Revision Tutorial Solution:

Animations, Transformations, Projections and Normalisation

1. *Animating Objects*

The transformation is in three parts:

Translate the coordinates so that the centre of the cube is at the origin
Scale the coordinates by 99/100
Translate the cube back to its original position

In matrices this becomes

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & -5 & -10 & 1 \end{pmatrix}\begin{pmatrix} 0.99 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 5 & 5 & 10 & 1 \end{pmatrix}$$

which multiplies out to

$$\begin{pmatrix} 0.99 & 0 & 0 & 0 \\ 0 & 0.99 & 0 & 0 \\ 0 & 0 & 0.99 & 0 \\ 0.05 & 0.05 & 0.1 & 1 \end{pmatrix}$$

2. *Viewing transformations*

The third vertical direction is $\boldsymbol{v} = \boldsymbol{w} \times \boldsymbol{u}$.

This can be evaluated using the cross product determinant which gives

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0.6 & -0.2 & 0.77 \\ 0.79 & 0 & 0.61 \end{vmatrix} = 0.122\,\hat{\mathbf{i}} + 0.9743\,\hat{\mathbf{j}} + 0.158\,\hat{\mathbf{k}} = \begin{pmatrix} 0.122 \\ 0.9743 \\ 0.158 \end{pmatrix}$$

and is pretty much upwards.

The transformation is:

$$\begin{pmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ -\mathbf{C.u} & -\mathbf{C.v} & -\mathbf{C.w} & 1 \end{pmatrix} = \begin{pmatrix} 0.79 & 0.122 & 0.6 & 0 \\ 0 & 0.974 & -0.2 & 0 \\ -0.61 & 0.158 & 0.77 & 0 \\ -1.8 & \text{-12.54} & -11.7 & 1 \end{pmatrix}$$

3. *Projection*

The combined transform (with the projection matrix second) is:

$$\begin{pmatrix} 0.79 & 0.122 & 0.6 & 0 \\ 0 & 0.974 & -0.2 & 0 \\ -0.61 & 0.158 & 0.77 & 0 \\ -1.8 & -12.54 & -11.7 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0.79 & 0.122 & 0.6 & 0.3 \\ 0 & 0.974 & -0.2 & -0.1 \\ -0.61 & 0.158 & 0.77 & 0.385 \\ -1.8 & -12.54 & -11.7 & -5.85 \end{pmatrix}$$

The coordinate (10, 10, 20, 1) projects to (-6.1, 1.58, 7.7, 3.85).

To convert this to a Cartesian coordinate we divide by the last ordinate to get (-1.58, 0.41, 2, 1). The value of $z = 2$ confirms that the point has projected into the viewing plane.

4. *Normalisation*

This can be done with ratios. In the diagram below, let **P** represent the projected point, i.e. (-1.58, 0.41) in world coordinates, and let **O** represent the origin.

In the world coordinate system the ratio of the distance from x to the right hand side to the total window width is

$$\frac{5-(-1.58)}{10} = 0.658$$

The same ratio must be preserved if we measure in pixels so $x_{pix} = 66$.

Similarly the y ratio is 0.459 so $y_{pix} = 46$.