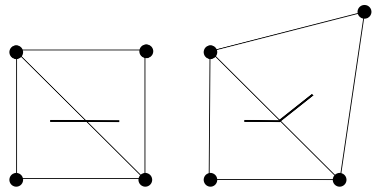
$$\mathbf{p}' = \gamma\mathbf{x}_1' + \alpha\mathbf{x}_2' + \beta\mathbf{x}_3'$$

Non-rigid transformation: Piecewise affine



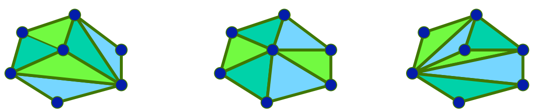
Non-rigid transformation: Piecewise affine

- Problem: Produces continuous deformations, but the deformation may not be smooth. Straight lines can be kinked across boundaries between triangles



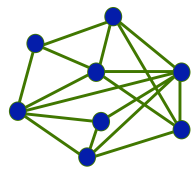
Triangulations

- A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.
- There are an exponential number of triangulations of a point set.



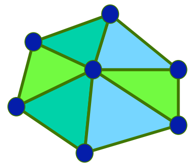
An O(n^3)* Triangulation Algorithm*

- Repeat until impossible:
  - Select two sites.
  - If the edge connecting them does not intersect previous edges, keep it.

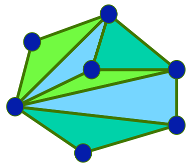


"Quality" Triangulations

- Let $\alpha(T) = (\alpha_1, \alpha_2, .., \alpha_{3t})$ be the vector of angles in the triangulation T in increasing order.
- A triangulation T_1 will be "better" than T_2 if $\alpha(T_1) > \alpha(T_2)$ lexicographically.
- The Delaunay triangulation is the "best"
  - Maximizes smallest angles

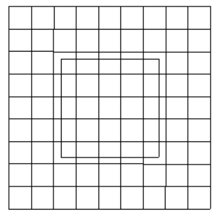


good

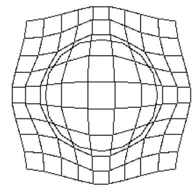


bad

Representing deformations

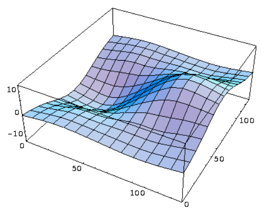


Before deformation

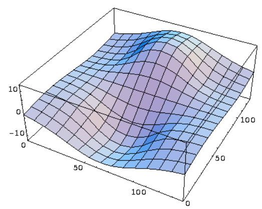


After deformation

Representing deformations



Displacement in the horizontal direction



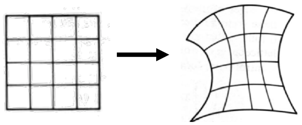
Displacement in the vertical direction

B-splines

- Free-Form Deformation (FFD) are a common technique in Computer Graphics for modelling 3D deformable objects
- FFDs are defined by a mesh of control points with uniform spacing
- FFDs deform an underlying object by manipulating a mesh of control points
  - control point can be displaced from their original location
  - control points provide a parameterization of the transformation

Free Form Deformation (FFD)

Deform space by deforming a lattice around an object



The deformation is defined by moving the control points

Imagine it as if the object were encased in rubber

Free Form Deformation (FFD)

The lattice defines a B-Spline volume

$$\mathbf{T}(u,v,w) = \sum_{ijk} \mathbf{p}_{ijk} B(u)B(v)B(w)$$

Compute lattice coordinates

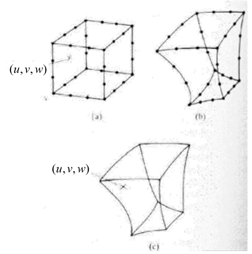
(u,v,w)

Alter the control points

$\mathbf{p}_{ijk}$

Compute the deformed points

$\mathbf{T}(u,v,w)$



FFDs using linear B-splines

- FFDs based on linear B-splines can be expressed as a 2D (3D) tensor product of linear 1D B-splines:

$$\mathbf{u}(x,y) = \sum_{l=0}^{1}\sum_{m=0}^{1} B_l(u)B_m(v)\phi_{i+l,j+m}$$

where

$$i = \left\lfloor \frac{x}{\delta_x} \right\rfloor,\ j = \left\lfloor \frac{y}{\delta_y} \right\rfloor,\ u = \frac{x}{\delta_x} - \left\lfloor \frac{x}{\delta_x} \right\rfloor,\ v = \frac{y}{\delta_y} - \left\lfloor \frac{y}{\delta_y} \right\rfloor$$

and B_i corresponds to the B-spline basis functions

$$B_0(s) = 1 - s$$

$$B_1(s) = s$$

FFDs using cubic B-splines

- FFDs based on cubic B-splines can be expressed as a 2D (3D) tensor product of cubic 1D B-splines:

$$\mathbf{u}(x,y) = \sum_{l=0}^{3}\sum_{m=0}^{3} B_l(u)B_m(v)\phi_{i+l,j+m}$$

where

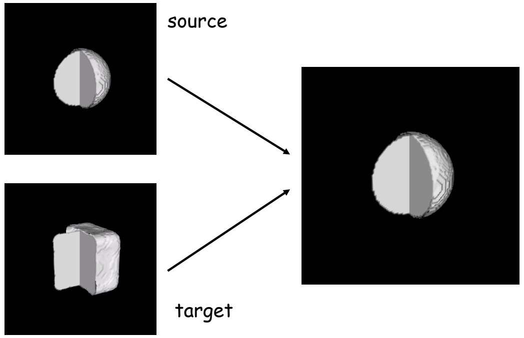
$$i = \left\lfloor \frac{x}{\delta_x} \right\rfloor - 1,\ j = \left\lfloor \frac{y}{\delta_y} \right\rfloor - 1,\ u = \frac{x}{\delta_x} - \left\lfloor \frac{x}{\delta_x} \right\rfloor,\ v = \frac{y}{\delta_y} - \left\lfloor \frac{y}{\delta_y} \right\rfloor$$

and B_i corresponds to the B-spline basis functions

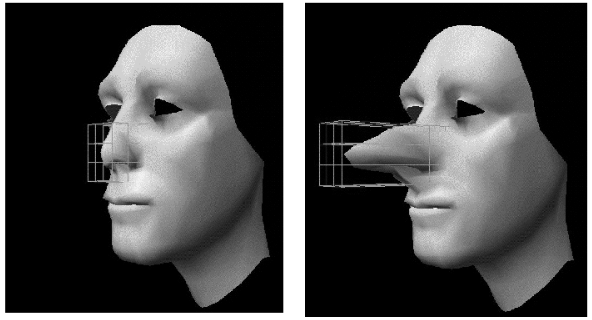
$$B_0(s) = (1-s)^3/6 \qquad B_2(s) = (-3s^3 + 3s^2 + 3s + 1)/6$$

$$B_1(s) = (3s^3 - 6s^2 + 4)/6 \qquad B_3(s) = s^3/6$$

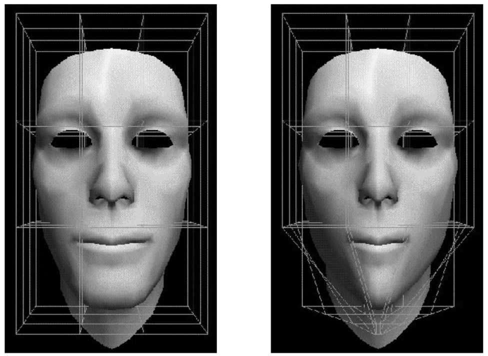
FFDs in 3D



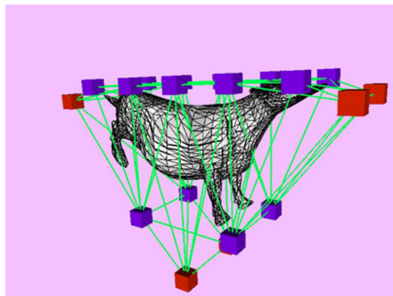
FFD Example



FFD Example

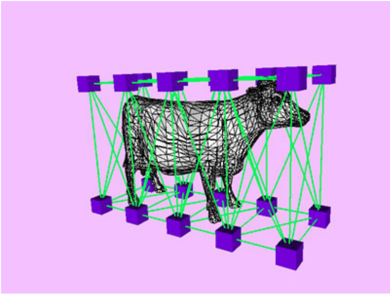


FFD: Examples



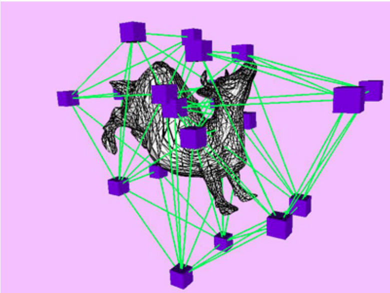
From "Fast Volume-Preserving Free Form Deformation Using Multi-Level Optimization" appeared in ACM Solid Modelling '99

FFD: Examples



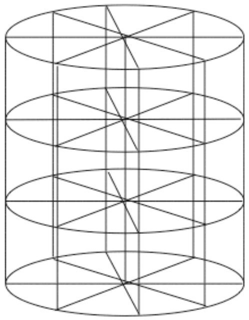
From "Fast Volume-Preserving Free Form Deformation Using Multi-Level Optimization" appeared in ACM Solid Modelling '99

FFD: Examples

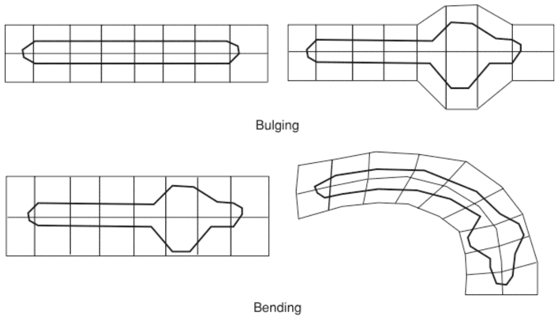


From "Fast Volume-Preserving Free Form Deformation Using Multi-Level Optimization" appeared in ACM Solid Modelling '99

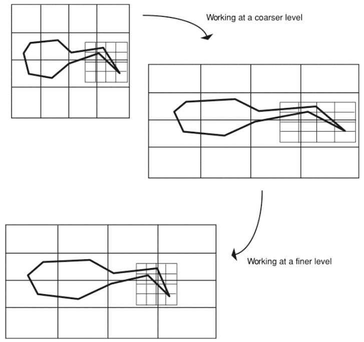
FFDs: alternate grid organizations



FFDs: Bulging & Bending



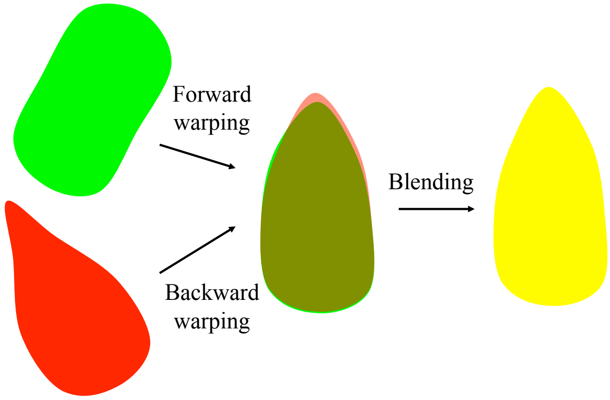
FFDs:hierarchical



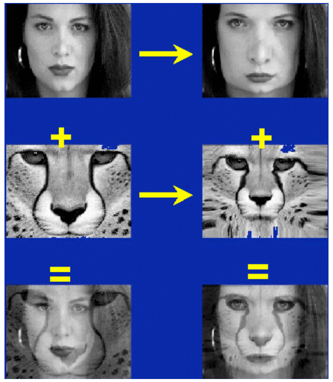
FFDs

- Used for warping:
  - Lee et al. (1997)
- Advantages:
  - Control points have local influence since the basis function has finite support
  - Fast
    - linear (in 3D: 2 x 2 x 2 = 8 operations per warp)
    - cubic (in 3D: 4 x 4 x 4 = 64 operations per warp)
- Disadvantages:
  - Control points must have uniform spatial distribution

Morphing = (warping)^2 + blending



Morphing = (warping)^2 + blending



Morphing

```
GenerateAnimation(Image_0, Image_1)
begin
  foreach intermediate frame time t do
        Warp_0 = WarpImage(Image_0, t)
        Warp_1 = WarpImage(Image_1, t)
        foreach pixel p in FinalImage do
                Result(p) = (1-t)Warp_0 + tWarp_1
        end
  end
end
```

Image Combination

- Determines how to combine attributes associated with geometrical primitives. Attributes may include
  - color
  - texture coordinates
  - normals
- Blending
  - cross-dissolve
  - adaptive cross-dissolve
  - alpha-channel blending
  - z-buffer blending

Image Combination: Cross-dissolve

- Blending with cross-dissolve:

$$I = (1-t)\cdot I_A + t\cdot I_B$$

  - intensities
  - RGB space
  - HSV space
  - texture space

Image Combination: Cross-dissolve

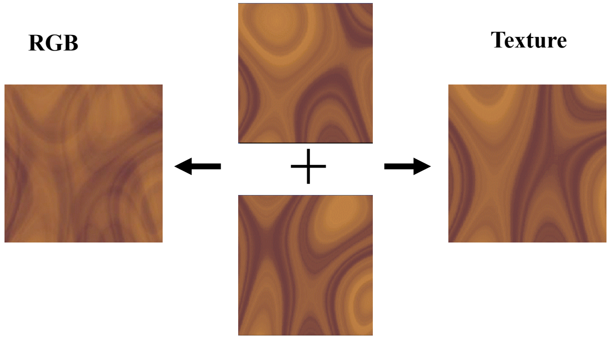


Image Combination: Adaptive cross-dissolve

- Adaptive cross-dissolve

$$I = (1 - w(\mathbf{p},\lambda)) \cdot I_A(\mathbf{p}) + w(\mathbf{p},\lambda) \cdot I_B(\mathbf{p})$$

  – similar to cross-dissolve but blending function depends on position in image

Image Combination: Alpha channel blending

- Blending using RGBA images

$$I = \alpha_a \cdot I_A + \alpha_b \cdot I_B$$

- Images are represented by quadruples:
  – R, G, B indicating color
  – Alpha channel encodes pixel coverage information
    – $\alpha = 0$ transparent
    – $0 < \alpha < 1$ semi-transparent
    – $\alpha = 1$ opaque

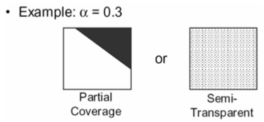
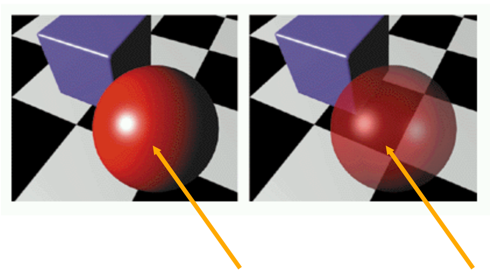


Image Combination: Alpha channel blending



$\alpha = 1$ $\alpha = 0.5$

Image Combination: Alpha channel blending

- Convention:
  – RGBA represents a pixel with color $C = (R,G,B)$ as

$$C = (\alpha r, \alpha g, \alpha b, \alpha)$$

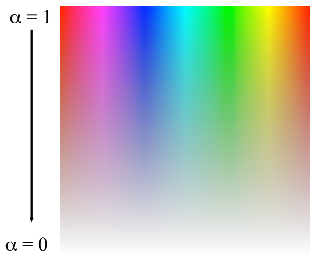
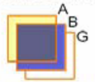


Image Combination: Alpha channel blending

- Suppose we put A over B over background G



  – How much of B is blocked by A?

    α_A

  – How much of B shows through A?

    $(1 - \alpha_A)$

  – How much of G shows through both A and B?

    $(1 - \alpha_A)(1 - \alpha_B)$

Image Combination: Alpha channel blending

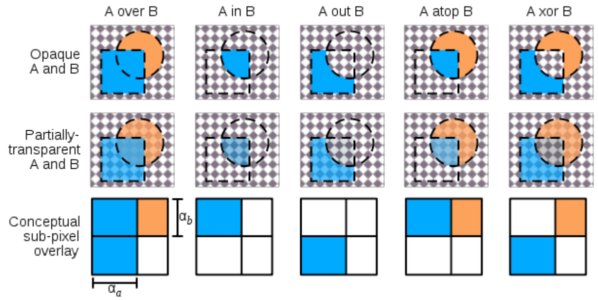


Image Combination: Alpha channel blending

- Example: C = A over B

  - For colors that are <u>not</u> premultiplied:
    - $C = \alpha_A A + (1-\alpha_A)\, \alpha_B B$
    - $\alpha = \alpha_A + (1-\alpha_A)\, \alpha_B$
  - For colors that <u>are</u> premultiplied:
    - $C' = A' + (1-\alpha_A) B'$
    - $\alpha = \alpha_A + (1-\alpha_A)\, \alpha_B$

A over B

Assumption: coverages of A and B are uncorrelated for each pixel

Image Combination: Z-buffer blending

- Blending using Z-buffer values:

$$I = \begin{cases} I_a & \text{if } z_a < z_b \\ I_b & \text{else} \end{cases}$$

  – defines an ordering

  – can be used for layering